Lecture 3
Efficiency wages

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Lecture outline:

Basic model of efficiency wages
   Possible reasons for efficiency wages
   Simple model
   Model interpretation
   Model implications

More general version of the efficiency-wages model
   Model set-up
   Example
   Model implications

Simple model of shirking

Gift exchange – fair wage hypothesis

Efficiency wages as a turnover reduction tool
   Retainment
Basic model of efficiency wages
Why may firms choose to pay higher than market-clearing wages?

- Nutrition – workers with higher wages may consume more food thus their health conditions improve and they are more productive (developing economies)
- Inability to monitor workers effort – high wage makes losing a job a real punishment for low effort (Shapiro-Stiglitz)
- Higher wages attract high skilled workers (with possibly higher reservation wages) and this increases average skill level of the workforce hired by the firm
- High wages build loyalty among workers (gift exchange) reducing turnover – high turnover may be costly due to substantial costs of hiring and firing. Higher wages my lower likelihood that workers will unionize (insiders-outsiders)
A historical example

- Raff and Summers (1987) conduct a case study on Henry Ford’s introduction of the five dollar day in 1914.
- Ford experience supports efficiency wage interpretations. Ford’s decision to increase wages so dramatically (doubling for most workers) is most plausibly portrayed as the consequence of efficiency wage considerations, with the structure being consistent, evidence of substantial queues for Ford jobs, and significant increases in productivity and profits at Ford.
- High turnover and poor worker morale appear to have played a significant role in the five-dollar decision.
- Ford’s new wage put him in the position of rationing jobs, and increased wages did yield productivity benefits and profits.
- There is also evidence that other firms emulated Ford’s policy to some extent, with wages in the automobile industry 40% higher than in the rest of manufacturing (Rae 1965, quoted in Raff and Summers).
Basic model of efficiency wages

Assume there is a large number, \( N \), of competitive firms. The representative firm aims at profit maximization:

\[
\pi = Y - wL, \tag{1}
\]

where \( Y \) is the firm’s output, \( w \) – the wage it pays and \( L \) is the amount of labour it hires. For simplicity, we neglect prices and express wages and output in real terms.

Firm’s output is not only a function of the number of workers, but also of their effort. Assume there are no other inputs:

\[
Y = F(eL), \quad F'(\cdot) > 0 \quad F''(\cdot) < 0. \tag{2}
\]
Crucial assumption of efficiency wages model is that the effort depends positively on wage. Consider first simple case (Solow, 1979) where the wage is the only determinant of the effort:

\[ e = e(w), \quad e'(\cdot) > 0. \] (3)

Finally, we assume that there are \( \bar{L} \) identical workers. Every worker supplies 1 unit of labour inelastically.

The problem of the firm may be rewritten as:

\[ \max_{L,w} \quad F(e(w)L) - wL. \] (4)
Unconstrained firm, chooses $L$ and $w$, according to the F.O.C.:

\begin{align*}
F'(e(w)L)e(w) - w &= 0, \quad (5) \\
F'(e(w)L)Le'(w) - L &= 0. \quad (6)
\end{align*}

We can rewrite (5) as:

\[ F'(e(w)L) = \frac{w}{e(w)}. \quad (7) \]

Substituting (7) into (6), and dividing by $L$ yields:

\[ \frac{we'(w)}{e(w)} = 1. \quad (8) \]
Equation (8) states that at the optimum, the elasticity of effort with respect to wage is 1. Remember that firm’s output is a function of effective labour \((eL)\), and that’s why firm wants to hire effective labour as cheaply as possible. When a firm hires a worker it obtains \(e(w)\) units of effective labour at a cost of \(w\). The cost per unit of effective labour is then \(w/e(w)\). When the elasticity of \(e\) w.r.t. \(w\) is 1, a marginal change in \(w\) does not change this ratio, thus this is the first-order condition for the problem of choosing \(w\) to minimize the cost of effective labour. The wage that satisfies (8) is known as the **efficiency wage**.
Interpretation – single firm perspective

\[ F'(e(w)L) = \frac{w}{e(w)}. \]  \tag{7}

Equation (7) states that the firm hires workers until the marginal product of effective labour equals its cost. This is analogous to the condition in a standard labour demand problem \((VMP_L = w)\).
Interpretation – economy-wide perspective

Let $w^*$ and $L^*$ denote values of $w$ and $L$ that satisfy (7) and (8). Since all firms are identical, they choose the same values of $w$ and $L$. Total labour demand is therefore $NL^*$. If labour supply, $\bar{L}$, exceeds this amount, firms are unconstrained in their choice of $w$. In this case the wage is $w^*$, employment is $NL^*$ and unemployment equals $\bar{L} - NL^*$. If $NL^* > \bar{L}$ then firms are constrained, wages are bid up to the point where demand and supply are in balance and there is no unemployment.
Figure 1. The determination of the efficiency wage
Implications of the basic efficiency-wages model

- Efficiency wages may give rise to unemployment
- Real wage does not respond to demand shifts, which influence employment only
- Costs of a unit of effective labour are constant, which implies price rigidity in price-setting firms
- In the long run, economic growth leading to an outward shift of labour demand will eventually lead to a decrease in unemployment to zero (because of real wage rigidity). Further increase in labour demand will push real wage up.
- In practice, we do not observe a downward trend of unemployment in the long run!
More general version of the efficiency-wages model
More general version of the efficiency-wages model

The wage may not be the only determinant of effort. If the firm chooses to pay higher wages to encourage workers to work harder but cannot monitor their effort perfectly, losing a job is the only punishment, when workers were caught shirking. The cost for a worker of being fired depends not only on the lost wage, but also on how easy is to find a new job, and what is the alternative wage. Therefore workers (not perfectly observable) effort should be higher, when the unemployment is high, and when the best possible alternative wage is low.

\[ e = e(w, w_a, u), \quad \frac{\partial e}{\partial w} > 0, \quad \frac{\partial e}{\partial w_a} < 0, \quad \frac{\partial e}{\partial u} > 0. \tag{9} \]
Empirical example


The question:
„How do you think the work effort of your employees would be affected if local unemployment were to rise?”

Increase: 86%.
Each firm is small relative to the economy and takes $w_a$ and $u$ as given. The representative firm’s optimization problem is the same as before, except that the effort is now determined also by $w_a$ and $u$.

The F.O.C. can be rearranged as:

$$F'(e(w, w_a, u)L) = \frac{w}{e(w, w_a, u)},$$  \hspace{1cm} (10)

$$\frac{w e'(w, w_a, u)}{e(w, w_a, u)} = 1.$$  \hspace{1cm} (11)
Interpretation

Assume the function $e(\cdot)$ is sufficiently well behaved that there is a unique optimal $w$ for given $w_a$ and $u$. Given this assumption, we have $w = w_a$ in equilibrium. This extended version shows that unemployment in equilibrium exhibits no trend in the long run, but also that shifts in labour demand appear to have large effects on unemployment in the short run.
Example (Summers, 1988)

Suppose the effort function is given by:

\[ e = \begin{cases} 
(\frac{w-x}{x})^\beta & \text{if } w > x \\
0 & \text{otherwise,} 
\end{cases} \quad (12) \]

\[ x = (1 - bu)w_a, \quad (13) \]

where \( 0 < \beta < 1 \) and \( b > 0 \), \( x \) is a measure of labour market conditions. If \( b = 1 \), \( x \) is the alternative wage multiplied by the fraction of workers being employed. If \( b < 1 \) then workers put less weight on unemployment – due to higher value of leisure or unemployment benefits. If \( b > 1 \) – workers put more weight to unemployment (long duration, small chances of finding new job, risk aversion). Equation (12) shows that the effort increases less than proportionately with \( w - x \) for \( w > x \).
Differentiating (12) leads to the condition that elasticity of effort w.r.t. wage is equal 1:

\[ \beta \frac{w}{[(w - x)/x]^{\beta}} \left( \frac{w - x}{x} \right)^{\beta-1} \frac{1}{x} = 1. \quad (14) \]

We can simplify this condition to get:

\[ w = \frac{x}{1 - \beta} = \frac{1 - bu}{1 - \beta} w_a \quad (15) \]

For small values of \( \beta \), \( 1/(1 - \beta) \approx 1 + \beta \). Therefore (15) implies that when \( \beta \) is small, firms offer a premium equal approximately the fraction \( \beta \) of the labour market conditions index, \( x \).
In equilibrium all firms pay the same wage, so $w = w_a$. Using this condition and (15) we get:

$$(1 - \beta)w_a = (1 - bu)w_a. \quad (16)$$

From this we can see the equilibrium unemployment rate as:

$$u^* = \frac{\beta}{b}, \quad (17)$$

which is obviously positive.
As equation (15) shows, each firm wants to pay more than $w_a$ if $u < u^*$, and less if $u > u^*$. In equilibrium all firms pay the same wages, so $u^*$ is equilibrium unemployment rate. Using this in the effort function (12) we can express the equilibrium effort by:

$$e^* = \left[ \frac{w_a - (1 - bu^*)w_a}{(1 - bu^*)w_a} \right]^\beta = \left( \frac{\beta}{1 - \beta} \right)^\beta$$

(18)
Finally, the equilibrium wage satisfies the condition that $F'(eL) = w/e$ equivalent to $w = eF'(eL)$. Total employment in equilibrium is $(1 - u^*)\bar{L}$, therefore each firm hires $(1 - u^*)\bar{L}/N$ workers. Equilibrium wage is therefore:

$$w^* = e^* F' \left( \frac{e^*(1 - u^*)\bar{L}}{N} \right).$$  \hspace{1cm} (19)
Model implications

- Equilibrium unemployment depends only on parameters of the effort function (see eq. 17), not the production function. An upward trend of production is not followed by a trend in unemployment.

- Relatively small values of $\beta$ – the elasticity of effort with respect to the premium that firms pay over the labour market conditions index, $x$ – may lead to nonnegligible unemployment. For $\beta = 0.06$ and $b = 1$ or $\beta = 0.03$ and $b = 0.5$, the unemployment rate is 6%.

- With $\beta = 0.03$ and $b = 0.5$, we get $u = 6\%$, but we can also see from (13) that the workers exert no effort at all, unless the firm pays at least 97% of the alternative wage. So efficiency-wage forces are very strong for these parameter values.
Model implications

- Using this outcome we can explain why younger people might have on average higher unemployment rates. They probably have higher value of leisure, which means a lower $b$. On the other hand they might also have higher elasticity of effort w.r.t. the premium that that firm pays over the labour market conditions index $x$. All this together according to (17) leads to higher unemployment rates.
Simple model of shirking
Simple model of shirking

Assumptions:

Providing effort $e$ causes for a worker a cost $\phi(e)$. This cost can be either real expenditure or loss of welfare (disutility – who likes to work hard???) that must be compensated by higher consumption, if the level of welfare is kept at the same level.

When a firm hires a worker, there is an agreement on the (minimum) efficiency $\hat{e}$ and the wage $\hat{w}$ for the worker. The worker is said to shirk, if (s)he works less efficiently than what was agreed on, $e < \hat{e}$. 
Simple model of shirking

In case of shirking, the worker is either caught and fired, or not caught and kept in the workplace despite of the too low productivity. To simplify the analysis, let’s take two assumptions as follows: the probability that one is caught for shirking in the firm is a constant $\theta \in (0, 1)$, and the elasticity of the workers efficiency cost is a constant $1/\varepsilon$:

$$
\phi(e) = \mu e^{1/\varepsilon}, \quad \frac{d\phi}{de} e = \frac{1}{\varepsilon}, \quad \mu > 0, \quad \varepsilon \in (0, 1). \quad (20)
$$

Because $\phi'' > 0$, the workers effort to improve its efficiency is increasingly difficult.
Simple model of shirking

Now we see the following:

- The workers income without shirking is equal to the wage in the firm, \( w \), minus efficiency cost \( \phi(e) \): \( w - \phi(e) \)

- The workers income with shirking is equal to the wage in the firm, \( w \), times the probability of not being caught for shirking, \( (1 - \theta) \), plus the expected wage outside the firm, \( v \), times the probability of being caught for shirking, \( \theta \): \( (1 - \theta)w + \theta v \)
Simple model of shirking

The worker shirks definitely, if income for that is greater than for nonshirking, \((1 - \theta)w + \theta v > w - \phi(e)\). Hence, if the employer wants to ensure that the worker does not shirk, there must be incentive comparability constraint imposed:

\[(1 - \theta)w + \theta v \leq w - \phi(e),\]

which is equivalent to:

\[w \geq v + \frac{\phi(e)}{\theta}.\]  

(21)

The firm will choose the lowest possible wage that satisfies this condition, so \(w = v + \phi(e)/\theta\).
Simple model of shirking

We can observe some interesting facts. The wage providing appropriate incentive not to shirk is higher if:

- wages paid outside the firm grow ($v \uparrow$)
- cost of effort is increasing ($\phi \uparrow$)
- the probability of being caught is low ($\theta \downarrow$)
Simple model of shirking

From the condition (21) we can state that: \( \phi(e) = \theta(w - v) \). If we use the definition of the cost function from (20) we get:

\[
e = \left( \frac{\theta}{\mu} \right)^\varepsilon (w - v)^\varepsilon
\]

Now if we take the efficiency wages condition that the elasticity of effort with respect to wages is equal 1, we get:

\[
\varepsilon \left( \frac{\theta}{\mu} \right)^\varepsilon (w - v)^{\varepsilon-1} w = 1
\]
Simple model of shirking

After some simplifications we get finally:

$$w = \frac{v}{1 - \varepsilon}.$$

If we adopt that $v = (1 - u)w + ub$, and assume that benefits are paid as a constant proportion of average wage, so that $b = \beta w$, we can calculate unemployment:

$$u^* = \frac{\varepsilon}{1 - \beta}$$

with the same conclusions as before.
Gift exchange – fair wage hypothesis
Gift exchange – fair wage hypothesis

Workers have some idea of a 'fair wage'. The fundamental concept here is the reference group theory. It says that our morale and willingness to perform depends importantly on what we get relative to what we see our likes getting. (Akerlof 1982, Akerlof, Yellen 1990).

The employer chooses $w$ to motivate workers to exert effort $e$, which will be chosen to maximize workers utility.

The workers effort depends on the relation between the fair wage ($\hat{w}$) and the actual wage ($w$) paid by the firm. The fair wage is higher the higher are expected wages paid by other firms ($w_a$), but it is lower if unemployment is high.
Empirical examples


This study reports evidence from a field experiment that was conducted to investigate the relevance of gift-exchange in a natural setting. In collaboration with a charitable organization we sent roughly 10,000 solicitation letters to potential donors. One third of the letters contained no gift, one third contained a small gift and one third contained a large gift. The treatment assignment was random. The results confirm the economic importance of gift-exchange. Compared to the no gift condition, the relative frequency of donations increased by 17 percent if a small gift was included and by 75 percent for a large gift.
Empirical examples


Reciprocity as a contract enforcement device. The requirement of a generally cooperative job attitude renders reciprocal motivations potentially very important in the labor process. If a substantial fraction of the work force is motivated by reciprocity considerations, employers can affect the degree of "cooperativeness" of workers by varying the generosity of the compensation package – even without offering explicit performance incentives.
Empirical examples


A field experiment testing the gift-exchange hypothesis inside a treeplanting firm paying its workforce incentive contracts. Firm managers told a crew of tree planters they would receive a pay raise for one day as a result of a surplus not attributable to past planting productivity. They compare planter productivity – the number of trees planted per day on the day the gift was handed out with productivity on previous and subsequent days of planting on the same block, and thus under similar planting conditions. We find direct evidence that the gift had a significant and positive effect on daily planter productivity, controlling for planter-fixed effects, weather conditions and other random daily shocks. Moreover, reciprocity is the strongest when the relationship between planters and the firm is longterm.
Therefore we can adopt the same notation for workers effort as before:

\[ e = e(w, \hat{w}) \quad \text{where} \quad \hat{w} = h(w_a, u), \]

or we can simply write:

\[ e = e\left(\frac{w}{w_a}, u\right). \tag{22} \]

This leads to the same conclusion, that in the optimum the wage will be chosen in such a way, to make the elasticity of workers effort w.r.t. wage equal 1.
Efficiency wages as a turnover reduction tool
So far we have studied motivating effects of higher wages. Now let’s turn to the problems concerning labour turnover. Firms may wish to retain its workers, or at least minimize turnover, because training and hiring new workers induces real costs for the firm. Paying higher wages is a tool to achieve this. Assume, that the workers quit rate is given by:

\[ q = q(w, w_a, u) \quad (q_w < 0, q_{w_a} > 0, q_u < 0). \]  

(23)

Whenever a quitter is replaced, the firm must incur training cost \( \theta \). The firms steady-state profits are:

\[ \pi = F(L) - [w + \theta q(w, w_a, u)] L, \]

(24)
Clearly, this expression is similar to the one used earlier (see eq. 4). It can be solved by first minimizing the cost per worker (the term in brackets) and then by choosing employment. The equilibrium unemployment is determined by setting $w = w_a$ and using quit rate function. It turns out that the equilibrium unemployment increases with relative training costs ($\theta/w$) or when exogenous turnover propensities rise.
Henry Ford example

Table 1. Annual Turnover and Layoff Rates (%) at Ford, 1913–1915

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<th>1913</th>
<th>1914</th>
<th>1915</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average employment</td>
<td>13 632</td>
<td>12 115</td>
<td>18 028</td>
</tr>
<tr>
<td>Turnover rate</td>
<td>370</td>
<td>54</td>
<td>16</td>
</tr>
<tr>
<td>Lay-off rate</td>
<td>62</td>
<td>7</td>
<td>0.1</td>
</tr>
</tbody>
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The annual turnover rate (ratio of separations to employment) equaled 370% in 1913. It means that on average 31% of the workers left each month. In other words, Ford had to hire 50 448 men during the year, in order to have average employment at the level of 13 623. Introduction of the five-dollar-day (the average wage at that time was $ 2.30 for 9 hours) in 1914 reduced turnover considerably.