Lecture 1
Macroeconomics of the Labour Market

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Lecture outline:

Introduction

Basic facts about the labour market performance

Basic model of the neoclassical theory of labour supply
  Choice between consumption and leisure
  The properties of labour supply
  Aggregate labour supply and Labour Force participation rate

Labour demand in the neoclassical model

Unemployment in neoclassical model
  Possible explanations for unemployment
  Difference in unemployment of skill groups in a neoclassical model
What is it going to be about...

Labour market macroeconomics:

1. Introduction and basic facts on labour market performance and basic model of neoclassical labour supply
2. The NAIRU model
3. Basic model of efficiency wages
4. Shapiro and Stiglitz model of efficiency wages
5. Wage bargaining and trade unions
6. Search and matching theory
What is it going to be about…

- Read suggested literature (see the syllabus) + make notes

- Problem set — sample problems

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Basic facts about the labour market performance
Basic facts about the labour market performance

Definition: unemployment is an economic condition marked by the fact that individuals actively seeking for jobs remain unhired.

Unemployment is a fact – much of the labour market theory is devoted to explain the existence and various aspects of unemployment (its reasons, duration, persistence, etc.)
Basic measures of the labour market performance

Each individual member of the working age population might be potentially in one three states:

- Not economically active (out of the labour market) – $N$
- Economically active, and:
  - Unemployed – $U$, or
  - Employed – $E$ (either employee or employer)

Three basic measures:

- The participation rate: $a = \frac{E+U}{N+E+U}$
- The employment rate: $e = \frac{E}{N+E+U}$
- The unemployment rate: $u = \frac{U}{E+U}$
Unemployment fluctuates over time but in the very long run seems to have no trend

Figure 1. Unemployment rate, USA 1960-2006.
Unemployment exhibits a lot of persistence

If we ran simple regression of the unemployment rate on its past value for a very long period of time, we would get the following result:

\[
U_t = 0.7139 + 0.8561U_{t-1}, \quad R^2 = 0.733 \quad (UK, 1856-2005)
\]

\[
U_t = 0.9919 + 0.8567U_{t-1}, \quad R^2 = 0.735 \quad (US, 1891-2005)
\]
Unemployment exhibits a lot of persistence

Take the time series representation of the unemployment rate:

\[ U_t = \alpha_0 + \alpha_1 U_{t-1} \]

then the long run equilibrium value \( \bar{U} = \frac{\alpha_0}{1 - \alpha_1} \) equals 4.96% for UK and 6.92% for US. Let’s find the convergence speed to the long run equilibrium.

\[
\begin{align*}
U_1 &= \alpha_0 + \alpha_1 U_0 \\
U_2 &= \alpha_0 + \alpha_1 \bar{U} = \alpha_0 + \alpha_1 (\alpha_0 + \alpha_1 U_0) \\
\vdots \\
U_t &= \alpha_0 [1 + \alpha_1 + \alpha_1^2 + \ldots + \alpha_1^{t-1}] + \alpha_1^t U_0
\end{align*}
\]
Unemployment exhibits a lot of persistence

Then if we calculate the sum of geometric series (convergence is guaranteed by $|\alpha_1| < 1$) and use the definition of $\bar{U}$, we can write:

$$U_t = \alpha_0 \frac{1 - \alpha_1^t}{1 - \alpha_1} + \alpha_1^t U_0$$

$$U_t = \bar{U}(1 - \alpha_1^t) + \alpha_1^t U_0$$

$$U_t - \bar{U} = \alpha_1^t (U_0 - \bar{U})$$

Suppose $U_0$ is not equal $\bar{U}$. How many periods of time ($T$) will it take to eliminate half of the difference between $U_0$ and $\bar{U}$?
Unemployment exhibits a lot of persistence

We can treat this $T$ as a measure of convergence speed (half-life):

$$U_T - \bar{U} \equiv \alpha_1 T (U_0 - \bar{U}) = \frac{1}{2} (U_0 - \bar{U}) \Rightarrow \alpha_1^T = \frac{1}{2}$$

Then by taking logs for both sides we get:

$$T = -\frac{\ln 2}{\ln \alpha_1}$$

For both UK and US is takes about 4.5 years!
Unemployment is countercyclical

Figure 2. Change in unemployment rate and GDP growth, USA 1970-2008.
Unemployment is different among countries...

Figure 3. Unemployment rates across countries, 1980-2006.
... and age and sex groups

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Unemployment durations might be quite long

Figure 4. Unemployment duration (months) across countries, 1970-2007.
Unemployment durations might be quite long

*Figure 5.* Long term unemployment (over 12 months) across countries, 1970-2007.
Basic model of the neoclassical theory of labour supply
Choice between consumption and leisure

- It is assumed that consumer derives utility from two sources: the quantity of consumed goods (aggregated to $C$ with price normalized to unity) and the amount of leisure (time devoted to other activities than work, denoted by $L$) – $U(C, L)$
- Total time available – $T$ (number of hours per day, month, year, etc.)
- Labour supply: $h = T - L$
- Market wage – $w$
- Non-wage income – $R$
Choice between consumption and leisure

The properties of utility function:

\[ U_C > 0, \quad U_L > 0, \quad U_{CC} < 0, \quad U_{LL} < 0, \quad U_{CL} > 0. \]

Budget constraint:

\[ C \leq w(T - L) + R \quad \Leftrightarrow \quad C + wL \leq wT + R \quad (1) \]

Maximization problem:

\[ \begin{align*}
\max_{C,L} & \quad U(C, L) \\
\text{s.t.} & \quad C + wL \leq R_0
\end{align*} \]
Choice between consumption and leisure

Lagrangian:

\[ \mathcal{L}(C, L, \lambda) = U(C, L) + \lambda(R_0 - C - wL) \]

F.O.C.:

\[ \mathcal{L}_\lambda = 0 : \quad R_0 - C - wL = 0 \]
\[ \mathcal{L}_C = 0 : \quad U_C(C, L) - \lambda = 0 \]
\[ \mathcal{L}_L = 0 : \quad U_L(C, L) - \lambda w = 0 \]
Solution

If we denote optimal values by $C^*$, $L^*$ and $\lambda^*$, from F.O.C. we get:

$$\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w \quad \text{with} \quad C^* + wL^* = R_0$$  \hspace{1cm} (2)

Intuition: optimal choice lies on the budget constraint and the slope of the indifference curve equals (for interior solution) the slope of the budget constraint. $L^*$ is defined implicitly by (2), so:

$$L^* = \mu(w, R_0).$$  \hspace{1cm} (3)
Choice between consumption and leisure

Figure 6. Trade-off between consumption and leisure and optimal choice.
Reservation wage

How can we guarantee the interior solution? When will the individual be willing to supply any labour to the market? Clearly, when the market wage is high enough to provide appropriate incentives to give up some leisure. The opportunity cost of leisure increases with market wage. In terms of our graphical representation, we can infer the minimum wage, at which the individual will be indifferent between entering the labour market or staying outside. This is called the reservation wage.
Reservation wage

The point $E$ has to lie to the left of point $A$, otherwise $L = T$ and $h = 0$. With convexity of utility curves, the $MRS_{CL} = \frac{U_L}{U_C}$ is decreasing to the south-east along the indifference curve.

$$h > 0 \iff w_A \equiv \frac{U_L}{U_C} \bigg|_{L=T} < w$$

$$w_A = \frac{U_L(R, T)}{U_C(R, T)}$$

The reservation wage depends on the value of $R$ and the shape of utility function. It determines the labour market participation.
The properties of labour supply – substitution and income effects

We are now in position to analyse some interesting comparative statics results. Primarily, we are interested in the influence of a non-wage increase impact on the labour supply and the wage increase impact on the labour supply. In order to do it, we can totally differentiate the F.O.C.: 

\[
dR + dw\ T - dC - dL\ w - dw\ L = 0
\]
\[
dC\ U_{CC} + dL\ U_{CL} - d\lambda = 0
\]
\[
dC\ U_{CL} + dL\ U_{LL} - d\lambda\ w - dw\lambda = 0
\]
ComparativeStatics

We can rewrite it more conveniently as:

\[-dC - dL \, w = -(T - L) \, dw - dR\]
\[-d\lambda + dC \, U_{CC} + dL \, U_{CL} = 0\]
\[-d\lambda \, w + dC \, U_{CL} + dL \, U_{LL} = \lambda dw\]

If we are now interested in the effect of a rise in a non-wage income \( R \) on \( L \), we can simply take \( dw = 0 \), and divide all three equations by \( dR \). This yields the following:

\[
\begin{bmatrix}
0 & -1 & -w \\
-1 & U_{CC} & U_{CL} \\
-w & U_{CL} & U_{LL}
\end{bmatrix}
\begin{bmatrix}
\frac{d\lambda}{dR} \\
\frac{dC}{dR} \\
\frac{dL}{dR}
\end{bmatrix}
= \begin{bmatrix}
-1 \\
0 \\
0
\end{bmatrix}
\]
Comparative statics

Now we can use the Cramer's theorem to solve for $dL/dR$:

$$\frac{dL}{dR} = \frac{1}{|H|} \begin{vmatrix} 0 & -1 & -1 \\ -1 & U_{CC} & 0 \\ -w & U_{CL} & 0 \end{vmatrix} = \frac{1}{|H|} \begin{vmatrix} -1 & -1 & U_{CC} \\ -1 & -w & U_{CL} \end{vmatrix}$$

$$= (-1) \cdot \frac{-U_{CL} + wU_{CC}}{|H|}$$
Comparative statics

Let’s check the value of the determinant of the bordered Hessian $\overline{H}$:

$$\det\overline{H} = -w^2 U_{CC} + 2wU_{CL} - U_{LL} > 0.$$ 

This means that:

$$\frac{dL}{dR} > 0.$$ 

Of course this result rests on the assumption that $U_{CL} > 0$, meaning that $L$ is treated as a normal good. Our result suggests that an increase of non-wage income is a disincentive to work, since the individual will opt for more leisure.
Comparative statics: effect of a rise in non-wage income

Figure 7. Effect of a rise in non-wage income.
Comparative statics: effect of a wage increase

In order to grasp the effect of a wage increase on labour supply we need to repeat our procedure once again, and divide all three equations by $dw$. The system becomes:

$$
\begin{bmatrix}
0 & -1 & -w \\
-1 & U_{CC} & U_{CL} \\
-w & U_{CL} & U_{LL}
\end{bmatrix}
\begin{bmatrix}
d\lambda/dw \\
dC/dw \\
dL/dw
\end{bmatrix}
=
\begin{bmatrix}
-(T - L) \\
0 \\
\lambda
\end{bmatrix}
$$
Comparative statics: effect of a wage increase

\[
\frac{dL}{dw} = \frac{1}{|H|} \begin{vmatrix} 0 & -1 & -(T - L) \\ -1 & U_{CC} & 0 \\ -w & U_{CL} & \lambda \end{vmatrix} = \\
= \frac{1}{|H|} \begin{vmatrix} -1 & 0 \\ -w & \lambda \end{vmatrix} + \frac{-(T - L)}{|H|} \begin{vmatrix} -1 & U_{CC} \\ -w & U_{CL} \end{vmatrix} = \\
= -\lambda - (T - L)(-U_{CL} + wU_{CC}) \frac{1}{|H|} = \\
= -U_C - (T - L)(-U_{CL} + wU_{CC}) \frac{1}{|H|} \geq 0
\]
Comparative statics: effect of a wage increase

The influence of a wage increase on the labour supply can be ambiguous:

- higher wage will increase labour supply (decrease demand for leisure) if substitution effect is stronger than income effect: i.e. when \((T - L)\) is small or when \(UC\) is large – somebody worked relatively small or had relatively low wage;

- higher wage will decrease labour supply (increase demand for leisure) if income effect is stronger than substitution effect: i.e. when when \((T - L)\) is large or when \(UC\) is small – somebody worked already for long or had high wage.
Comparative statics: effect of a wage increase

Figure 8. Effect of a wage increase.
Individual labour supply

*Figure 9. Individual labour supply.*
Supplementary constraints

Our analysis of labour supply is limited in a number of ways:

► it divides time only into two activities: work and leisure. It is possible that household production is a substitute to market production and time devoted to it could also play a role in determining labour supply

► more sophisticated models will emphasize the role of the labour supply being decided at a household level – bargaining power and comparative advantage issues (with the same or differentiated utility functions for household members)

► intertemporal issues (life cycle and retirement decisions)

► other costs of participation (transport costs, child care, etc.)

► discrete time choices (time of work offered is in most cases not a continuous variable)
Supplementary constraints

Reality of life is far from flexible number of hours devoted to work:

- Assume that an individual has a choice of working during $T - L_f$ hours or not working at all.
- Let point $E$ represent the unconstrained optimum. If $E$ lies to the left of $E_f$ the individual will agree to work for $T - L_f$ hours – he or she would even have liked to work more.
- If point $E$ is to the right of $E_f$, then the individual will take up the job only if the point $E_A$ – the intersection of indifference curve passing through $A$ and the budget line – lies to the left of $E_f$: in this case he or she would obtain higher utility from work than from not participating.
- If point $E_A$ lies to the right of $E_f$ then individual will not participate (involuntary nonparticipation) – he or she would like to work for $T - L^*$ hours at current wage, but faces constraints that keep them from doing so.
Supplementary constraints

Figure 10. Constraints on hours of work.
Aggregate labour supply and Labour Force participation rate

- Given constraints on the number of hours devoted for work, each individual has choice to work for $h = T - L_f$ hours or not work at all.
- In a large population, reservation wages are probably different due to utility differences and different non-wage incomes; $w_A \in [0, +\infty)$.
- Distribution of reservation wages is represented by a cumulative distribution function $\Phi(\cdot)$. By definition, $\Phi(w)$ shows the proportion of population for which reservation wage is lower than current wage $w$ – it is the participation rate.
- $N\Phi(w)$ – aggregate labour supply.
- Since $d\Phi(w)/dw > 0$, then the elasticity of aggregate labour supply is also positive if $N$ is constant.
- Decision to participate is sensitive to many incentives, not necessarily only wage.
Labour demand in the neoclassical model
Labour demand in the neoclassical model

The demand for labour is a demand for factors of production derived from demand for final goods. Firms in a perfectly competitive market use a short run production function (with predetermined capital stock and given technology) to maximize profits:

$$Y = AF(L, \bar{K}),$$

$Y$ – real output, $\bar{K}$ – given capital stock, $L$ – amount of labour employed, $A$ – technology. Standard assumptions about the production function $F(\cdot)$ are adopted:

$$F_L > 0, \quad F_K > 0, \quad F_{LL} < 0, \quad F_{KK} < 0, \quad F_{LK} = F_{KL} > 0.$$
Labour demand in the neoclassical model

Firms aim at maximizing short-run profits:

$$\pi = PY - WL$$

Max \( L \) \( \pi = PAF(L, K) - WL \).

$$\frac{d\pi}{dL} = 0, \quad \Rightarrow \quad AF_L(L, K) = W/P. \quad (4)$$

Demand for labour \( L^D \) is defined implicitly by (4).
Labour demand in the neoclassical model

Let's take $w = W/P$ as a real wage. Then by totally differentiating (4) we get:

$$d(AF_L(L^D, \bar{K})) = dw$$

$$F_L \, dA + A(F_{LL} \, dL^D + F_{LK} \, d\bar{K}) = dw$$

By rearranging terms we obtain:

$$dL^D = -\frac{F_L}{AF_{LL}} \, dA - \frac{F_{LK}}{AF_{LL}} \, d\bar{K} + \frac{1}{AF_{LL}} \, dw$$
Labour demand in the neoclassical model

Properties of labour demand:

▶ since $F_{LL} < 0$, then an increase of real wage reduces demand for labour

▶ since $F_{LK} > 0$, then an increase in capital stock increases demand for labour

▶ since $F_L > 0$, then an improvement in technology increases demand for labour
Labour demand in the neoclassical model

Figure 11. Labour demand in the neoclassical model.
Unemployment in neoclassical model
Neoclassical labour market

Neoclassical labour market is characterized by:

- Homogenous workers
- Perfect competition
- Market clearing, flexible wages
- Perfect information
- No involuntary unemployment

Is neoclassical model enough to explain the performance of the labour market today?
Possible explanations for unemployment

If wages do not clear the market, so that there exists unemployment, why don’t they fall?

- There is no problem – some people just don’t want to work anyway
- Wage rigidity (especially downward) – minimum wage legislation
- Efficiency wages – it may be profitable for the firms to pay higher wage in equilibrium than market-clearing wage
- Bargaining power of labor unions (insiders-outsiders)
- Heterogeneity of workers and jobs – search for jobs and matching a worker with suitable job is costly and takes time
Can we still explain the difference in unemployment of skill groups using neoclassical model?

Skilled and unskilled labor in the production function:

\[ Y = G(L_s, L_u, K) = G(L_s, L_u, 1) \equiv F(L_s, L_u), \]

with \( F_u \equiv \frac{\partial F}{\partial L_u} > 0 \), \( F_s \equiv \frac{\partial F}{\partial L_s} > 0 \), \( F_{uu} \equiv \frac{\partial^2 F}{\partial L_u^2} < 0 \), and \( F_{ss} \equiv \frac{\partial^2 F}{\partial L_s^2} < 0 \).

Representative firm chooses two types of labour:

\[
\max_{L_s, L_u} \pi \equiv PF(L_s, L_u) - W_s L_s - W_u L_u.
\]
Maximization of profits yields standard condition for the value of marginal product of labour equal real wage (for skilled and unskilled labour):

\[ F_u(L_u, L_s) = \frac{W_u}{P} \equiv w_u \]

\[ F_s(L_u, L_s) = \frac{W_s}{P} \equiv w_s \]

These two conditions lead to labour demand functions, \( L_u^D \) and \( L_s^D \). By totally differentiating these two conditions, we get:

\[
\begin{bmatrix}
  dL_s \\
  dL_u
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
  F_{uu} & -F_{su} \\
  -F_{su} & F_{ss}
\end{bmatrix} \begin{bmatrix}
  dw_s \\
  dw_u
\end{bmatrix}
\]

where \( \Delta \equiv F_{ss}F_{uu} - F_{su}^2 > 0 \).
Therefore demand functions can be written as follows:

$$L^D_s = L^D_s(w_s, w_u), \quad L^D_u = L^D_u(w_s, w_u).$$

Clearly, the "own" real wage effects are negative, because of diminishing marginal products:

$$L^D_{ss} \equiv \frac{\partial L^D_s}{\partial w_s} = \frac{F_{uu}}{\Delta} < 0, \quad L^D_{uu} \equiv \frac{\partial L^D_u}{\partial w_u} = \frac{F_{ss}}{\Delta} < 0.$$

What are the "cross" real wage effects? Additional assumption – skilled and unskilled labour are treated as (imperfect) gross substitutes, so that these cross effects are positive. If unskilled labour becomes more expensive, the demand for skilled increases and vice versa.
In order to close the model as simply as possible, supply curves of the two types of labour are both assumed to be inelastic:

\[ L^S_s = \overline{L}_s \]

\[ L^S_u = \overline{L}_u \]
Implications

- With flexible wages, both types are fully employed (equilibrium skill premium, $w_s/w_u$)
- With a binding, skill-independent, minimum wage $\bar{w}$ the unskilled will experience unemployment. How to cure it?
  - abolish minimum wage (incomes distribution problems)
  - subsidize unskilled work
  - let government hire unskilled workers
  - train unskilled workers to become skilled (investment in human capital may pay for itself)
Graphical representation

Figure 12. Labour market equilibrium for skilled and unskilled
Articles
