Lecture 5
Search and matching theory

Leszek Wincenciak, Ph.D.

Warsaw University

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Lecture outline:

Introduction

- Even in the absence of marked changes in overall employment, there are simultaneous processes of job creation and destruction, reaching 20% of total employment in manufacturing during a year
- Workers are searching for the best jobs
- Firms are looking for the best workers
- Searching for job or a worker and matching takes time and is costly
- This leads to frictional unemployment

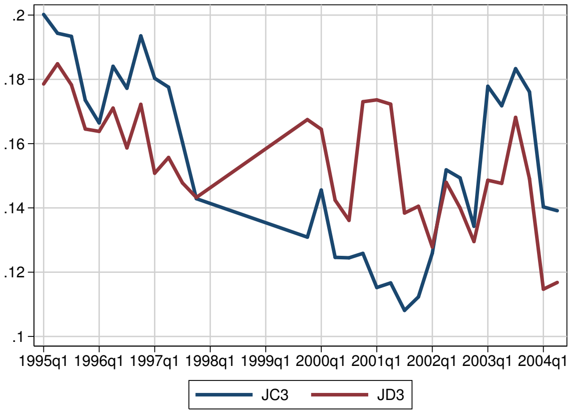

Figure 1. Job creation and destruction rates in Polish manufacturing, LFS data

Search and matching theory

Frictional unemployment

- Firms create job openings (vacancies)
- Workers search for jobs
- Match of a worker and a vacancy results in a productive job
- Matching is not coordinated (workers and firms dedicate time and resources to find a suitable match)
- Probability that a firm or a worker find the partner depends on a relative number of vacant jobs and unemployed workers
- Labor supply (L) = unemployed + employed
- Labor demand = filled jobs + vacancies

- Total number of unemployed workers – uL
- Total number of vacancies – vL
- Total number of matches between unemployed workers and vacant firms in each unit of time – mL
- The process of matching is summarized by a matching function, which expresses the number of newly created jobs (mL) as a function of the number of unemployed workers (uL) and vacancies (vL):

$$mL = m(uL, vL) \tag{1}$$

The matching function, assumed to be increasing in both arguments, can be thought of as similar to aggregate production function. Workers and vacant jobs can be viewed as productive inputs which produce a match, which results in a productive job. Creation of employment requires presence of both unemployed workers and vacant jobs – $m(0,0) = m(0,vL) = m(uL,0) = 0$. In order to have a constant unemployment rate in a growing economy, we need the matching function to exhibit a constant returns to scale (empirical evidence seems to support this assumption).

In case of CRS matching function, we can write:

$$m = \frac{m(uL, vL)}{L} = m(u, v). \tag{2}$$

The function $m(\cdot)$ determines the flow of workers who find a job and who exit the unemployment pool within each time interval. Consider the case of an unemployed worker: at each moment in time, the worker will find a job with probability $p = m(\cdot)/u$. With constant returns to scale for $m(\cdot)$ we may thus write:

$$\frac{m(u, v)}{u} = m\left(1, \frac{v}{u}\right) \equiv p(\theta), \tag{3}$$

where $\theta = v/u$ is called the labor market tightness.

The instantaneous probability p that a worker finds a job is positively related to the tightness of the labor market which is measured by θ, the ratio between the number of vacancies and unemployed workers. An increase in θ, reflecting a relative abundance of vacant jobs relative to unemployed workers, leads to an increase in p.

The average length of an unemployment spell is given by $1/p(\theta)$, and is thus inversely related to θ. Similarly, the rate at which a vacant job is matched to a worker may be expressed as:

$$\frac{m(u,v)}{v} = m\left(1, \frac{v}{u}\right)\frac{u}{v} = \frac{p(\theta)}{\theta} \equiv q(\theta), \tag{4}$$

a decreasing function of the vacancy/unemployment ratio. An increase in θ reduces the probability that a vacancy is filled and $1/q(\theta)$ measures the average time that elapses before a vacancy is filled.

The dependence of p and q on θ captures the dual externality between agents in the labor market: an increase in the number of vacancies relative to unemployed workers increases the probability that a worker finds a job ($dp(\cdot)/dv > 0$), but at the same time it reduces the probability that a vacancy is filled ($dq(\cdot)/dv < 0$).

The dynamics of unemployment

The dynamics of unemployment

Changes in unemployment result from a difference between the flow of workers who lose their job and become unemployed, and the flow of workers who find a job. The inflow into unemployment is determined by the 'separation rate' which we take as given for the moment: at each moment in time a fraction s of jobs (corresponding to a fraction $1 - u$ of the labor force) is hit by a shock that reduces the productivity of the match to zero: in this case the worker loses her job and returns to the pool of unemployed, while the firm is free to open up a vacancy in order to bring employment back to its original level. Given match destruction rate s, jobs therefore remain productive for an average period of $1/s$.

Given these assumptions we can now describe the dynamics of the number of unemployed workers. Since L is constant, $d(uL)/dt = \dot{u}L$ and hence:

$$\begin{aligned} \dot{u}L &= s(1-u)L - p(\theta)uL \\ \Rightarrow \dot{u} &= s(1-u) - p(\theta)u. \end{aligned} \tag{5}$$

The dynamics of the unemployment rate depend on the tightness of the labor market θ: at a high value for the ratio of vacancies to unemployed workers, workers easily find a job leading to a large flow out of unemployment.

From equation (5) we can immediately derive the steady state relationship between the unemployment rate and θ:

$$u = \frac{s}{s + p(\theta)}. \tag{6}$$

Since $p'(\cdot) > 0$, the properties of the matching function determine a negative relation between θ and u: a higher value of θ corresponds to a larger flow of newly created jobs. In order to keep unemployment constant, the unemployment rate must therefore increase to generate an offsetting increase in the flow of destroyed jobs.

To obtain job creation and destruction rates, we may divide the flows into and out of employment by the total number of employed workers $(1-u)L$. The rate of destruction is simply equal to s, while the rate of job creation is given by $p(\theta)[u/(1-u)]$.

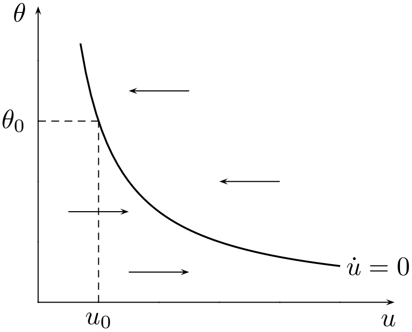

Figure 2. Dynamics of the unemployment rate

The steady-state relationship (6) is illustrated graphically in the left panel of Figure 5: to each value of θ corresponds a unique value for the unemployment rate. Moreover, the same properties of $m(\cdot)$ ensure that this curve is convex. For points above or below $\dot{u} = 0$, the unemployment rate tends to move towards the stationary relationship: keeping θ constant at θ_0, a value $u > u_0$ causes an increase in the flow out of unemployment and a decrease of the flow into unemployment, bringing u back to u_0.

Moreover, given u and θ, the number of vacancies is uniquely determined by $v = \theta u$, where v denotes the number of vacancies as a proportion of the labor force. The picture on the right hand side of the figure shows the curve $\dot{u} = 0$ in (v, u) space. This locus is known as the Beveridge curve, and identifies the level of vacancies v_0 that corresponds to the pair (θ_0, u_0) of the left hand panel of the Figure 5.

It is important to note that variations in the labor market tightness are associated with a movement along the curve $\dot{u} = 0$, while changes in the separation rate s or the efficiency of the matching process (captured by the properties of the matching function) correspond to movements of the curve $\dot{u} = 0$.

For example, an increase in s or a decrease in the matching efficiency cause an upward shift of $\dot{u} = 0$. Equation (6) gives a first steady state relationship between u and θ. To find the actual equilibrium values, we need to specify a second relationship between these variables. This second relationship can be derived from the behavior of firms and workers on the labor market.

Job creation by firms

Job creation by firms

The crucial decision of firms concerns the supply of jobs on the labor market. The decision of a firm whether to create a vacancy depends on the expected future profits over the entire time horizon of the firm, which we assume to be infinite. Formally, each individual firm solves an intertemporal optimization problem taking as given the aggregate labor market conditions which are summarized by θ, the labor market tightness. Individual firms therefore disregard the effect of their decisions on θ, and consequently on the matching rates $p(\theta)$ and $q(\theta)$. To simplify the analysis, we assume that each firm can offer at most one job. If the job is filled, the firm receives a constant flow of output equal to y. Moreover, it pays a wage w to the worker and it takes this wage as given. The determination of this wage will be described later on.

On the contrary, if the job is not filled, the firm incurs a flow cost c, which reflects the time and resources invested in the search for suitable workers. Firms therefore find it attractive to create a vacancy as long as its value, measured in terms of expected profits, is positive; in the opposite case, the firm will not find it attractive to offer a vacancy and will exit the labor market. The value that a firm attributes to a vacancy (denoted by V) and to a filled job (J) can be expressed using the asset equations. Given a constant real interest rate r, we can express these values as:

$$rV(t) = -c + q(\theta(t))(J(t) - V(t)) + \dot{V}(t), \tag{7}$$

$$rJ(t) = (y - w(t)) + s(V(t) - J(t)) + \dot{J}(t). \tag{8}$$

(7) and (8) are explicit functions of time. The flow return of a vacancy is equal to a negative cost component $(-c)$, plus the capital gain in case the job is filled with a worker $(J - V)$, which occurs with probability $q(\theta)$, plus the change in the value of the vacancy itself $(\dot{V})$. Similarly, (8) defines the flow return of a filled job as the value of the flow output minus the wage $(y - w)$, plus the capital loss $(V - J)$ in case the job is destroyed, which occurs with probability s, plus the change in the value of the job $(\dot{J})$.

Subtracting (7) from (8) yields the following expression for the difference in value between a filled job and a vacancy:

$$\begin{aligned} r(J(t) - V(t)) = &(y - w(t) + c) \\ &- [s + q(\theta(t))](J(t) - V(t)) \\ &+ (\dot{J}(t) - \dot{V}(t)). \end{aligned} \tag{9}$$

Now, if we focus on steady state equilibria we can impose $\dot{V} = \dot{J} = 0$ in equations (7) and (8). Moreover, we assume free entry of firms and as a result $V = 0$: new firms continue to offer vacant jobs until the value of the marginal vacancy is reduced to zero. Substituting $V = 0$ in (7) and (8) and combining the resulting expressions for J, we get:

$$\left.\begin{array}{c} J = c/q(\theta) \\ J = (y-w)/(r+s) \end{array}\right\} \Rightarrow y - w = (r+s)\frac{c}{q(\theta)}. \qquad (10)$$

Equation (7) gives us the first expression for J. According to this condition the equilibrium value of a filled job is equal to the expected costs of a vacancy, that is the flow cost of a vacancy c times the average duration of a vacancy $1/q(\theta)$.

The second condition for J can be derived from (8): the value of a filled job is equal to the value of the constant profit flow $y - w$. These flow returns are discounted at rate $r + s$ to account for both impatience and the risk that the match breaks down. Equating these two expressions yields the final solution (10), which gives the marginal condition for employment in a steady state equilibrium: the marginal productivity of the worker (y) needs to compensate the firm for the wage w paid to the worker and for the flow cost of opening a vacancy.
The latter is equal to the product of the discount rate $r + s$ and the expected costs of a vacancy $c/q(\theta)$.

This last term is just like an adjustment cost for the firm's employment level. It introduces a wedge between the marginal productivity of labor and the wage rate, which is similar to the effect of the hiring costs. However, in this model the size of the adjustment cost is endogenous and depends on the aggregate conditions on the labor market. In equilibrium, the size of the adjustment costs depend on the unemployment rate and on the number of vacancies, which are summarized at the aggregate level by the value of θ. If, for example, the value of output minus wages $(y - w)$ increases, then vacancy creation will become profitable $(V > 0)$ and more firms will offer jobs. As a result, θ will increase, leading to a reduction in the matching rate for firms and an increase in the average cost of a vacancy and both these effects tend to bring the value of a vacancy back to zero.

Finally, notice that equation (10) still contains the wage rate w. This is an endogenous variable. Hence the 'job creation condition' (10) is not yet the steady state condition which together with (6) would allow us to solve for the equilibrium values of u and θ. To complete the model we need to analyze the process of wage determination, to which we now turn.

Wage determination and the steady state

Wage determination

The process of wage determination that we adopt here is based on the fact that the successful creation of a match generates a surplus. That is, the value of a pair of agents that have agreed to match (the value of a filled job and an employed worker) is larger than the value of these agents before the match (the value of a vacancy and an unemployed worker). This surplus has the nature of a monopolistic rent and needs to be shared between the firm and the worker during the wage negotiations. Here we shall assume that wages are negotiated at a decentralized level between each individual worker and her employer. Since workers and firms are identical, all jobs will therefore pay the same wage.

Wage determination

Let E and U denote the value that a worker attributes to employment and unemployment, respectively. The joint value of a match (given by the value of a filled job for the firm and the value of employment for the worker) can then be expressed as $J + E$, while the joint value in case the match opportunity is not exploited (given by the value of a vacancy for a firm and the value of unemployment for a worker) is equal to $V + U$. The total surplus of the match is thus equal to the sum of the firm's surplus, $J - V$, and the worker's surplus, $E - U$:

$$(J + E) - (V + U) \equiv (J - V) + (E - U). \tag{11}$$

Wage determination

The match surplus is divided between the firm and the worker through a wage bargaining process. We take their relative bargaining strength to be exogenously given. Formally, we adopt the assumption of Nash bargaining. This assumption is common in models of bilateral negotiations. It implies that the bargained wage maximizes a geometric average of the surplus of the firm and the worker, each weighted by a measure of their relative bargaining strength (Nash maximand). In our case the assumption of Nash bargaining gives rise to the following optimization problem:

$$\max_{w} (J - V)^{1-\beta}(E - U)^{\beta}, \tag{12}$$

where $0 \leq \beta \leq 1$ denotes the relative bargaining strength of the worker.

Wage determination

Given that the objective function is a Cobb-Douglas one, we can immediately express the solution (the first order conditions) of the problem as:

$$E - U = \frac{\beta}{1-\beta}(J - V) \Rightarrow E - U = \beta[(J - V) + (E - U)]. \quad (13)$$

The surplus that the worker appropriates in the wage negotiations $(E - U)$ is thus equal to a fraction β of the total surplus of the job.

Similar to what is done for V and J in (7) and (8), we can express the values E and U using the relevant asset equations (reintroducing the dependence on time t):

$$rE(t) = w(t) + s(U(t) - E(t)) + \dot{E}(t) \tag{14}$$

$$rU(t) = z + p(\theta)(E(t) - U(t)) + \dot{U}(t). \tag{15}$$

For the worker the flow return on employment is equal to the wage plus the loss in value if the worker and the firm separate, which occurs with probability s, plus any change in the value of E itself; the return on unemployment is given by the imputed value of the time that a worker does not spend working, denoted by z, plus the gain if she finds a job and the change in the value of U. Parameter z includes the value of leisure and/or the value of alternative sources of income including possible unemployment benefits. It is assumed to be exogenous and fixed.

Restricting attention to steady state equilibria, so that $\dot{E} = \dot{U} = 0$, we can derive the surplus of the worker $E - U$ directly from (14) and (15).

$$E - U = \frac{w - z}{r + s + p(\theta)}. \tag{16}$$

According to (16) the surplus of a worker depends positively on the difference between the flow return during employment and unemployment $(w - z)$ and negatively on the separation rate s and on θ: an increase in the ratio of vacancies to unemployed workers increases the exit rate out of unemployment and reduces the average length of an unemployment spell.

Using (16) and noting that in steady state equilibrium

$$J - V = J = \frac{y - w}{r + s},$$

we can solve the expression for the outcome of the wage negotiations given by (13) as:

$$\frac{w - z}{r + s + p(\theta)} = \frac{\beta}{1 - \beta} \frac{y - w}{r + s}.$$

Rearranging terms, and using (10), we obtain the following equivalent expressions for the wage:

$$w - z = \beta[(y + c\theta - w) + (w - z)] \quad (17)$$

$$\Rightarrow w = z + \beta(y + c\theta - z). \quad (18)$$

Equation (17) is the version in terms of the flows of equation (13): the flow value of the worker's surplus, i.e. the difference between the wage and alternative income z, is a fraction β of the total flow surplus. The term $y - w + c\theta$ represents the flow surplus of the firm, where $c\theta$ denotes the expected cost savings if the firm fills a job. Moreover, the wage is a pure redistribution from the firm to the worker. If we eliminate the wage payments in (17) we obtain the flow value of the total surplus of a filled job $y + c\theta - z$, which is equal to the sum of the value of output and the cost saving of the firm minus the alternative costs of the worker. Finally, equation (18) expresses the wage as the sum of the alternative income and the fraction of the surplus that accrues to the worker.

It can easily be verified that the only influence of aggregate labor market conditions on the wage occur via θ, the ratio of vacancies to unemployed workers. The unemployment rate u does not have any independent effect on wages. The explanation is that wages are negotiated after a firm and a worker meet. In this situation the match surplus depends on θ, as we saw above. This variable determines the average duration of a vacancy, and hence the expected costs for the firm if it would continue to search. The determination of the equilibrium wage completes the description of the steady state equilibrium.

The equilibrium can be summarized by equations (6), (10) and (18) which we shall refer to as BC (*Beveridge curve*), JC (*job creation condition*) and W (*wage equation*):

$$u = \frac{s}{s + p(\theta)} \qquad (BC) \qquad (19)$$

$$w = y - (r + s)\frac{c}{q(\theta)} \qquad (JC) \qquad (20)$$

$$w = (1 - \beta)z + \beta(y + c\theta) \qquad (W) \qquad (21)$$

For a given value of θ, the wage is independent of the unemployment rate. The system can therefore be solved recursively for the endogenous variables u, θ and w. Using the definition for θ we can then solve for v. The last two equations jointly determine the equilibrium wage w and the ratio of vacancies/unemployed θ, as is shown in the left panel of Figure 6. Given θ, we can then determine the unemployment rate u, and consequently also v, which equate the flows into and out of unemployment (the right hand panel of the figure).

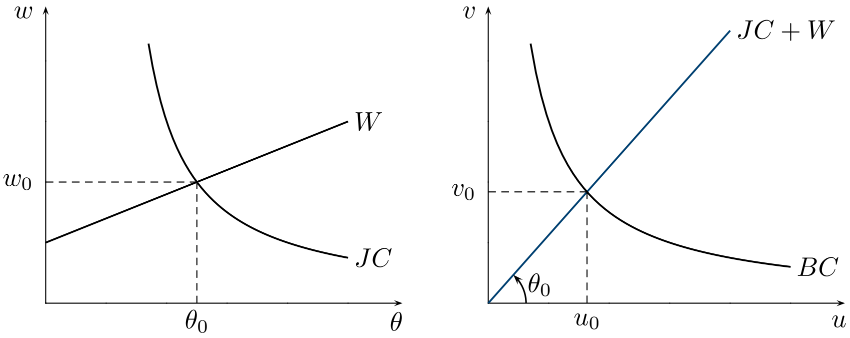

Figure 3. Equilibrium of the labor market with frictional unemployment

Comparative statics

Comparative statics

This dual representation facilitates the static comparative analysis, which is intended to analyze the effect of changes in the parameters on the steady state equilibrium.

Scenario 1: Assume, that we observe an increase in unemployment benefits, a component of z, or an increase in the relative bargaining strength of workers β.

Comparative statics, $z \uparrow$ or $\beta \uparrow$

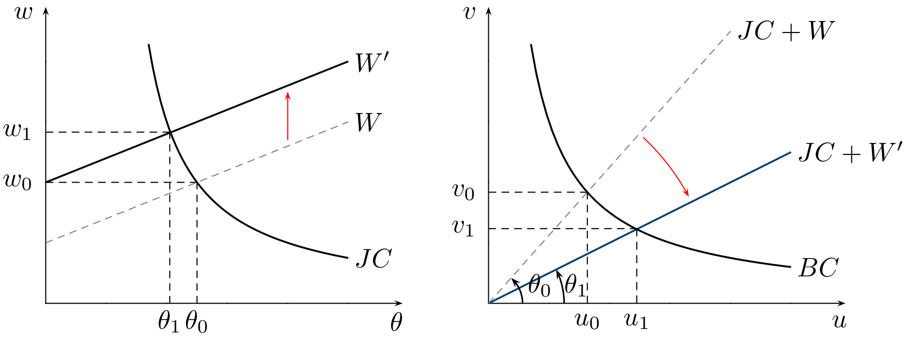

Figure 4. The effects of an increase in z or β

Interpretation

As a result of an increase in unemployment benefits (captured by z) or an increase in the bargaining power of the workers (captured by β) the wage curve defined by (21) shifts upwards. This causes an increase in the wage and a reduction in the labor market tightness, θ. This reduction, along the Beveridge Curve (BC), is accompanied by an increase in u and a reduction in v.

Scenario 2: Consider now an adverse aggregate shock, resulting in a decrease in y.

Comparative statics, $y \downarrow$

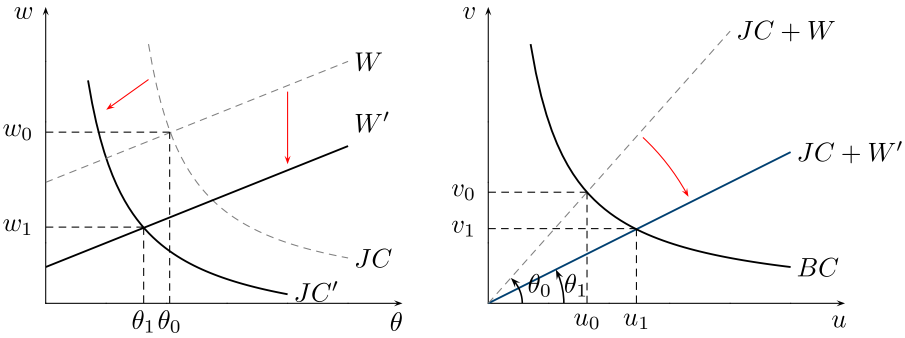

Figure 5. The effects of an adverse aggregate shock

A reduction in y moves both JC and W schedules downwards. This results in reduction of the wage but has an ambiguous effect on θ. However, formal analysis shows that in a stationary equilibrium θ also decreases (provided that $0 \leq \beta \leq 1$). At the same time the curve BC does not shift, so that the unemployment rate must increase while the number of vacancies v is reduced.

Scenario 3: Consider now a 'reallocative' shock, i.e. the increase in the separation rate s.

Comparative statics, $s \uparrow$

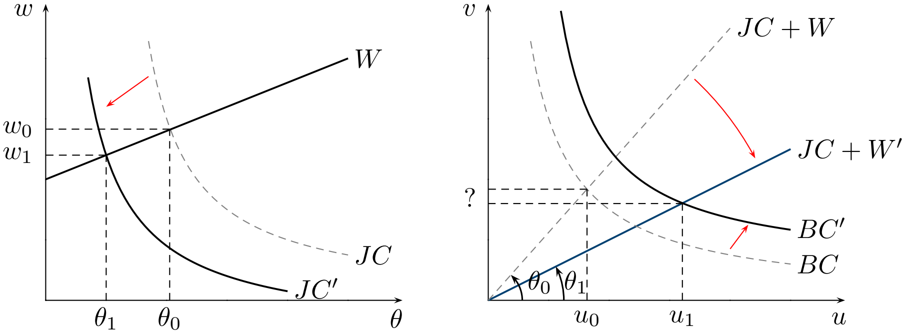

Figure 6. The effects of an adverse reallocative shock