Delayed Overshooting Puzzle in Structural Vector Autoregression Models

Klodiana Istrefi∗, Balazs Vonnak †

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Abstract

The delayed overshooting of the exchange rate to a monetary policy shock is a typical puzzle found in many VAR studies. Some authors argue that it is a phenomenon due to improper identification of monetary policy shocks, like Cholesky ordering. In this paper, we investigate to what extent various identification restrictions in structural VARs are able to recover the true effect of monetary policy on the exchange rate. We do this under a controlled experiment where we simulate two sets of artificial observations from an open economy DSGE model of Adolfson, Lassen, Linde, and Villani (2008). In one case the data generating process features delayed overshooting; in the other it does not. We estimate VAR models and identify monetary policy shocks by Cholesky ordering and sign restrictions. Our results show that Cholesky type restrictions, although not successful in recovering the true monetary shock, do not produce a delayed overshooting puzzle in case the data generating process does not contain it. For the case where the delayed overshooting is a feature of the true model, all the identification schemes under investigation recover it correctly.

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Keywords: Monetary Policy; Exchange Rate; DSGE; Vector Autoregressions; Cholesky Decomposition; Sign restrictions.

∗Ph.D student in Economics, Goethe University Frankfurt, Email: Klodiana.Istrefi@hof.uni-frankfurt.de
†Magyar Nemzeti Bank, Email: vonnakb@mnb.hu
Introduction

It is a common finding in exchange rate econometrics that the implications of theoretical exchange rate models are not supported by the data. One of the most famous empirical puzzles is the delayed overshooting puzzle, the hump-shaped impulse response of exchange rate to a monetary policy shock found in VARs, first introduced in Eichenbaum and Evans (1995) and supported, among others by Peersman and Smets (2003) and Scholl and Uhlig (2007). However, recent work in the literature suggests that most of the puzzles or the non-sense results are due to improper identification strategies used in VAR analyses. This is the case mostly with the recursive VARs. Studies that employ this identification strategy usually assume that either the exchange rate does not react to a monetary policy shock or that monetary policy does not take in account exchange rate surprises, depending on the ordering (the latter is more common). When a more careful and plausible identification strategy is used, the results are either inconclusive or puzzles seem to disappear (see, among others, Kim and Roubini (2000), Faust and Rogers (2003) and Bjornland (2009)).

These results emerge from identified-VAR analyses which try to recover a monetary policy shock from a set of observed variables for which the data generating process and the underlying shocks are not known. Practice shows that identification strategies are then considered proper if they deliver what are perceived as reasonable results. In this paper we investigate if the delayed overshooting puzzle is an artifact of the identification assumptions in structural VAR models. We do this under a controlled experiment, where the data generating process (DGP) in the economy, the underlying structural shocks and their effects on the variables are known. This experiment involves two versions of a DSGE model: one that does not produce delayed overshooting after a surprise monetary policy shock and another one that produces such an effect on the exchange rate. The experiment is designed as follows: first, artificial data are simulated from a small open economy DSGE model, which are then fed to VARs with various identification schemes. The responses of the economy to the identified monetary policy shock with SVARs are then compared to the corresponding true responses from the DSGE model. Results from the estimated SVARs are compared for two cases. In one case they are estimated on a large number of observations and in the other case on a short sample of observations, under a Monte Carlo exercise.

The ability of the SVAR model in recovering the true underlying monetary policy shock in the simulated data is tested for the following identification strategies: identification using Cholesky decomposition, identification using a combination of zero and sign restrictions and identification with DSGE model - consistent sign restrictions only. With respect to our data generating process, the Cholesky decomposition imposes improper restrictions as they exclude contemporaneous
reaction of certain variables to specific shocks contrary to the structure of the DGP. On the other hand, sign-identified VARs allow for simultaneous reactions, but restrict the sign of the reaction, in line with the DGP. Through this experiment, we are able to see if identification with Cholesky is insufficient to isolate the monetary policy shock and if improper restrictions it imposes lead to artificial puzzles, especially for the exchange rate. Furthermore, we can test (at least with respect to our DGP) how results from structural VARs change if we move from improper to model-consistent restrictions.

We generate the artificial data using a medium scale DSGE model for a small economy, estimated for Sweden, by Adolfson et al. (2008). This is not an incidental choice. First of all, the model as in Adolfson et al. (2008) represents a new Keynesian small open economy model in the tradition of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), widely used in central banks for policy analysis and forecasting. This model is in the same spirit as the pool of DSGE models developed for small open economies and has well-documented empirical properties. Furthermore, this model provides an example of how DSGE models are built to replicate some of the puzzles found in empirical work. More specifically, Adolfson et al. (2008) present a modified uncovered interest rate parity (UIP) condition that allows for a negative correlation between the risk premium and the expected change in the exchange rate. Under this modification the DSGE model delivers the hump-shaped impulse response of exchange rate to a monetary policy shock found in VARs. Adolfson et al. (2008) offer two versions of their model, one estimated with the standard UIP condition and one with the modified UIP condition. For our purpose, we use both versions of the model.

It is important to stress that under this experiment we do not build the VAR representation of the DSGE model\(^1\). Instead, we work with only a subset of the variables from the DSGE model\(^2\), which are then fed to VARs, as it is usually done in empirical work when the DGP is not known. We use the most common variables employed in such analysis for a small open economy: production, consumer prices and interest rate for both foreign and domestic economy and the nominal exchange rate. Foreign economy variables are treated as exogenous. Results show that identified VARs are able to discriminate between different data generating processes and that the identification schemes with DSGE model-consistent restrictions are more successful in recovering the effects of the underlying monetary policy shock. Interestingly, when data are abundant, these results show that Cholesky-type restrictions, although not successful in recovering the true monetary shock, do not produce a delayed overshooting in case the data generating process does not contain it. For the case where the delayed overshooting is a feature of the DGP, all the identification

\(^2\)We are aware of the issues raised when trying to approximate the data generating process behind a DSGE model with a finite VAR with omitted variables. Given these issues we do not expect a perfect match of the responses from the estimated SVARs and the DSGE.
schemes under investigation recover it correctly. This is in contrast to what the recent literature seemed to converge, assigning the delayed overshooting puzzle to improper restrictions in the Cholesky identification. On small sample, all the identification strategies under discussion tend to suggest a short delayed response of the exchange rate when there is no delay in the DGP or a less delayed response than the true response. However, when facing sampling uncertainty, recursive identification yields less robust results compared to sign identification approaches, despite the fact that the latter contain identification uncertainty as well.

We check for the robustness of the results with two other data generating processes, with a DSGE models as in Medina and Soto (2007) and as in Jaaskela and Jennings (2011) and arrive at the same conclusions. Furthermore, we test the same identification strategies with real data from the Swedish economy. Estimates on actual data are closer to the small sample simulation results with no delayed overshooting from Adolfson et al. (2008). As it was the case with the simulated data, the result regarding the response of the exchange rate is robust to all three identification schemes used, casting some doubt on whether it is justified to apply a modified UIP in the Adolfson et al. (2008).

Controlled experiments in similar fashion have been recently used in the literature mostly to assess the effects of the monetary policy on output in the case of closed economies. For the open economy case, Jaaskela and Jennings (2011) have used a DSGE model estimated for Australia to assess the ability of VARs in identifying monetary policy shocks, finding that recursive identification produces puzzles while identification with sign restrictions does not, provided that a sufficient number of shocks are identified. Relative to their work, we focus on the exchange rate and contribute to the literature by performing the experiment with data generated from an empirically plausible DSGE model with a richer structure. With this model we have the advantage to test the performance of VAR identification strategies for both, the case when the DGP features delayed overshooting of the exchange rate and the case when it does not. Furthermore, our approach compares favorably as it provides insights into the asymptotic and small sample properties of the identification strategies used. In contrast to Jaaskela and Jennings (2011) we use minimal restrictions to identify the effect of the monetary policy shock on the exchange rate, a situation that is more plausible to occur in practice, when the econometrician may not have apriori valid assumptions to identify many of the shocks present in the data.

The paper proceeds as follows: Section 1 describes shortly the Adolfson et al. (2008) model that we use as a data generating process. Section 2 presents the steps of the controlled experiment and the results from the VAR analysis under different identification schemes. Section 3 concludes.

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The model as DGP

To generate the data we use a medium scale DSGE model as in Adolfson et al. (2008). This model represents a new Keynesian small open economy model in the tradition of Christiano et al. (2005) and Smets and Wouters (2003), widely used in central banks for policy analysis and forecasting. This model is in full operational use at the Sveriges Bank. The model features a number of nominal and real frictions such as sticky prices and sticky wages, incomplete exchange rate pass-through, habit persistence and investment adjustment cost. Adolfson et al. (2008) is a version of the model developed in Adolfson et al. (2007) with the main difference being in the specification of the uncovered interest rate parity (UIP) condition. The first version of the model employs a standard UIP condition while the latest version of the model employs a modified UIP condition. The modified version allows for a negative correlation between the risk premium and the expected change in the exchange rate which then allows the DSGE model to deliver a hump-shaped response of the exchange rate to a monetary policy shock, thus replicating the delayed overshooting of the exchange rate commonly found in empirical works.

The domestic economy in Adolfson et al. (2008) is comprised of households, four types of firms working on the domestic and external sector, a government and a monetary authority. Households choose consumption, work effort and holding of real balances to maximize their utility. They consume domestic and imported goods, own physical capital, domestic and foreign assets. They supply labor monopolistically and set their wages subject to á la Calvo rigidity. On the production side, there are four different firms: domestic goods firms, importing consumption, importing investment, and exporting firms. They all produce a differentiated good and have monopolistic power when setting prices. Price setting in all these sectors is subject to á la Calvo rigidities.

Monetary authority is assumed to follow a Taylor rule, with short term interest rate reacting to deviations of CPI inflation from the inflation target, to output gap and to the real exchange rate. The foreign economy is modeled exogenously, resembling the typical situation of a small open economy with no power in the world market. Both foreign economy and fiscal policy are modeled and estimated exogenously as VAR processes out of the DSGE model. The model is rich in terms of the exogenous shocks, containing 21 of them. Among these shocks we mention here preference shocks to consumption and labor efforts, technology shocks (a unit root and a stationary technology shock), four sector specific mark-up shocks, a monetary policy shock, shocks related with fiscal policy, a risk premium shock and shocks coming from the foreign economy.

Adolfson et al. (2008) estimate both versions of the model on Swedish data using Bayesian techniques for the period 1980Q1-2004Q4. When exploring the consequences of the modification for empirical coherence their results show a preference for the model featuring the modified
UIP condition. The modification seems to be important especially for the model’s forecasting performance.

1.1 Standard and modified UIP condition

Let’s denote the model with standard UIP condition as M1 and the model with modified UIP condition as M2. In Adolfson et al. (2008), households hold cash, domestic bank deposits and foreign bonds. Saving in domestic deposits pays a gross nominal rate of $R_t$ while saving in foreign bonds pays a gross interest rate of $R^*_t$ adjusted for a risk premium of holding the foreign bonds\footnote{The introduction of risk premium is needed to ensure a well-defined steady state for open economy models (see Schmitt-Groh and Uribe (2003)) for more details.}.

In M1, the risk premium is a function of net foreign asset position of the domestic households, $a_t \equiv (S_t B_t)/P_t z_t$. In M2, risk premium specification contains an additional term, namely the expected change in the exchange rate, $E_t S_{t+1}/S_{t-1}$, with $S_t$ and $B_t$ being the nominal exchange rate and bond holding, respectively. Adolfson et al. (2008) base and justify this modification on the empirical findings of strong negative correlations between risk premium and expected change in the exchange rates (the so-called the forward premium puzzle). In M2 risk premium is given by:

$$\Phi(a_t, S_t, \tilde{\phi}_t) = \exp(-\tilde{\phi}_s(a_t - \tilde{a}) - \tilde{\phi}_s(\frac{E_t S_{t+1}}{S_t} \frac{S_t}{S_{t-1}} - 1) + \tilde{\phi}_t)$$ (1)

where $\tilde{\phi}_t$ is the risk premium shock. In M1, risk premium has a similar specification with $\tilde{\phi}_s = 0$.

Given the specification of the risk premium in M2, combining the first order condition for cash and for foreign bond holdings from the maximization problems of the households, we derive the following modified UIP condition (in log-linearized form):

$$\hat{R}_t - \hat{R}^*_t = (1 - \tilde{\phi}_s)E_t \Delta \hat{S}_{t+1} - \tilde{\phi}_s \Delta \hat{S}_t - \tilde{\phi}_a \hat{a}_t + \tilde{\phi}_t$$ (2)

while the standard UIP condition in M1 is of the form:

$$\hat{R}_t - \hat{R}^*_t = E_t \Delta \hat{S}_{t+1} - \tilde{\phi}_a \hat{a}_t + \tilde{\phi}_t$$ (3)

Figure 1 illustrates the effect of an exogenous monetary policy shock\footnote{Note that here we show the responses to CPI and nominal exchange rate while Adolfson at al. (2008) present in Figure 1 the impulse responses to CPI inflation and real exchange rate.} to output, consumer prices, nominal interest rate and nominal exchange rate, from M1 and M2. In both cases, the contractionary shock induces a gradual decline of output and prices and an appreciation of the...
exchange rate on impact. Under the model with standard UIP condition, the sharp appreciation of the exchange rate is followed by a gradual depreciation. However, under the modified UIP, the exchange rate reaches its peak appreciation only after several quarters, resembling the delayed overshooting. This change is due to the larger persistence that the modified UIP condition assumes. The negative correlation between risk premium and the expected change of the exchange rate, induces the households to require a lower risk-adjusted $R^*_t$ on the foreign bond holdings. But since the returns on domestic and foreign assets should equalize, a larger appreciation of exchange rate is required.

(a) DGP with standard UIP

(b) DGP with modified UIP

Figure 1: IRFs to a monetary policy shock

Notes: The line in red denotes the median impulse response from the estimated DSGE model as in Adolfson et al. (2008). A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters. In vertical axis, for each variable the log deviations from the steady state.
2 VAR analysis

To examine the effect of monetary policy in real life problems using identified-VAR modeling, the econometrician needs to recover the monetary policy shock from the variables she can observe. In this case the econometrician does not know the true data generating process but uses, in the best case, a priori justifiable identifying restrictions to be able to recover such shocks. Practice has shown that the identification strategy in VARs is considered to be proper when they are able to deliver what are perceived as reasonable results (see for example Uhlig (2005) for a discussion). Instead, when using simulated data from a DSGE model, we know the data generating process in the economy, the underlying structural shocks and their effect on the variables. Assuming that we don’t know the model coefficients, we can select a list of variables from this economy, estimate VAR models on them and then investigate which of the VAR identifying restrictions used in the literature is able to recover the true underlying structural shocks.

In the following we run exactly this experiment. First we solve the estimated DSGE model using calibrated parameters and the posterior median of estimated parameters as in Adolfson et al. (2008) (see Table A.1 in the Appendix A). Then we simulate the model and with a selected subset from the simulated variables we estimate structural VARs and investigate their performance in recovering the monetary policy shock and its effect in the exchange rate. Their performance is judged by comparing VAR responses with the true responses from the DSGE model. The subset of the DSGE model variables chosen for VAR estimation is comprised of the following variables: foreign output, y*, foreign consumer prices, p*, foreign interest rate, i*, domestic output, y, domestic consumer prices, p, domestic nominal interest rate, i, and the nominal exchange rate, s. This set represents the most common set of variables used in the empirical literature of monetary policy analysis for small open economies. We estimate the VARs using variables in levels and adopt a Bayesian approach when making statistical inference. The frequency of the data is quarterly.

In real life the econometrician does not have the luxury to conduct VAR analysis using long series of data for different reasons. In cases when long samples of data exist, most probably there are regime changes, structural breaks and estimating a VAR on these data would be in contradiction with the principle that VARs should be estimated on sample periods that define reasonably constant parameter regimes. With simulated data from a DSGE model we don’t face this problem so we are able to explore the asymptotic properties of our estimation strategies. For the long sample size VAR analysis we use a simulated sample of 10,000 periods. Since VAR analyses are typically made on much shorter samples, we investigate the small sample properties of our estimators as well. For the small sample size SVAR analysis, we use 500 sets of simulated data, each having 100 observations.
2.1 Identification of monetary policy shock

The reduced-form VAR model has the following presentation:

\[ A(L)z_t = \epsilon_t \]  

where \( z_t \) is a n-dimensional vector of variables, \( A(L) \equiv I + A_1 L - A_2 L^2 - \ldots - A_p L^p \) is the autoregressive lag order polynomial and \( \epsilon_t \) represents the reduced-form errors with covariance matrix, \( \Sigma_\epsilon \). \( A(L) \), \( \epsilon_t \) and \( \Sigma_\epsilon \) can be estimated consistently with standard estimation methods. However we are interested in recovering the corresponding structural shocks \( u_t \) which relate to reduced-form errors through the following relationship: \( u_t = S\epsilon_t \), with \( E(\epsilon_t u_t') = \Sigma_u = I_n \). To this purpose we need to recover the elements of \( S \). Noting that \( \epsilon_t = S^{-1}u_t \), we can see that \( \Sigma_\epsilon = S^{-1}E(u_t u_t')S^{-1'} = S^{-1}S^{-1'} \). Since \( \Sigma_\epsilon \) is known (estimated), we can uniquely identify on maximum \( n(n+1)/2 \) from \( n^2 \) elements of \( S^{-1} \). For exact identification we should impose additional \( n(n-1)/2 \) restrictions on \( S^{-1} \).

The literature of structural vector autoregressions has continuously evolved in finding valid identifying restrictions which vary from short and long run zero restrictions to sign restrictions. In the following, we revisit some of the identifying assumptions used in the literature and investigate their ability in recovering the true underlying monetary policy shock in the simulated data. More specifically, we will investigate the following identification strategies: identification using the Cholesky decomposition, identification using a combination of zero and sign restrictions and identification with sign restrictions only. With respect to our data generating processes, Cholesky decomposition imposes improper restrictions as they exclude contemporaneous reaction of certain variables to specific shocks contrary to the structure of the DGP. On the other hand, sign-identified VARs allow for simultaneous reactions but restrict the sign of the reaction. Through this experiment we are able to see if Cholesky is insufficient to isolate the monetary policy shock and if improper restrictions it imposes lead to unexpected results, especially for the exchange rate. Furthermore, we can test (at least with respect to our DGP) how results from structural VARs change if we move from improper to model-consistent restrictions.

Let \( z_t = (y_t, p_t, i_t, s_t) \) be the block of the endogenous variables for the small open economy and \( z_t^* = (y_t^*, p_t^*, i_t^*) \) the block of the exogenous variables. With Cholesky identification we impose a recursive ordering of the shocks and assume that a monetary policy and an exchange rate shock will not affect contemporaneously the GDP and CPI. Furthermore, an exchange rate shock is assumed to not affect interest rates as well (see for example Eichenbaum and Evans (1995)). Being ordered last, an exchange rate shock will have a delayed effect on output, prices and interest rates. Such a

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6For a recent survey on structural VARs and identification assumptions refer to Kilian (2011).
slow exchange rate pass-through to macroeconomy can be justified\(^7\) by the presence of nominal rigidities in the economy. These (identifying) assumptions provide enough restrictions for the Cholesky decomposition to be applied. Such case belongs to the class of fully identified VAR models where all shocks are identified but we are interested only in the identification of the monetary policy shock. Under the Cholesky identification the relationship between the reduced-form errors and structural shocks is presented as below:

\[
\begin{pmatrix}
    \epsilon^y_t \\
    \epsilon^p_t \\
    \epsilon^i_t \\
    \epsilon^s_t
\end{pmatrix} =
\begin{bmatrix}
    x & 0 & 0 \\
    x & x & 0 \\
    x & x & x \\
    x & x & x
\end{bmatrix}
\times
\begin{pmatrix}
    u^1_t \\
    u^2_t \\
    u^{MP}_t \\
    u^4_t
\end{pmatrix}
\]

The hybrid identification (zero and sign restrictions) allows for contemporaneous responses between monetary policy and exchange rate based on the idea that exchange rate developments are important for monetary policy decisions in small open economies (see McCallum (1994), Cushman and Zha (1997) and Vonnak (2010)). This identification strategy combines zero restrictions on the contemporaneous effect of monetary policy on output and prices with sign restrictions on the response of interest rate and exchange rate to this shock. In reality, sign restrictions on the responses of interest rate and nominal exchange rate are justified by the general postulation that a monetary contraction is associated with an increase in interest rates and an exchange rate appreciation on impact. By restricting the sign of the exchange rate response on the impact period only, by construction, we avoid exchange rate puzzles which are immediate depreciations of the exchange rate to a monetary policy shock. Still, after this period the exchange rate can react freely in whatever direction, keep appreciating or depreciate. In other words, the sign restriction we impose does not affect the timing of the peak appreciation of the exchange rate. Under these identifying assumptions\(^8\) the relationship between the reduced form errors and structural shocks is presented as below:

\[
\begin{pmatrix}
    \epsilon^y_t \\
    \epsilon^p_t \\
    \epsilon^i_t \\
    \epsilon^s_t
\end{pmatrix} =
\begin{bmatrix}
    x & x & 0 & x \\
    x & x & 0 & x \\
    x & x & + & x \\
    x & x & - & x
\end{bmatrix}
\times
\begin{pmatrix}
    u^1_t \\
    u^2_t \\
    u^{MP}_t \\
    u^4_t
\end{pmatrix}
\]

The pure sign restriction identification relaxes the contemporaneous rigidity in GDP and CPI to the monetary policy shock.\(^9\) Although the sign restrictions we set above on interest rate and exchange rate are model consistent, from the DSGE model we know there are four other shocks that have the same sign impact on them, namely the markup shocks for imported consumption and

\(^7\)With respect to interest rate even this assumption is hard to justify (McCallum(1994)) and we relax it later.
\(^8\)Details about the sign restriction algorithm are found in Appendix B.
\(^9\)With real data, delayed CPI and GDP reaction to monetary policy may not be a bad assumption for monthly data.
investment, the asymmetric technology shock and the consumption preference shock. Therefore, allowing for contemporaneous reaction of all the variables and restricting only the sign of interest rate and exchange rate on impact is not enough to discriminate the monetary policy shock between the five shocks. However, in comparison with the other shocks, the monetary policy shock has a unique (negative) sign impact on the response of output and consumer prices. Under these assumptions the relationship between the reduced form errors and structural shocks is presented as below:

\[
\begin{pmatrix}
\epsilon^y_t \\
\epsilon^p_t \\
\epsilon^i_t \\
\epsilon^a_t
\end{pmatrix} =
\begin{pmatrix}
x & x - x \\
x & x - x \\
x & x + x \\
x & x - x
\end{pmatrix}
\times
\begin{pmatrix}
u^1_t \\
u^2_t \\
u^{MP}_t \\
u^4_t
\end{pmatrix}
\]

In the following, we show the evidence from the estimated VARs for each of the identification scheme in parallel. We summarize and interpret the evidence from structural VARs mostly based on the posterior medians of impulse responses and the corresponding 16th and 84th percentiles. Although some authors have expressed reservations for the usage of the median response for inference when structural disturbances are identified with sign restrictions, we treat it as a summary of the impulse response distribution for each horizon. In order not to be subject to criticism, when we formulate statements about the shape of the responses or other characteristics that relate not only to one single horizon, we use additional statistics that present the information we are interested in a statistically, meaningful way. More specifically, we treat the results carefully especially when interested about the shape of the response of the exchange rate to the VAR-identified monetary policy shock. In these cases, we complement the discussion with further evidence coming from the distribution of the quarter with the highest appreciation of the exchange rate (distribution of the peak location) and from the excess return, given a monetary policy shock. These statistics convey correct information about the presence or not of a delayed overshooting of the exchange rate.

10Inoue and Kilian (2011) highlight the fact that the vector of the posterior median responses suffers from two shortcomings: it has no structural economic interpretation, as it is a summary statistic of responses from different structural models which rarely correspond to the response of any of those (admissible) models, and it is not a well-defined statistical object. See as well Fry and Pagan (2011).

11Excess return is calculated as the difference between interest rate spread and the expected change of exchange rate, given a monetary policy shock.
2.2 Results and Discussion

2.2.1 Long sample

Using the long sample of the simulated data and the above identifying strategies we estimate structural VAR(2) for both data generating processes in hand. Figure 2 reports the impulse responses of output, consumer prices, interest rates and the nominal exchange rate to a monetary policy shock identified with each of the identification strategies (black line) and the true responses from the DSGE model (red line) for the case the data are generated from the model with standard UIP condition. Figure 3 presents the same information for the model with the modified UIP condition.

Figure 2: DGP with standard UIP: IRFs to a monetary policy shock identified with SVARs

Notes: The solid line in black denotes median impulse response from the estimated VAR(2), large sample size (T=10,000), and the shaded area the corresponding 68 percent error band. The line in red denotes the median impulse response from the estimated DSGE model as in Adolfson et al. (2008). A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.

In general, we observe a poor match between the effects of a monetary policy shock identified
with Cholesky and the true effects\textsuperscript{12} from the underlying monetary policy shock, in both data generating processes. When compared to the responses from the DSGE model, a monetary policy shock identified with Cholesky restrictions produces the so-called "price puzzle", familiar in the VAR literature of monetary policy analysis. In both panels, the reaction of the interest rate appears overestimated and slightly more persistent. Furthermore, the response of output is rightly recovered by Cholesky assumptions, which is a consistent result between the two SVAR estimations.

\textbf{Figure 3:} DGP with modified UIP: IRFs to a monetary policy shock identified with SVARs

\textit{Notes:} The solid line in black denotes median impulse response from the estimated VAR(2), large sample size (T=10000), and the shaded area the corresponding 68 percent error band. The line in red denotes the median impulse response from the estimated DSGE model as in Adolfson et al. (2008). A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.

Regarding the exchange rate, even though its response appears less persistent than the true one, its direction is correctly recovered for both data generating processes. More specifically, in the first panel of Figure 2 we observe an appreciation of the exchange rate on impact, followed by a continued depreciation thereafter, in line with what the underlying model with standard UIP condition predicts (although a much faster depreciation). In the first panel of Figure 3, the

\textsuperscript{12}Again, we are aware of the issues raised when trying to approximate the data generating process behind a DSGE model with a finite VAR with omitted variables. Given these issues we do not expect a perfect match of the responses from the estimated SVARs and the DSGE.
appreciation of the exchange rate goes on for more quarters following the shock, in line with the delayed overshooting the underlying model with modified UIP condition would predict.

Compared to the Cholesky identification, usage of sign restrictions for identification introduces identification uncertainty in our VAR analysis as under this strategy we identify a set of impulse responses satisfying the restrictions. This is visible from the wider uncertainty bands around the median response for both VARs estimated with hybrid restrictions and with sign restrictions only. Still, under both these identification strategies, the direction and the persistence of VAR responses are closer to the true responses than under the Cholesky identification. The “price puzzle” is avoided with the sign restriction identification while in all other cases the true impulse responses are contained by or very close to the uncertainty band. Under these two identification strategies, quantitatively, the nominal exchange rate response from the structural VAR appears overestimated in comparison with the true response. However, the appreciation shows up immediate in panel (b) and (c) in Figure 2, and delayed in respective panels of Figure 3, in line with what the underlying data generating processes would suggest.

Since we cannot rely on the information from the median impulse for the shape of the response of the exchange rate to a monetary policy shock, we investigate the response of the nominal exchange rate through quarters by calculating the posterior distribution of the quarter where exchange rate response is the lowest (the timing of the peak appreciation). Figure 4 shows this statistic, for all three identification strategies and for both data generating processes. The first panel ((a), (b), (c)) shows the results from the SVARs with data from M1 and the second panel ((d), (f), (g)) shows the results from the SVARs with data from M2.

Figure 4: Posterior distribution of peak appreciation of exchange rate to a monetary policy shock
Results show that in case the data generating process is the model without delayed overshooting, all peak appreciations suggested by the identified VAR with Cholesky appear in the first quarter. With sign restrictions, although the uncertainty of the responses has increased when compared with the Cholesky, results regarding the posterior distribution of the peak appreciation remain robust; about 90 percent of peak appreciations occur immediately. In case the data generating process is the model with delayed overshooting, Cholesky identification predicts that about 70 percent of the peak appreciations occur in the fourth quarter, and the rest in the fifth quarter. In this case, the structural VAR with Cholesky correctly identifies a delayed appreciation of the exchange rate although a less persistent one when compared to the DSGE model where peak appreciations occur in the sixth and the seventh quarter. With sign restrictions, the distribution of the peak is centered on the sixth quarter.

To sum up, from the analysis with long samples (with minimized estimation uncertainty) we observe that even though identification with Cholesky assumes improper restrictions with the respect to both data generating processes that we use, it captures correctly the exchange rate response to a Cholesky-identified “monetary policy shock”. The delayed overshooting of the exchange rate shows up correctly in the case of the structural VAR estimated with data simulated from the DSGE model that assumes a delayed overshooting. In case the underlying model does not assume it, the Cholesky identification does not produce the puzzle artificially. This is an interesting result taking in account that the recent literature\textsuperscript{13} has been converging to the conclusion that the delayed overshooting puzzle may be an artifact of improper restrictions one imposes with Cholesky identification.

**Predictable excess return:** In the following we investigate the implications the above identified VAR estimates have for the predictable excess return (PER), namely the difference between interest rate spread and the expected change of exchange rate, given a monetary policy shock. PER can be considered as a measure of the violation of the UIP condition. For both DSGE and VAR responses, we calculate the predictable excess return (in annualized terms) as:

\[
PE_{Rt} = i_t - i_t^* - 4\Delta s_{t+1} \tag{5}
\]

Figure 5 reports the estimates of predictable excess return from long sample VARs estimated with simulated data from M1 (first panel) and M2 (second panel). Within each panel we present the results pertaining to our three identification schemes. As we can see, given a monetary policy shock, the DSGE responses from the model with standard UIP condition assume a PER equal to zero while the model with modified UIP assumes a positive PER. On the other side, the estimated

\textsuperscript{13}See Kim and Roubini (2000), Bjornland (2009) and Bjornland and Halvorsen (2010).
VARs suggest a persistent negative excess return for the first case and a positive to negative excess return for the second one. Excess returns (either positive or negative) appear especially overestimated under sign-identified VARs. As we saw before, sign restriction overestimates the exchange rate reaction, therefore it causes the underestimation of excess return in the case of no delayed overshooting (since the depreciation is faster). In the case with delayed overshooting, it causes an overestimation in the beginning, since the appreciation is bigger.

Figure 5: True DSGE vs. VAR estimated excess returns

Notes: First panel, VAR simulated from DSGE model with standard UIP and second panel VAR simulated from DSGE model with modified UIP. The solid line in black denotes the VAR estimated PER and the shaded area in light blue indicates its 68 percent confidence region. The line in red denotes the true PER from the DSGE model.

2.2.2 Monte Carlo Exercise

In order to investigate the small sample properties of our SVAR estimations, we repeat the experiment as above using 500 samples of simulated data with size of 100 observations. For each sample of data we estimate the three types of structural VARs as discussed previously, with the aim to identify the underlying monetary policy shock and its effect on the exchange rate. In more detail, for each identification strategy, we follow these steps:

Step 1: For each simulated sample, estimate a reduced form VAR(2), draw randomly from the VAR posterior and from the rotation matrix, compute the corresponding responses, and keep the draw if all the restrictions are satisfied. Compute the median of the responses, the posterior peak appreciation and excess return based on 1000 successful draws.

Step 2: Given the results for each sample, compute the all-samples response distribution
(median and 16-84th percentiles of the responses across all samples), the distribution (the 16-84 percent percentiles) of the sample median responses and respective statistics related with peak appreciation and excess return.

Figure 6 presents the impulse responses for the case the data are simulated from M1 and Figure 7 presents the results corresponding to the DGP from M2. As one might expect, given sampling uncertainty, the confidence bands (light blue area) show higher uncertainty about the responses of the variables to the identified monetary policy shock, for all identification strategies. Particularly, increased uncertainty is quite evident for responses under the Cholesky identification.

![Figure 6: DGP with standard UIP: IRFs to a monetary policy shock identified with SVARs](image)

Notes: The solid line in black denotes the pointwise median of impulse responses from all samples. The shaded area in light blue indicates the 16th and 84th percentile region. The deep blue area corresponds to the middle 68 percent distribution of the pointwise median responses from all samples. The line in red denotes the true impulse response from the DSGE model. A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.

With a large sample, the median responses were precisely estimated under Cholesky while on small sample there is a lot of uncertainty around the estimates. This suggests that Cholesky identification is quite sensitive to sampling uncertainty compared to sign restriction identification,
which is surprising since the latter approach contains identification uncertainty as well\textsuperscript{14}. The distribution of sample medians are almost as wide as the all-sample distribution which means that bulk of the estimation uncertainty stems from the choice (or realization) of the particular sample, not from the uncertainty within one sample. Put differently, when we estimate the impulse responses on a small sample, the estimates will be unreliable even if we have narrow confidence bands.

Figure 7: DGP with modified UIP: IRFs to a monetary policy shock identified with SVARs

Notes: The solid line in black denotes the pointwise median of impulse responses from all samples. The shaded area in light blue indicates the 16th and 84th percentile region. The deep blue area corresponds to the middle 68 percent distribution of the pointwise median responses from all samples. The line in red denotes the true impulse response from the DSGE model. A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.

Qualitatively, the results are comparable with the long sample results. The “price puzzle” is present for identification schemes with zero restrictions. The true DSGE responses are either contained within the uncertainty bands or lie close to their bounds. The Cholesky identification is particularly uncertain in the case when there is delayed overshooting in the data generating process, as except from the interest rate, the distributions lie quite symmetrically around the horizontal axis. This uncertainty is reflected even in the distribution of the peak appreciation of the exchange

\textsuperscript{14}Canova and Paustian (2010) show as well that sample uncertainty is small to identification uncertainty
rate. In the case of no delayed overshooting about 80 percent of peak appreciations with Cholesky appear in the first and second quarter while in the other case the distribution is dispersed between the first and the tenth quarter.

On the other hand, the results from small sample does not seem to alter significantly from the long sample results for the structural VARs using combined zero - sign restricted identification and pure sign restrictions (panel b and c in Figures 6 and 7). With respect to the exchange rate, respectively, about 90 and 70 percent of the sample median peak appreciations correspond to the first quarter (Figure 8, (b) and (c)) in case of the DGP without delayed overshooting. In the other case, the distribution of the peaks falls between the third and the fifth quarter. When compared with the true distribution and with the long sample results we observe that in short sample the distribution moves away from the true value. In case the delayed overshooting is not an effect of the monetary policy shock, the SVARs suggest a delayed peak response (up to one quarter after the shock) and in case delayed overshooting is there, the SVARs suggest a less delayed one.

Figure 8: Peak appreciation of exchange rate to a monetary policy shock

Figure 9 reports the estimates of the excess return from short sample VARs. Increased uncertainty around response functions is translated in excess returns not different from zero in the first panel, for all the identification schemes. In the second panel, hybrid and pure sign restrictions predict overestimated positive excess returns following a monetary policy shock.

The MC exercise under the controlled experiment shows that Cholesky identification, given sampling uncertainty, may produce misleading results for the response of the exchange rate. In contrast, when estimation uncertainty is not present, the delivered results are in line with what
the DGP would suggest. Still in reality, the econometrician would not be able to avoid estimation uncertainty given the available length of the series. Nevertheless, there are examples in the recent literature showing that employing estimation methods that allow to use more information than the standard VAR approach (while still using a Cholesky recursive identification strategy to identify a monetary policy shock), do not find a delayed overshooting or if so, a short delay, up to two quarters. For example Barigozzi et al (2010) find no delayed overshooting when estimating a Structural Dynamic Factor Model on a panel of euro area data. For a panel of industrial countries Binder et al. (2010) find a delayed response of about three months when using a model that allows for simultaneous multi-country adjustments (Global VAR setup) in response to a monetary policy shock. If not allowing for such adjustments, the identification with Cholesky would suggest a delayed response of exchange rate for about 20 months.

2.3 VAR with actual data - Sweden

In the following we investigate what do the above identification schemes suggest for the monetary policy shock and its effects when using real data for a small open economy. We perform this investigation on data for Sweden as the SVAR analysis above was based on a DSGE model estimated for this economy. To this purpose we estimate identified VARs using quarterly data for the period

---

15Australia, Canada, France, Germany, Italy, Japan, New Zealand, United Kingdom and the United States
1993 - 2010, with $z^{\text{Sweden}}_t = (GDP_t, CPI_t, R_t, S_t)$ being the vector of endogenous variables and $z^*_t = (GDP^*_t, CPI^*_t, R^*_t)$ the block of exogenous variables. We have chosen this period to avoid the break in monetary policy regime that Sweden experienced in the beginning of the year 1993. In this year Sweden switched from a fixed exchange rate regime to an inflation targeting regime. The choice of the variables is related with the idea of a small open economy where the foreign economy is given exogenously. Regarding the foreign economy, in line with the data used for estimation in Adolfson et al. (2008), we use foreign weighted variables across Sweden’s largest trading partners. As variable for the exchange rate for Sweden, $S_t$, we use the Effective Total Competitiveness Weights nominal exchange rate.

![Figure 10: Sweden: Impulse Responses to a monetary policy shock](image)

**Notes**: The solid line in black denotes median impulse response from the estimated VAR. The shaded area indicates the 16th and 84th percentile response from the estimated VAR. A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.

Figure 10 shows the impulse responses of output, consumer prices, interest rates and nominal effective exchange rate to a monetary policy shock identified with Cholesky recursive, hybrid restrictions and pure sign restrictions, respectively. In general, for all the identification strategies, we observe a muted response of output and in the case of sign restrictions this response is even

\[16\] All variables are in logs except for interest rates. We provide a more detailed description of the data we use in the Appendix B.
not statistically different from zero. The response of prices exhibits the “price puzzle” as with the simulated data under identifications with zero restrictions. Exchange rate seems to be the variable that reacts more persistently to the monetary policy shock identified with any of the identification schemes used. Under all of them, a monetary contraction is followed by an immediate appreciation of the exchange rate. Still, under Cholesky the peak appreciation appears to be dispersed between the first and the third quarter while the sign restriction identification suggests a peak response of the exchange rate in the second quarter. All identifications suggest an initial positive excess return, lasting for about a year with Cholesky and about half less under the other identifications.

When compared with the results from the controlled experiment, the responses from the real data fall in-between the results from SVARs with M1 and M2 short sample data. SVARs with real data suggest a very short lived delayed overshooting, up to the second quarter. Still, from the small sample analysis with simulated data we saw that the distribution of the peak response tends to move away from the true value, either suggesting a short delayed response when there is no delay in DGP or a less delayed response than the true response. Estimates on actual data seem to be closer to the small sample simulation results with no delayed overshooting, casting some doubt on the results from Adolfson et al. (2008) which suggest a delayed overshooting of 7 quarters for the Swedish economy. As it was the case with the simulated data, the result regarding the response of the exchange rate is robust to all three identification schemes used.

![Figure 11: Posterior distribution of peak response of exchange rate to a monetary policy shock](image1.png)

![Figure 12: Predictable Excess Return for Sweden](image2.png)

**Notes:** The solid line in black denotes predictable excess return from the estimated VAR, given a monetary policy shock. The shaded area indicates the 16th and 84th percentile PER from the estimated VAR.
Interestingly, a careful look at the literature suggests that the findings\textsuperscript{17} regarding the response of the exchange rate to a monetary policy shock, at least for the case of Sweden, appear robust to different specifications of the VAR, the choice of variables and to different identifications schemes. Adolfson, Andersson, Lassen, Linde, and Villani (2006) show that a contractionary monetary policy shock leads to a delayed overshooting of the real exchange rate for at least four quarters for Sweden. This result comes from the estimation of a Bayesian VAR(4) with a Cholesky recursive identification (with GDP growth, CPI inflation, nominal interest rate and real exchange rate as endogenous variables). Bjornland (2009) investigates this issue for a number of countries like Australia, Canada, New Zealand and Sweden by employing an identification strategy that takes in account the neutrality of monetary policy in real variables in the long run while allowing for a contemporaneous relationship between interests rates and real exchange rates. Results for Sweden show a delayed appreciation up to the third quarter. Bjornland and Halvorsen (2010) provide evidence for a delayed overshooting until the second quarter, both under a Cholesky and a Cholesky - sign identification\textsuperscript{18}.

3 Conclusion

Recent work in the literature suggests that delayed overshooting puzzle or other non-sense results are due to improper identification strategies used in VAR analyses. In this paper, we investigate to what extent various identification restrictions in structural VARs are able to recover the true effect of monetary policy on the exchange rate under a controlled experiment. We estimate VAR models using two sets of artificial observations from an open economy DSGE model of Adolfson et al. (2008) and identify monetary policy shocks by Cholesky ordering, by a combination of zero and sign restrictions and by sign restrictions only. With respect to our data generating process, the Cholesky decomposition imposes improper restrictions while sign-identified VARs allow for model-consistent sign restrictions.

Our results show that when data are abundant, Cholesky type restrictions, although not successful in recovering the true monetary shock, do not produce a delayed overshooting puzzle in case the data generating process does not contain it. For the case where the delayed overshooting is a feature of the true model, all the identification schemes under investigation recover it correctly. This result is in contrast to what the recent literature seemed to converge, assigning the delayed overshooting puzzle to improper restrictions in the Cholesky identification. However, all identifica-

\textsuperscript{17}Judging by the shape of the impulse responses only.

\textsuperscript{18}Under this identification, the recursive identification as in Cholesky is combined with sign restriction on the reaction of the exchange rate to a monetary policy shock. This identification allows a contemporaneous reaction between interest rates and real exchange rate, similarly to ours.
tion strategies employed in this experiment showed to be misleading when faced with sampling uncertainty. Cholesky-identified estimates appeared to be more affected to this uncertainty.

To check for the generality of the results we performed the same experiment using two other small open economy DSGE models as data generating processes, namely a DSGE model estimated for Chile as in Medina and Soto (2007) and a DSGE model estimated for Australia as in Jaaskela and Jennings (2011). Results\(^{19}\) from identified VARs using simulated data from these models are consistent with what we discussed above. Furthermore, we test the same identification strategies with real data from the Swedish economy. Estimates on actual data seem to be closer to the small sample simulation results with no delayed overshooting from Adolfson et al. (2008). As it was the case with the simulated data, the result regarding the response of the exchange rate is robust to all three identification schemes used.

Overall, we believe our exercise adds new contribution to the empirical literature of exchange rate dynamics and the exchange rate channel of monetary transmission mechanism. It provides a test on the claims of recent literature that delayed overshooting of the exchange rate is an outcome of improper identification restrictions in VARs. For econometricians it provides more insight into the performance of different identification strategies. Results suggest that improper restrictions set with the recursive identification scheme may be, but are not necessarily the source for puzzling outcomes. Nevertheless, given estimation uncertainty, identification of monetary policy shocks with Cholesky decomposition should be avoided. This paper provides another evidence in favor of the Dornbusch overshooting model and calls for caution for researchers working with DSGE models and trying to incorporate delayed overshooting as an “empirical fact”.

References


\(^{19}\)A short description for each of these models and corresponding results are shown in the Appendix C and D.


A The model as DGP: Adolfson et al. (2008)

The domestic economy in Adolfson et al. (2008) is comprised of households, four types of firms working on the domestic and the external sector, a government and a monetary authority. The foreign economy is modeled exogenously, resembling the typical situation of a small open economy with no power in the world market. In the following we present a description of the main characteristics, decisions and constraints of each of the agents in this economy for the version of the model with standard UIP.

A.1 Households

A continuum of households consumes domestic and imported goods, own capital and financial assets in the form of domestic deposits, foreign bonds and cash balances. A representative household $j$, maximizes the following utility function:

$$
E_0^j \sum_{t=0}^{\infty} \beta^t \left[ \zeta^c_t (C_{j,t} - bC_{j,t-1}) - \zeta^h_t A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} + A_q \left( \frac{Q_{j,t}}{z_{t-1}} \right)^{1-\sigma_q} \right]
$$

(A.1)

subject to the budget constraint given by:

$$
M_{j,t+1} + S_t B_{j,t+1}^* + P_t C_{j,t+1}(1+\tau^c_t) + P_t^d I_{j,t} + P_t K_{j,t}
= R_{t-1}(M_{j,t} - Q_{j,t}) + Q_{j,t} + (1-\tau^h_t)\Pi_t + (1-\tau^p_t)\left( \frac{W_{j,t}}{1+\tau^w_t} h_{j,t} \right)
+ (1-\hat{\tau}^k_t) R_t^k K_{j,t}^* + R_{t-1}^* \Phi \left( \frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1} \right) S_t B_{j,t}^*
+ \left[ (R_{t-1} - 1)(M_{j,t} - Q_{j,t}) + \left( R_{t-1}^* \Phi \left( \frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1} \right) - 1 \right) S_t B_{j,t}^* + B_{j,t}^*(S_t - S_{t-1}) \right]
+ TR_t + D_{j,t}
$$

(A.2)

with $C_{j,t}$, $h_{j,t}$ and $Q_{j,t}$ being the level of consumption, the work effort and real balances for the household $j$, respectively. Consumption and work efforts are subject to consumption preference shocks, $\zeta^c_t$ and labor supply preference shocks, $\zeta^h_t$. Households supply differentiated labor service and set their wage, subject to $\hat{\tau}$ la Calvo rigidity. Under such rigidity, households index their wages to past CPI inflation, current inflation target and to technology growth. Households earn $R_{t-1}$ interest rate on domestic deposits and a risk-adjusted pre-tax gross rate interest, $R_{t-1}^* \Phi \left( \frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1} \right)$, in foreign bond holdings.
Aggregate consumption and total investment are given by a CES index of domestically produced and imported goods, as below:

\[ C_t = \left[ (1 - \omega_c)^{1/\eta_c} (C^d_t)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c} (C^m_t)^{(\eta_c-1)/\eta_c} \right]^{\eta_c/\eta_c - 1} \]  
(A.3)

\[ I_t = \left[ (1 - \omega_i)^{1/\eta_i} (I^d_t)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i} (I^m_t)^{(\eta_i-1)/\eta_i} \right]^{\eta_i/\eta_i - 1} \]  
(A.4)

where \( C^d_t, C^m_t \) and \( I^d_t, I^m_t \) are domestic and imported consumption and investment goods, respectively. The share of imports on consumption and investment is given by \( \omega_c \) and \( \omega_i \), while \( \eta_c, \eta_i \) represent the elasticity of substitution between domestic and foreign goods.

Households own physical capital, \( K_t \), and decide how much of it to rent to the domestic firms given costs of adjusting the investment rate, \( \bar{S}(I_t, I_{t-1}) \). The law of motion for \( K_t \) is given by:

\[ K_{t+1} = (1 - \delta)K_t + \Upsilon_t \left( 1 - \bar{S}(I_t, I_{t-1}) \right) \]  
(A.5)

with \( \Upsilon_t \) being a stationary investment-specific technology shock.

Each household solves its maximization problem with respect to consumption, cash holdings, physical capital, investments and foreign bond holdings. Combining the f.o.c for cash and for foreign bond holdings (after log-linearization) we get the following uncovered interest parity condition:

\[ \hat{\hat{R}}_t - \hat{\hat{R}}^*_t = E_t \Delta \hat{S}_{t+1} - \tilde{\phi}_a \hat{a}_t + \tilde{\phi}_t \]  
(A.6)

A.2 Firms

There are four different firms: domestic goods firms, importing consumption, importing investment, and exporting firms. They all produce a differentiated good and have monopolistic power when setting prices. Price setting is subject to à la Calvo rigidities. The four final goods are each a CES (constant elasticity of substitution) composite of respective differentiated goods. For the final domestic good, the CES composite would be as follows:

\[ Y_t = \left[ \int_0^1 (Y_{i,t})^{1/\lambda^d_i} \, di \right]^{\lambda^d_i}, \quad 1 \leq \lambda^d_i < \infty, \]  
(A.7)
with \( \lambda^d_t \) being the time-varying flexible-price markup shock. The demand for firm \( i \)'s differentiated product follows:

\[
Y_{i,t} = \left( \frac{P_{d,i,t}}{P_{d,i,t}} \right)^{-\lambda^d_t} Y_t
\]

(A.8)

Domestic goods firms produce their differentiated goods using capital and labor inputs under the following technology:

\[
Y_{i,t} = z_{t}^{1-\alpha} \epsilon_{t} K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} - z_{t} \phi
\]

(A.9)

where \( K_{i,t} \) is the capital stock, \( H_{i,t} \) the homogeneous labor hired by the \( i \)th firm, \( z_{t} \) a unit-root technology shock common to the domestic and foreign economies and \( \epsilon_{t} \) is a domestic covariance stationary technology shock. Cost minimization for an intermediate firm \( i \) is given by:

\[
MC_{d,t} = \frac{1}{(1-\alpha)} \frac{1}{\alpha} \left( R_{t}^{k} \right)^{\alpha} \left[ W_{t} (1 + \nu (R_{t-1} - 1)) \right]^{1-\alpha} \frac{1}{(z_{t})^{1-\alpha}} \epsilon_{t}
\]

(A.10)

where \( R_{t}^{k} \) is gross nominal rental rate of capital, \( R_{t-1} \) the gross nominal interest rate, and \( \nu \) the fraction of the wage bill that a firm \( i \) finances in advance through loans to financial intermediary.

The price setting problem is the same for all the four firms, resulting in four specific Phillips curve equations determining inflation for domestic consumption, import consumption, import investment and export sectors. As an example we give the price setting decision of the domestic goods firms. Each of these firms is subject to price stickiness a la Calvo where with a probability, \( (1 - \xi_{d}) \), the firm can reoptimize its price, \( P_{t}^{d,new} \), in any period. Firms that cannot reoptimize every period, index the new price to the current inflation target and to last period’s inflation rate. Optimization problem of an individual firm is then:

\[
\max_{P_{t}^{d,new} \in \mathbb{W}_{t}} E_{t} \sum_{s=0}^{\infty} (\beta \xi_{d})^{s} v_{t+s} \left( (\pi_{t} \pi_{t+1} \cdots \pi_{t+s-1}) \frac{\alpha}{\alpha} (\pi_{t+1} \pi_{t+2} \cdots \pi_{t+s})^{1-\alpha} P_{t}^{d,new} \right) Y_{i,t+s} - MC_{d,t}^{i,t+s} Y_{i,t+s} + z_{t} \Delta_{t} \phi_{t}
\]

The log-linearized Phillips curve for the domestic good producing firm is:
\[
(\hat{\pi}_t^d - \hat{\pi}_t^e) = \frac{\beta}{1 + \kappa_d \beta} (E_t \hat{\pi}_{t+1}^d - \rho_e \hat{\pi}_t^e) + \frac{\kappa_d}{1 + \kappa_d \beta} (\hat{\pi}_{t-1}^d - \hat{\pi}_t^e) \\
- \frac{\kappa_d (1 - \rho_e) \hat{\pi}_t^e}{1 + \kappa_d \beta} + \frac{(1 - \xi_d) (1 - \beta \xi_d)}{\xi_d (1 + \beta \kappa_d)} (m^{\kappa_d} - \hat{\lambda}_t^{\kappa_d})
\]

(A.12)

Firms that produce differentiated importing consumption and investment goods, use a brand naming technology to convert the homogeneous goods bought in the world market at price \(P_t^*\), with marginal cost \(S_t\). The exporting goods firms use this technology to convert homogeneous domestic goods in differentiated goods to sell in the world market, with marginal cost \(P_t^d/S_t\). Since these firms face nominal rigidities a la Calvo when setting prices in local currency, a short run incomplete exchange rate pass-through is induced.

A.3 Monetary Policy

Central bank is assumed to follow an instrument rule, where short term interest rate is adjusted in response to deviations of CPI inflation from the time-varying inflation target, the output gap (measured as actual minus trend output), the real exchange rate, \(\hat{x}_t\), and the interest rate set in the previous period. In log-linearized form, the rule is given as:

\[
\hat{R}_t = \rho_{R,t} \hat{R}_{t-1} + (1 - \rho_{R,t}) \left[ \hat{\pi}_t^e + r_{\pi,t} (\hat{\pi}_{t-1}^e - \hat{\pi}_t^e) + r_{\pi,t} \hat{y}_{t-1} + r_{x,t} \hat{x}_{t-1} \right] + r_{\Delta \pi,t} \Delta \hat{\pi}_t^e + r_{\Delta y,t} \Delta \hat{y}_t + \epsilon_R
\]

(A.13)

The government consumes domestic good, collects taxes from households and transfers back to them any surplus or fiscal deficit. The government together with the foreign economy are estimated ahead of the DSGE model with identified VAR models and then exogenously put in the the DSGE model.

In equilibrium the following constraints should hold for clearance in the final goods market, the foreign bond market, and the loan market for working capital:

\[
C_t^d + I_t^d + G_t + C_t^e + I_t^e \leq z_t^{1-\alpha} \epsilon_t \alpha K_t^\alpha H_t^{1-\alpha} - z_t \phi
\]

(A.14)

\[
S_t B_t^* = S_t P_t^s (C_t^e + I_t^e) - S_t P_t^s (C_t^m + I_t^m) + R_{t-1}^s \Phi(a_{t-1}, \tilde{\phi}_{t-1}) S_t B_t^*
\]

(A.15)

\[
\nu W_t H_t = \mu_t M_t - Q_t
\]

(A.16)

Since the economy in this model (both versions) is subject to a unit root technology shock, \(z_t\), the model is made stationary by scaling the real quantities with it. The model is log-linearized
around a constant steady state. Both model M1 and M2 are estimated with Bayesian techniques using Swedish data for the period 1980Q1-2004Q4 using a vector of 15 observables.

A.4 Prior and Posterior distributions as in Adolfson et al. (2008)

Table A.1: Prior and Posterior distributions as in Adolfson et al. (2008)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior type</th>
<th>Mean</th>
<th>Standard UIp mean</th>
<th>Standard Modified UIp mean</th>
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<tr>
<td>Calvo domestic prices</td>
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<tr>
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<tr>
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<tr>
<td>Inv. Specif. Tech. Shock</td>
<td>$\rho_{yt}$</td>
<td>beta</td>
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<tr>
<td>Asymm. Tech. Shock</td>
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<td>invgamma</td>
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<tr>
<td>Unit root tech. Shock</td>
<td>$\rho_{ch}$</td>
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<td>0.63</td>
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<tr>
<td>Labor supp. Shock</td>
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<td>beta</td>
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<tr>
<td>Risk premium shock</td>
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<td>0.85</td>
<td>0.93</td>
</tr>
<tr>
<td>Unit root tech. Shock</td>
<td>$\rho_{zt}$</td>
<td>beta</td>
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<td>0.13</td>
</tr>
<tr>
<td>Stationary tech. Shock</td>
<td>$\mu_{zt}$</td>
<td>beta</td>
<td>0.85</td>
<td>0.63</td>
</tr>
<tr>
<td>Inv. Specif. Tech. Shock</td>
<td>$\sigma_{yt}$</td>
<td>invgamma</td>
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<td>0.13</td>
</tr>
<tr>
<td>Asymm. Tech. Shock</td>
<td>$\sigma_{yt}$</td>
<td>invgamma</td>
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<td>$\rho_{ch}$</td>
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<td>0.63</td>
</tr>
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<td>0.25</td>
</tr>
<tr>
<td>Risk premium shock</td>
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</tr>
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<td>0.77</td>
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<td>Imp. cons. Markup shock</td>
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<td>Imp. inv. Markup shock</td>
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<td>Inflation response</td>
<td>$\bar{\sigma}_{x}$</td>
<td>truncnormal</td>
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<td>1.71</td>
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<tr>
<td>Output response</td>
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<td>normal</td>
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<td>0.11</td>
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<td>Diff. Output response</td>
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<td>normal</td>
<td>0.06</td>
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<tr>
<td>Monetary policy shock</td>
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<td>$\sigma_{\sigma}$</td>
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<td>0.08</td>
</tr>
</tbody>
</table>

Log Marginal Likelihood: -2268.33 -2252.37
B Empirical analysis

B.1 Data description for Sweden

The main source of the data is the OECD database and when not, it is stated along the variable.

$GDP_t$ - Gross Domestic Product, chained volume estimates, s.a. with X11 ARIMA.

$CPI_t$ - Consumer Price Index, 2005=100.

$R_t$ - Three month interbank rate, per cent per annum.

$S_t$ - Nominal effective exchange rate (TCW), 1992 = 100. Source: Riksbank.

$GDP_t^*$ - Foreign GDP (TCW). Source: OECD and authors calculations.

$R_t^*$ - Foreign interest rate (TCW). OECD and authors calculations.

$CPI_t^*$ - Foreign Consumer Price Index (TCW). OECD and authors calculations.

TCW - Total Competitiveness Weights index, a set of trade weights for 20’s largest trade partners of Sweden: IMF.

B.2 Zero and Sign restriction methodology

We follow Uhlig (2005) and identify structural shocks based on sign restrictions with a Bayesian re-estimation algorithm. This algorithm works as follows. First, if $S_1$ and $S_2$ are two decomposition of $\sum \epsilon$, then there is an orthogonal matrix $O$ such that $S_2 = S_1 O$. We then make random draws from VAR posterior and from uniform distribution for $O$, calculate the corresponding responses, and keep the draw if all the sign restrictions are satisfied. We repeat these procedure until we reach 1000 successful draws. This will give us a random sample of responses, from which statistics, such as mean, median and percentiles can be calculated. We present the median and 16-84 percentiles of the responses, based on the successful draws.

We generate random orthogonal matrices following Reppa (2009). Given a random draw from the distribution of the covariance matrix, $\sum \epsilon$, and a random orthogonal matrix $O$, the identification matrix $S$ can be calculated as $S = PO$, where $P$ is the Cholesky factor of $\sum \epsilon$. Random orthogonal matrices are generated from the QR-decomposition of a matrix, whose elements are independent standard normal variables. QR-decomposition is efficiently implemented in MATLAB, and it can easily be extended to the case of zero restrictions. However, imposing zero restriction on some responses effectively decreases the dimensionality of the possible decompositions of $\sum \epsilon$, and this should be taken into account when generating the random matrices $O$. A straightforward algorithm to produce an orthogonal matrix $O$ that satisfies the zero restrictions is the following: calculate the $j$th column of $O$ as the normalized residual of the linear projection of the $j$th column of $X$ onto
the previous columns of O and the corresponding rows of P.

Let us denote by $P_j$ the rows of P, orthogonal to shock $j$. Next, define:

$$A_j = \begin{pmatrix} P_j' & O_{j-1} \end{pmatrix} \in \mathbb{R}^{n \times (k_j+j-1)}$$  \hspace{1cm} (B.1)

where $O_{j-1}$ is the first $j-1$ columns of O. The residual from projecting $X_{.,j}$, the $j^{th}$ column of X on the columns of $A_j$ is given by:

$$r_j = X_{.,j} - A_j (A_j' A_j) A_j' X_{.,j}$$  \hspace{1cm} (B.2)

and the $j^{th}$ column of O will be $r_j / \|r_j\|$. 

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C  The model as DGP: Medina and Soto (2007)

The model in Medina and Soto (2007) is a small-open economy DSGE model estimated using Chilean quarterly data for the period 1987:1-2005:4. This model represents a new Keynesian small open economy model in the tradition of Christiano et al. (2005), Altig, Christiano, Eichenbaum, and Linde (2003) and Smets and Wouters (2003). While it shares common features to them like nominal rigidities in wages and prices, habit formation in consumption, investment adjustment cost and incomplete exchange rate pass-through in short run, the production sector is extended to an intermediate tradable sector and a commodity-exporting sector. The model in Medina and Soto (2007) assumes a standard UIP condition. Monetary policy is given as a Taylor type rule, with the nominal interest rate responding to the deviation of core CPI inflation from target and to output gap.

Figure C.1: Chile, long sample: IRFs to a monetary policy shock

Notes: The solid line in black denotes median impulse response from the estimated VAR using data simulated from the model in Medina and Soto (2007). The dotted lines indicate the 16th and 84th percentile response from the estimated VAR. The red line indicate the true DSGE responses to the monetary policy shock. A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.
Figure C.2: Chile, MC exercise: IRFs to a monetary policy shock

Notes: The solid line in black denotes median impulse response from the estimated VAR using data simulated from the model in Medina and Soto (2007). The dotted lines indicate the 16th and 84th percentile response from the estimated VAR. The red line indicate the true DSGE responses to the monetary policy shock. A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.

Figure C.3: Chile: Posterior distribution of peak response of exchange rate to a monetary policy shock

Notes: Long sample results in (a, b, c) and MC exercise results in (d, e, f).
Notes: The solid line in black denotes predictable excess return from the estimated VAR, given a monetary policy shock. The dotted lines indicate the 16th and 84th percentile PER from the estimated VAR. The line in red denotes the true PER from the estimated DSGE model. Horizontal axis is lag horizon in quarters. Long sample results in (a, b, c) and MC exercise results in (d, e, f).

D The model as DGP: Jaaskela and Jennings (2011)

The model in Jaaskela and Jennings (2011) is a small-open economy DSGE model estimated using Australian quarterly data for the period 1984:Q1-2009:Q4. This model represents a new Keynesian small open economy model in the tradition of Gali and Monacelli (2005). The model in Jaaskela and Jennings (2011) assumes a standard UIP condition. Monetary policy is given as a Taylor type rule, with the nominal interest rate responding to the deviation of CPI inflation from target and to output gap.
Figure D.1: Australia, long sample: IRFs to a monetary policy shock

Notes: The solid line in black denotes median impulse response from the estimated VAR using data simulated from the model in Jaaskela and Jennings (2011). The dotted lines indicate the 16th and 84th percentile response from the estimated VAR. The red line indicate the true DSGE responses to the monetary policy shock. A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.
Figure D.2: Australia, small sample: IRFs to a monetary policy shock

Notes: The solid line in black denotes median impulse response from the estimated VAR using data simulated from the model in Jaaskela and Jennings (2011). The dotted lines indicate the 16th and 84th percentile response from the estimated VAR. The red line indicate the true DSGE responses to the monetary policy shock. A decrease in exchange rate implies appreciation. Horizontal axis is lag horizon in quarters.

Figure D.3: Australia: Posterior distribution of peak response of exchange rate to a monetary policy shock

Notes: Long sample results in (a, b, c) and MC exercise results in (d, e, f).
Figure D.4: Australia: Predictable Excess Return

Notes: The solid line in black denotes predictable excess return from the estimated VAR, given a monetary policy shock. The dotted lines indicate the 16th and 84th percentile PER from the estimated VAR. The line in red denotes the true PER from the estimated DSGE model. Horizontal axis is lag horizon in quarters. Long sample results in (a, b, c) and MC exercise results in (d, e, f).