The Term Structure of the Welfare Cost of Uncertainty

Pierlauro Lopez1,*

University of Lugano and Universitat Pompeu Fabra

Abstract

The marginal cost of consumption fluctuations has a term structure that is a simple transformation of the term structures of equity and interest rates. I use recent evidence extracted from index option markets to infer a downward-sloping and volatile term structure of welfare costs. On average, cashflow stability is a macroeconomic priority and short-run stability is a greater priority than long-run stability. I find that at the margin the elimination of one-year ahead cashflow volatility is worth 15 percentage points of additional growth. This number compares to a marginal cost of all consumption fluctuations of about 2 percentage points. Over time, the term structure of welfare costs varies substantially. Return predictors reveal the states that drive the term structure of welfare costs and thereby signal its current position and future developments. Finally, the link between welfare costs and risk premia can make the case for risk premia targeting as a welfare-enhancing policy regime.

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1. Introduction

How much consumption growth are people willing to trade against a marginal stabilization of the consumption stream? The marginal welfare cost of consumption fluctuations (Alvarez and Jermann, 2004) answers this important question in economics, which goes back at least to Lucas (1987). I decompose the marginal cost of uncertainty into a term structure. This decomposition allows for studying how cashflow uncertainty at different horizons contributes to the total cost of fluctuations (proposition 1). The term structure of welfare costs allows for understanding both the tradeoff between growth and macroeconomic stabilization, and the tradeoff between cashflow stabilization at different periodicities.

*Present address: Faculty of Economics, University of Lugano, via Giuseppe Buffi 13, CH-6900 Lugano, Switzerland. Phone: 0041 78 675 2363. Fax: 0041 58 666 4647.

Email address: lopexp@usi.ch; pierlaurolopez@gmail.com (Pierlauro Lopez)

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In line with the insight that asset market data reveal the marginal cost of fluctuations (Alvarez and Jermann, 2004), I then show how the components of the term structure of welfare costs are tightly linked to the risk premia on market dividend strips (proposition 2); the term structure of welfare costs is a simple transformation of the term structures of equity and interest rates (e.g., studied in Lettau and Wachter, 2007, 2011; Binsbergen, Brandt and Koijen, 2012a; Binsbergen, Hueskes, Koijen and Vrugt, 2012b). This link allows for a measure of the cost of fluctuations that is directly observable, at least over the last two decades. My approach requires only the absence of arbitrage opportunities and does not require the specification of consumer preferences.

A theoretically important relationship is the one between the cost of fluctuations and the equity premium, as both are measures of the premium people command to shoulder aggregate risk. I discuss conditions under which the cost of fluctuations equals the equity premium (propositions 3 and 4). The equality holds if the term structures of equity and interest rates are both flat. While these conditions represent a natural benchmark and would allow for a quantification of the cost of uncertainty over a large sample, they are restrictive, both empirically and theoretically, and must therefore be taken with caution. On the one hand, the long-run risk literature emphasizes how local alternatives, against which we have no statistical power, to the conditions granting the equality can be crucial to make sense of other asset pricing facts (Bansal and Yaron, 2004; Koijen, Lustig, Nieuwerburgh and Verdelhan, 2010). On the other hand, recent evidence about the term structures of equity and interest rates challenges the trivial term structure properties required for the equality between the cost of fluctuations and the equity premium (Binsbergen, Brandt and Koijen, 2012a; Binsbergen, Hueskes, Koijen and Vrugt, 2012b; Boguth, Carlson, Fisher and Simutin, 2012).

In the empirical section, I use evidence about the term structure of equity extracted from index option markets and about the term structure of interest rates to infer the cost of fluctuations as a function of their periodicity. I find that these costs are large, volatile and have non-trivial term structure features. The point estimates, reported in figure 1a, suggest a downward-sloping term structure of welfare costs; people command a larger premium to shoulder short-term cashflow uncertainty than to shoulder long-term uncertainty. The premium at one-year frequency is 15.2% and its volatility is at least half that number. It is natural to compare this number to the equity premium, which averages 2.7% over the last two decades and has a standard deviation of at least three fourths of that number.

This evidence of a downward-sloping term structure of welfare costs helps to identify a model to capture and quantify the entire term structure. I study the implications of today’s leading consumption-based asset pricing models for the term structures. I consider the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the long-run risk model under limited information of Croce, Lettau and Ludvigson (2012), the ambiguity averse multiplier preferences of Barillas, Hansen and Sargent (2009), and the rare disasters model of Gabaix (2012). Although these models do not study the term structure of welfare costs directly, they have implications for it and are calibrated to match many other asset pricing facts. Unfortunately, from a structural perspective, replicating a downward-sloping term structure of equity and an upward-sloping term structure of interest rates is problematic (Lettau and Wachter).

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2See Cochrane (2011) and Ludvigson (2012) for a survey of the main asset pricing facts and of the progress state-of-the-art asset pricing models have made in explaining them.
I therefore have to discard a structural explanation and turn to reduced-form models. In this regard, Lettau and Wachter (2011) offer a parsimonious model designed to capture a downward-sloping term structure of equity, an upward-sloping term structure of interest rates, and time-varying risk premia. The quantitative implications of the model, reported in figure 1b, are a marginal cost of lifetime fluctuations of about 2.4%—which compares to an equity premium of about 7%. Over a horizon of up to 10 years, a marginal increase in uncertainty costs more than 10 percentage points of annual growth per unit of uncertainty, as measured by the conditional standard deviation of consumption over the relevant horizon. These numbers compare to much smaller marginal benefits of long-run stabilization.

Thus, on average, cashflow stability is a macroeconomic priority, especially in the short run. Over time, the tradeoff among short-run, long-run stabilization and growth revealed by the term structure of welfare costs varies substantially, because excess returns are predictable (Cochrane, 2011). Consequently, return predictors reveal the state that drives the time-variation in the term structure of welfare costs and thereby signal the current and future macroeconomic priorities. For example, in the model of Lettau and Wachter (2011), the cost of fluctuations at different periodicities is driven by one state, the price of risk, which is perfectly revealed by price-dividend and consumption-dividend ratios and by the risk-free rate. In the model, positive discount-rate news signal an increase in the cost of short-run fluctuations that slowly decays across maturities and over time.

Finally, from a welfare perspective, the cost of fluctuations decreases as the mean and the volatility of the individual components of its term structure decrease. This result can make the case for a risk premia targeting regime as a welfare-enhancing policy. I discuss whether a policy-maker should target the cost of fluctuations at different periodicities. A regime that targets either the mean or the volatility of the term structure components, which I refer to as risk premia targeting, is unambiguously desirable provided it is neutral on the mean growth rate of consumption and on the level of any additional factors that affect utility (proposition 5).

1.1. Literature review

Lucas (1987) opens the literature on the welfare cost of fluctuations. The controversy around Lucas’s original treatment, surveyed in Lucas (2003) and Barlevy (2005), revolves around two points: the choice of preferences and the definition of the stable consumption stream the consumer is offered. Lucas (1987) chooses a log utility representative consumer model and defines the cycle in consumption as its deviations from a deterministic growth trend. He then calculates a cost of fluctuations of 0.01-0.05% of aggregate consumption—a small amount that would make policies pursuing macroeconomic stability a low priority.

The first issue with this calculation is that most likely we have to abandon the power utility choice. In fact, within the representative-consumer power utility model, the gains from stabilizing fluctuations are small for the same reason that the model predicts a small equity premium. Consumption is a fairly smooth series, so that a model with a low and constant risk aversion can only predict that the representative consumer does not fear the typical empirical fluctuations much. However, precisely this prediction makes the model controversial (which is just the equity premium puzzle of Mehra and Prescott [1985]). Financial markets suggest that the observed smoothness
of aggregate consumption is not enough to conclude that any further stabilization policy is a low priority. Therefore, a large literature studies the cost of fluctuations under different preferences and find model-based estimates that go from virtually zero to more than 20% \cite{Barlevy2005}. Against this background, \cite{Alvarez2004} represent a breakthrough, as they manage to move the game to a preference-free environment by showing how we can directly use asset market data to measure the cost of fluctuations at the margin.

The other issue is the definition of the stable consumption stream that is hypothetically offered to the representative consumer. \cite{Lucas1987}, along with the majority of the literature, defines it as the deterministic trend in consumption. \cite{Alvarez2004} additionally consider the stochastic trend in a particular decomposition of the consumption process.

\cite{Alvarez2004} provide two main lessons about the estimate of the cost of fluctuations. A model that is consistent with the observed equity premium and takes the deterministic trend in consumption as the stable stream increases \cite{Lucas1987}'s estimates by two orders of magnitude (e.g., \cite{Tallarini2000}). If you consider instead as the stable stream some stochastic trend in consumption the estimates can be much closer to \cite{Lucas1987}'s. They conclude that what people really dislike is the low-frequency volatility in consumption.

My approach builds on and complements the analysis of \cite{Alvarez2004}. On the one hand, I stick to their preference-free setting and link the cost of fluctuations to a richer set of financial market evidence. On the other, I take stable consumption to mean the stabilization of an arbitrary set of coordinates of the consumption process around their conditional expectation. The term structure of the cost of fluctuations answers the question ‘How much compensation do people command to bear n-year ahead cashflow uncertainty?’ This question compares to ‘How

much compensation do people command to bear business-cycle volatility in the entire cashflow process?", which is the one studied by \cite{Alvarez and Jermann}. Their answer depends a lot on the parametric assumptions about the moving-average filter that separates the trend and the business-cycle components of the cashflow process. The question I am interested in is nonparametric and complements the exercise of \cite{Alvarez and Jermann}.

Against this background, it is natural to compare my finding of a downward-sloping term structure of welfare costs to the conclusion of \cite{Alvarez and Jermann} that long-run fluctuations are the fluctuations that people fear the most. Our results are not necessarily inconsistent, because \cite{Alvarez and Jermann} focus on the volatility of the entire consumption process at a given spectral frequency, whereas I focus on the entire volatility in consumption at a given horizon.

Finally, I take a slight departure from the original definition and express the marginal benefits of stabilization in terms of extra cashflow growth, as opposed to a uniform increase in lifetime cashflows. Along with still measuring the cost of fluctuations, my notion of welfare cost measures the tradeoff between growth and consumption stabilization and is therefore of more direct interest to economic policy-makers facing tradeoffs between economic growth and macroeconomic stability.

2. The term structure of the cost of uncertainty

People live in a stochastic world, have finite resources and decide how to allocate them across time. Identical risk-averse consumers \(i \in [0, 1]\) have time-\(t\) preferences \(U_t = E_t U(C_t, X_t)\), where \(C \equiv \{C_{t+n}\}_{n=1}^{\infty}\) is consumption and \(X \equiv \{X_{t+n}\}_{n=1}^{\infty}\) is any other factor that influences utility. Without loss of generality, I let factor \(X_t\) depend on aggregate consumption \(C_t = \int_0^1 C_t \text{di}\) but not on individual consumption \(C_i\). Since there is a continuum of agents each of which has zero mass, this modelling strategy allows me to ask an individual how much he would pay in exchange for less cashflow uncertainty without thereby having to affect all aggregate quantities, including in particular factor \(X_t\).

Financial markets are without arbitrage opportunities and people can trade in the financial market the full set of zero-coupon bonds and the full set of single market dividend payments, so-called dividend strips.

I am interested in measuring the cost of consumption fluctuations, i.e., how much consumption growth a consumer is willing to trade against a stable consumption stream. Let \(\bar{C}_{t+n}\) denote the consumption level that is offered to the \(i\)th individual at time \(t+n\), which I refer to as stable consumption. Then, I parametrize stable consumption as \(\bar{C}_{t+n}(\theta) = \theta E_t C_{t+n} + (1 - \theta) C_{t+n}\), where \(\theta \in [0, 1]\) indexes a convex combination of ex-ante and ex-post consumption and represents the fraction of ex-post consumption uncertainty that is removed.

**Definition** (Marginal cost of uncertainty). In line with \cite{Alvarez and Jermann} (2004), I define the
cost of fluctuations as \( \{L_t\} \) in

\[
E_t U \left( \left( 1 + L_t^N (\theta)^n C_{t+n} \right)_{n \in \mathcal{N}}, \{C_{t+n}\}_{n \in \mathcal{N}}, \{X_{t+n}\}_{n=1}^{\infty} \right) = E_t U \left( \left( \theta E_t C_{t+n} + (1 - \theta) C_{t+n} \right)_{n \in \mathcal{N}}, \{C_{t+n}\}_{n \in \mathcal{N}}, \{X_{t+n}\}_{n=1}^{\infty} \right)
\]

(1)

where the index set \( \mathcal{N} \subset \mathbb{N} \equiv \{1, ..., \infty\} \) indicates which coordinates of consumption are stabilized and allows for focusing on any window of interest.

Two particularly interesting quantities are the total cost \( L_t^N (1) \), which measures how much extra growth the elimination of all cashflow uncertainty is worth, and the marginal cost \( L_t^N \equiv \left. \frac{\partial L_t^N (\theta)}{\partial \theta} \right|_{\theta=0} \), which measures the current assessment of how much extra growth a marginal stabilization is worth.

I assume enough smoothness in preferences to guarantee that \( L_t^N \) is a differentiable map on \( \theta \in [0, 1] \). Then, differentiating (1) with respect to \( \theta \), I find

\[
L_t^N = \frac{\sum_{n \in \mathcal{N}} E_t (M_{t,t+n}) E_t (C_{t+n}) - E_t (M_{t,t+n} C_{t+n})}{\sum_{n \in \mathcal{N}} n E_t (M_{t,t+n} C_{t+n})}
\]

(2)

where \( M_{t,t+n} = \frac{\partial U_t}{\partial C_{t+n}} \) is the \( n \)-period stochastic discount factor. Note how \( D_t^{(n)} = E_t M_{t,t+n} C_{t+n} \) is the no-arbitrage price of a \( n \)-period dividend strip, and \( E_t M_{t,t+n} \) is the no-arbitrage price of a \( n \)-period zero-coupon bond.

Under no-arbitrage, equation (2) expresses the marginal cost of uncertainty at all coordinates \( n \in \mathcal{N} \) as a function of the price of a claim to the trend in consumption and of the price and duration of a claim to the entire stream of future consumption—the market portfolio.

The numerator of (2) is the welfare cost derived by Alvarez and Jermann (2004). To separate the trend from the short-run component, Alvarez and Jermann construct a moving average of past consumption. Unsurprisingly, the price of the claim to the trend in cashflows depends a lot on the moving-average filter assumed. They find in fact that under a stochastic trend the cost of fluctuations is two orders of magnitude smaller than under a deterministic trend specification. Here I propose a different possibility, one that looks at the cost of fluctuations in a way that bypasses the need to specify the moving-average filter and instead focuses on the cost of fluctuations at different periodicities.

**Definition** (Term structure of the cost of uncertainty). Consider the singleton set \( \mathcal{N} = \{n\} \), for \( n = 1, 2, ..., \) and consider the marginal costs \( \ell_t^{(n)} = \left. \frac{\partial L_t^{(n)} (\theta)}{\partial \theta} \right|_{\theta=0} \). The marginal cost \( \ell_t^{(n)} \) is the marginal cost of uncertainty when only the \( n \)th coordinate of consumption is stabilized. Then, by equation (2), it follows that

\[
\ell_t^{(n)} = \frac{1}{n} \left( \frac{E_t (M_{t,t+n}) E_t (C_{t+n})}{E_t (M_{t,t+n} C_{t+n})} - 1 \right)
\]
The motivation for calling the map \( l_t : n \mapsto l_t^{(n)} \) a term structure of the marginal cost of uncertainty is given by proposition 1. Given the prices of dividend strips \( \{D^{(n)}_t\} \) and the term structure components \( \{l^{(n)}_t\} \) you can compute the marginal cost \( L^N_t \) for any coordinate set \( N \subset \mathbb{N} \).

**Proposition 1.** The marginal cost of uncertainty within any window of interest \( N \), \( L^N_t \), is the linear combination of the term structure components \( \{l^{(n)}_t\} \) defined by

\[
L^N_t = \sum_{n \in N} \omega_n l^{(n)}_t
\]

where the weights \( \omega_n \equiv \frac{n D^{(n)}_t}{\sum_{n \in N} D^{(n)}_t} \) are positive and such that \( \sum_{n \in N} \omega_n = 1 \).

Unconditionally, the relation is

\[
L^N_t = \sum_{n \in N} \omega_n l^{(n)}_t
\]

where \( \omega_n = \frac{n D^{(n)}_t}{\sum_{n \in N} D^{(n)}_t} \).

2.1. The term structures of equity, interest rates, and welfare costs

Proposition 2 shows how the term structure of welfare costs is a simple transformation of the term structures of equity and interest rates. The unconditional term structure of welfare costs depends both on the average term structure and on a measure of uncertainty in the term structure components—the entropy measure \( V(X) \equiv 2[\ln E(X) - E(\ln X)] \).

**Proposition 2.** The \( n \)th component of the term structure of welfare costs is the risk premium for holding to maturity a portfolio long on a \( n \)-period dividend strip and short on a \( n \)-period zero-coupon bond. The term structure of welfare costs is the transformation of the term structures of equity and interest rates

\[
l^{(n)}_t = \frac{1}{n} \ln E R^{e,(n)}_{t+n}
\]

\[
= \frac{1}{n} \ln E \exp \left( \sum_{j=1}^{n} r^{e,(j)}_{d,t+n-j+1} - r^{e,(j)}_{b,t+n-j+1} \right)
\]

\[
l^{(n)}_t = E(l^{(n)}_t) + \frac{1}{2n} V(\exp n l^{(n)}_t)
\]

where \( r^{e,(n)}_{d,t} \) and \( r^{e,(n)}_{b,t} \) are the excess returns on a \( n \)-period dividend strip and zero-coupon bond, respectively.

Disregarding second-order terms, the three term structures are linked by the recursion

\[
l^{(n)}_t = \frac{n-1}{n} l^{(n-1)}_t + \frac{1}{n} E(r^{e,(n)}_{d,t} - r^{e,(n)}_{b,t})
\]
with boundary condition \( f^{(0)} = 0 \).

Equation (7) provides the link between the term structures of equity and interest rates and the term structure of welfare costs; the cost of \( n \)-period ahead fluctuations is the average of the last \( n \) dividend strip premia relative to zero-coupon bonds with equal maturity. Because the entropy measure is always positive, equation (5) shows how \( l^{(n)}(t) \geq \frac{1}{n} E(r^{(n)}_{t+1}) \), i.e., the first-order approximation in equation (7) is a lower bound for the actual cost of \( n \)-period ahead fluctuations.

There is a powerful intuition behind these formulas. At the margin, people would trade \( l^{(n)}(t) \) points of growth against the elimination of a fraction \( \theta \) of the aggregate cashflow uncertainty around the \( n \)th consumption coordinate, \( C_{t+n} \). Proposition 2 shows how this tradeoff is precisely the one offered by the financial market. In fact, by holding to maturity a portfolio long on a \( n \)-period dividend strip and short on a \( n \)-period zero-coupon bond people can experience an average growth rate of \( \frac{1}{n} \ln E_t R^{(n)} e^{(n)}_{t+1} \) by shouldering a volatility of \( var(r^{(n)}_{t+1}) = var(c^{(n)}_{t+n}) \). Therefore, the cost of \( n \)-year ahead consumption uncertainty must be \( l^{(n)}(t) = \frac{1}{n} \ln E_t R^{(n)} e^{(n)}_{t+1} \).

2.2. Relationship between the cost of uncertainty and the equity premium

A theoretically important relationship is the one between the cost of uncertainty and the equity premium, as both are measures of the premium people command to shoulder aggregate risk.

For the entire term structure of the cost of uncertainty—i.e., for all \( L^N, N \subset \overline{N} \)—to equal the equity premium you need extreme conditions. Namely, we require flat term structures of equity and interest rates. In essence, the term structure of equity is flat if shocks to the cashflow opportunity set (e.g., shocks to expected cashflow growth and to cashflow volatility) are either absent or unpriced; the term structure of interest rates is flat in economies in which the state driving the risk-free rate is either absent or unpriced, so that the expectations hypothesis of bond valuation holds.

Under assumptions 1 and 2, we have the equality of the cost of uncertainty and the equity premium whenever \( E_t r^{(n)}_{d,t+1} = E_t r^{(n)}_{d,t+1} \).

Assumption 1. Let either \( E_t r^{(n)}_{d,t+1} \geq E_t r^{(1)}_{d,t+1} \) or \( E_t r^{(n)}_{d,t+1} \leq E_t r^{(1)}_{d,t+1} \), for all \( n > 1 \).

Assumption 2. The expectations hypothesis of bond valuation holds, \( E_t r^{(n)}_{d,t+1} = 0 \), for all \( n \).

Assumption 1, which describes a very weak monotonicity in the term structure, does not appear restrictive, as it holds in every model among today’s leading consumption-based asset pricing models. The expectations hypothesis in assumption 2 is problematic (Fama and Bliss [1987]; Campbell and Shiller [1991]; Piazzesi and Swanson [2008]) but it works reasonably well on average.

I thus study the relationship between the one-period welfare cost, \( l^{(1)}(t) \), and the equity premium. Proposition 3 links the one-period welfare cost and the equity premium and shows how their difference is controlled by the systematic risk in the market dividend yield. Proposition 4 under the

\[10\] Note that if the term structure of equity is flat, then it must be at the level of the equity premium, and if the term structure of interest rates is flat, then it must be flat at zero, since \( E_t r^{(1)}_{b,t+1} = 0 \).
weak structure about the stochastic discount factor in assumption\(^3\) studies conditions under which the two quantities are equal\(^{11}\).

**Proposition 3.** Let a representative agent, lognormal environment without arbitrage opportunities in the financial market. By proposition\(^2\) the cost at time \(t\) of fluctuations at time \(t+1\), \(f_t^{(1)}\), equals the premium on a strip that pays off aggregate consumption next period. Therefore, the distance between \(f_t^{(1)}\) and the equity premium equals

\[
\delta \text{cov}_t(m_{t+1}, dp_{t+1})
\]

where \(dp\) is the log dividend-price ratio of the market portfolio and \(1/\delta\) is the unconditional market return.

**Assumption 3.** Preferences \(U_t\) conform to the generic stochastic discount factor

\[
m_{t+1} = -\rho_t - \gamma_t \sum_{j=0}^{\infty} \delta_j (E_{t+1} - E_t) \Delta c_{t+j+1}
\]

with \(\delta_0 = 1\).

**Assumption 4.** Either the market price-dividend ratio is constant, or consumption is a random walk and news to consumption growth and to the price-dividend ratio are orthogonal.

**Proposition 4.** Let a representative agent, lognormal environment without arbitrage opportunities in the financial market. Let assumptions\(^1\),\(^2\) and\(^3\). Then, the distance between the one-period welfare cost and the equity premium is

\[
\text{cov}_t(m_{t+1}, dp_{t+1}) = -\gamma_t \sum_{j=0}^{\infty} \delta_j \text{cov}_t((E_{t+1} - E_t) \Delta c_{t+j+1}, dp_{t+1})
\]

Moreover, let assumption\(^4\). Then, the cost of uncertainty \(L^N_t\) equals the equity premium, for any coordinate set \(N \subset \overline{N}\).

To evaluate the restrictiveness of the conditions listed in proposition\(^4\), I study them in some of today’s leading consumption-based asset pricing models\(^{12}\). The online appendix works out the details of each model.

\(^{11}\)The stochastic discount factor in equation\(^8\) is fairly general. For example, it embeds the preferences studied by Mehra and Prescott\,(1985); Epstein and Zin\,(1989); Campbell and Cochrane\,(1999); Bansal and Yaron\,(2004); Hansen and Sargent\,(2005); Barillas, Hansen and Sargent\,(2009); Lettau and Wachter\,(2007, 2011).

\(^{12}\)Note that, from an empirical perspective, the conditions in assumption\(^4\) that grant the equality between the cost of fluctuations and the equity premium are consistent with some stylized facts. The online appendix shows how they are approximately true in the restricted information set made by the market price-dividend ratio, dividend growth and market returns (studied in depth by Cochrane\,(2008)). However, if the information set is not the true one, then the stylized facts supporting assumption\(^4\) are likely to break down; indeed, the term structure evidence in section\(^5\) suggests this fragility, because it rejects the equality between the cost of uncertainty and the equity premium.
Although they are not restrictive for some asset pricing models (notably, the models of Barillas et al., 2009 and Gabaix, 2012 in examples 2.4 and 2.5 predict flat term structures), the conditions listed in proposition 4 to grant the equality between welfare costs and the equity premium are particularly severe for the long-run risk explanation discussed in example 2.3. In fact, within a long-run risk setting, the random walk in consumption would imply that the main component of the long-run risk explanation is absent, while the unitary elasticity of substitution implies that the long-run risk component is not priced. Both features kill the mechanism that generates many interesting asset pricing facts within that framework. Conversely, we can read the results of the long-run risk literature as showing how small departures from the random walk in consumption or the unitary elasticity of substitution break down the equality between the cost of uncertainty and the equity premium, which would therefore rest on fragile grounds.

Example 2.1 (Log utility). Consider preferences captured by log utility, \( U_t = \ln(C_t) + \beta E_t U_{t+1} + 1 \). Under log utility, the market portfolio has the convenient property that the price-dividend ratio of the market portfolio is constant. Therefore, by proposition 4, the welfare cost of uncertainty equals the equity premium.

Example 2.2 (Campbell and Cochrane (1999) habits). In the habit formation model of Campbell and Cochrane (1999) the utility function \( U(C_t, X_t) = (C_t - X_t)^{1-\gamma} + \beta E_t U_{t+1} \), where \( X_t \) represents an external habit that is a nonlinear function of past consumption such that the log stochastic discount factor has shape \( m_{t+1} = -\rho_t - \gamma_t(E_{t+1} - E_t)\Delta c_{t+1} \). The nonlinearity is calibrated to ensure that the precautionary savings effect largely offsets the intertemporal substitution effect in the determination of the risk-free rate, thus avoiding the risk-free rate puzzle of Weil (1989). At the same time a large and time-varying risk-aversion coefficient, \( \gamma_t \), matches a high and volatile equity premium. Under these conditions, the distance between the one-period welfare cost and the equity premium is given by \( \gamma_t \text{cov}_t(\Delta c_{t+1}, dp_{t+1}) \).

Thus, under the further assumption that consumption growth and the price-dividend ratio are conditionally orthogonal, the distance (9) collapses to zero. Then, by proposition 4, the welfare cost of uncertainty equals the equity premium.  

Example 2.3 (Epstein and Zin (1989) preferences). Consider preferences as described by the non-expected utility \( U_t = (1-\beta)C_t^{1-\rho} + \beta E_t U_{t+1}^{1-\gamma} \), where \( E_t U_{t+1}^{1-\gamma} \) is the certainty equivalent of utility at time \( t+1 \) evaluated through the expected utility function \( v(x) = x^{1-\gamma} \). Parameter \( \rho \) is the intertemporal elasticity of substitution and \( \gamma \) controls the risk aversion. An example are the recursive preferences of Epstein and Zin (1989).

I follow Restoy and Weil (2011), who use a version of the loglinearized Campbell-Shiller identity and a lognormal no-arbitrage pricing framework with conditionally homoskedastic consumption growth to show that, up to first-order, the market price-dividend ratio and the stochastic discount factor are functions of future consumption growth. Then, if consumption growth follows the generic

\[ \text{The baseline model of Campbell and Cochrane cannot however replicate the orthogonality assumption, because it has only one structural shock; this simple shock structure imposes a perfect correlation between dividend growth and dividend yields.} \]
\[ \begin{align*}
\zeta_{t+1} &= A \zeta_t + B e_{t+1} \\
\Delta c_{t+1} &= \mu + C \zeta_t + D e_{t+1}
\end{align*} \]

where \( \zeta \) is a state vector and \( e \sim WN(0, I) \), then the distance between the cost of uncertainty and the equity premium is

\[ \delta \text{cov}_t(m_{t+1}, d p_{t+1}) = (1 - \rho) \left( \gamma D + (\gamma - \rho) \delta C[I - \delta A]^{-1}B \right) \left( \delta C[I - \delta A]^{-1}B \right)' \]

where \( 1/\delta \) is the steady-state value of the market return.

The price-dividend ratio is constant if the intertemporal elasticity of substitution, \( \rho \), is unity or if consumption is a random walk (\( C = 0 \)). In these two cases, the distance (9) collapses to zero. Then, by proposition 3, the welfare cost of uncertainty equals the equity premium.

**Example 2.4** (Barillas, Hansen and Sargent (2009) ambiguity averse multiplier preferences). Consider agents who have a wish for robustness against some misspecification in the transition equation for the states of the economy. I follow Hansen and Sargent (2005) in modelling the misspecification through a nonnegative martingale \( G_t \) that distorts the probability distribution \( P \) implied by the transition equation for the states of the economy. The stochastic process \( G_t \) in turn implies a factor \( g_{t+1} \), defined recursively as \( G_{t+1} = g_{t+1}G_t \) with \( G_0 = 1 \), that distorts the conditional transition probability measure. The ambiguity-averse agent then evaluates the objective function by drawing the worst-case scenario about the misspecification and penalizes the objective by a function of relative entropy, which is strictly greater than zero unless there are no distortions \( P \)-almost everywhere. I can write the objective function as

\[ V_0 = \min \{ g_{t+1} \} \sum_{t=0}^{\infty} \beta^t \left[ \ln(C_t) + \beta \theta E_t^P \left[ g_{t+1} \ln g_{t+1} \right] \right] \]

where parameter \( \theta \) represents the agent’s aversion to model misspecification and where \( Q \) represents the distorted probability measure, whose Radon-Nikodym derivative with respect to probability measure \( P \) is \( G_t \). It follows that the optimized function \( g_{t+1} \) is the Esscher transform of probability measure \( P \), which implies the optimized value function

\[ V_t = \ln(C_t) - \beta \theta \ln E_t^P \exp \left\{ - \frac{V_{t+1}}{\theta} \right\} \]

Value function (11) is observationally equivalent to Epstein and Zin (1989) preferences with unitary intertemporal elasticity of substitution. Therefore, as in example 2.3, the price-dividend ratio is constant and the cost of uncertainty equals the equity premium.

**Example 2.5** (Gabaix (2012) variable rare disaster model). Although the preferences in the rare disasters model of Gabaix (2012) are not a special case of (8), the relationship between the one-period welfare cost and the equity premium can be easily studied. Gabaix assumes power utility and that log consumption growth falls by an amount \( b_{t+1} \) in the event of a disaster at time \( t + 1 \).
These assumptions imply the log stochastic discount factor

\[ m_{t+1} = \begin{cases} 
-\delta & \text{with probability } 1 - p_t \\
-\delta - \gamma b_{t+1} & \text{with probability } p_t 
\end{cases} \]

where \( p_t \) is the (potentially time-varying) probability of a disaster at time \( t + 1 \). Under the baseline calibration, the model implies flat term structures of equity and interest rates (a point also made by Binsbergen, Brandt and Koijen [2012a]), and therefore the equality between the equity premium and the cost of uncertainty.

Example 2.6 (Lettau and Wachter [2007, 2011] reduced-form model). Lettau and Wachter (2007, 2011) assume an essentially affine exponential-Gaussian setting in which shocks to dividend growth \((E_{t+1} - E_t)\Delta d_{t+1}\) is the only pricing factor, the price of risk \( x_t \) is linear in the states of the economy, and shocks to the price of risk are uncorrelated to cashflow shocks, i.e., \( \text{cov}_t(\Delta d_{t+1}, x_{t+1}) = 0 \). If on top of that consumption is a random walk, we can easily verify that the term structure of welfare costs is flat. Moreover, the term structure of interest rates is flat if shocks to the state that drives the risk-free rate are either absent (as in Lettau and Wachter [2007]) or unpriced, i.e., if \( \text{cov}_t(\Delta d_{t+1}, r_{t+1}) = 0 \). If both the term structures of welfare costs and interest rates are flat, then the term structure of equity must be flat, and therefore the welfare cost of uncertainty equals the equity premium at all periodicities.

3. Empirics of the cost of uncertainty

Suppose that in the market there are a riskless security and a full set of put and call European options whose underlying is the market index. In absence of arbitrage opportunities, put-call parity holds as

\[ C_{t,t+n} - P_{t,t+n} = P_t - P_t^{(n)} - X E_t M_{t,t+n} \tag{12} \]

where \( C_{t,t+n} \) and \( P_{t,t+n} \) are the prices at time \( t \) of a call and a put European options on the market index with maturity \( n \) and strike price \( X \), \( E_t M_{t,t+n} \) is the price of a \( n \)-period zero-coupon bond, \( P_t = E_t \sum_{j=1}^{n} M_{t,t+j} D_{t+j} \) is the price of the market portfolio, and \( P_t^{(n)} = E_t \sum_{j=1}^{n} M_{t,t+j} D_{t+j} \) is the price of \( n \)-period equity. Since the only unknown in equation (12) is the price of short-term equities, \( P_t^{(n)} \), we can synthetically replicate the prices of dividend strips via

\[ D_t^{(n)} = P_t^{(n)} - P_t^{(n-1)} \]

3.1. Empirical results

The evidence about dividend strips is taken from Binsbergen, Brandt and Koijen (2012a), who synthesize the evidence from the option market on the S&P500 index. Zero-coupon bond prices with maturities from one to five years are taken from the Fama-Bliss dataset (available from CRSP).\(^{14}\) I consider Fama-Bliss data from January 1996 to October 2009 to maintain comparability.

\(^{14}\)By the definition of the term structure of the cost of uncertainty, I should consider the term structure of TIPS, rather than the term structure of nominal bonds revealed by Fama-Bliss data. However, TIPS data for short-duration real
with the Binsbergen-Brandt-Koijen data. I consider the dividend strip data, thereby disregarding the
difference between aggregate consumption and dividends, because I am working in an endowment
economy, in which the two concepts coincide. Although definition (1) is in terms of consumption, I
show in the appendix how in a production economy the definition is in terms of dividends. This
result has the convenient consequence that the term structure of welfare costs links to the observable
term structure of equity, rather than to the unobservable term structure of consumption equity, even
in a production economy.

Figures 2a and 2b compute the cumulated return on an investment strategy that each period goes
long by a dollar on a strip and short by a dollar on the riskless asset—I take it to be the one-month
Treasury bill. The average risk premium on short-term dividend strips is large and positive. The
average risk premium over the available dataset is 15.3%, 8.9%, and 3.1% for a strategy that goes
long in the 1-year, 1.5-year and 2-year dividend strips, respectively, and 0.1%, 0.9%, 1.7%, 2.3%,
and 2.7% for a strategy that goes long in 1-year to 5-year zero-coupon bonds, respectively. The
point estimates of average excess returns for the strips compare to a 2.7% average excess return of
the market index.

Figures 2c and 2d plot the point estimates for the term structures of equity, interest rates, and
welfare costs. I also compute bootstrapped critical values corresponding to a 5-percent size for
the means of $l_t^{(n)}$ and $l_t^{(n)} - r_{t+1}^{em}$, $n = 1, 2$, and include them in the plot. I can reject both a flat term
structure of welfare costs (figure 2d) and that the first two components are zero (figure 2c). The
evidence suggests a downward-sloping term structure of welfare costs, driven both by a downward-
sloping term structure of equity and by an upward-sloping term structure of interest rates. Table 1
reports the point estimates for the term structure of welfare costs.

Moreover, the components of the term structure of welfare costs are volatile. To facilitate a
comparison with the literature, I replicate in table [1] the predictive regressions of Binsbergen, Brandt
and Koijen (2012a) for the average cumulated excess returns $\frac{1}{n} \sum_{t=1}^{n} r_{t+1}^{c,(n)}$, $n = 1, 2$, and for the market
excess return $r_{t+1}^{em}$. Since excess returns are forecastable, the cost of uncertainty varies over time.

How much do excess returns, hence the cost of uncertainty, vary? Considerably; table 1 shows how
the standard deviation of expected returns is 0.5-0.75 times the already large level. Moreover,
note that we are likely missing some important return predictors. Therefore, since the variance of
expected returns is increasing in the number of predictors, the estimates in table 1 underestimate the
actual volatility of the cost of uncertainty. The cost of one-year ahead cashflow uncertainty is huge
at some juncture of the business cycle.

The evidence thus indicates that some systematic stabilization policy to smooth the cost of
bonds are available only starting in 2003 (e.g., Gürkaynak, Sack and Wright 2010). I take therefore the term structure
of interest rates from Fama-Bliss data as a proxy for the relevant term structure of interest rates, a choice that biases the
estimates by the size of the inflation risk premium. This approximation error does not seem however to affect much the
quantitative estimates of the term structure of welfare costs because the contribution of the term structure of interest
rates is much smaller than the contribution of the term structure of equity. Most importantly, the upward slope in the
term structure of nominal interest rates seems to hold also for the term structure of real interest rates (Gürkaynak, Sack
and Wright 2010).

The true statistical power to these predictive regressions is given by the Campbell-Shiller identity, by an argument
along the lines of Cochrane (2008). The predictability results in table 1 acquire greater significance when compared to
the predictability of the market return, which is well-established in the literature (e.g., Cochrane 2011).
uncertainty likely is a macroeconomic priority, especially in the short run and at some junctures of the business cycle.

\[
\begin{array}{cccccccc}
\text{Dependent variable} & r_{t-1} & r_{t+1} & \frac{r_{t+2}}{2} & r_{t+1}^m & r_{t+2}^m & r_{t+3}^m & r_{t+4}^m \\
\text{Constant} & -0.28 & -0.06 & -0.04 & -0.65 & -0.56 & -0.67 & 0.13 & 0.17 & 0.16 \\
 & [0.23] & [0.29] & [0.35] & [0.18] & [0.20] & [0.18] & [0.19] & [0.18] & [0.19] \\
pd_t & -0.65 & -0.69 & -0.03 & -0.69 & -0.03 & -0.69 & -0.03 & -0.69 & -0.03 \\
 & [0.30] & [0.17] & [0.05] & [0.18] & [0.05] & [0.18] & [0.05] & [0.18] & [0.05] \\
pd_{t-1} & -0.32 & -0.61 & -0.04 & -0.61 & -0.04 & -0.61 & -0.04 & -0.61 & -0.04 \\
 & [0.39] & [0.18] & [0.04] & [0.18] & [0.04] & [0.18] & [0.04] & [0.18] & [0.04] \\
pd_t & -0.29 & -0.71 & -0.04 & -0.29 & -0.71 & -0.29 & -0.71 & -0.29 & -0.71 \\
 & [0.47] & [0.16] & [0.04] & [0.47] & [0.16] & [0.47] & [0.16] & [0.47] & [0.16] \\
R^2 & 0.0521 & 0.0128 & 0.0089 & 0.1186 & 0.0920 & 0.1094 & 0.0056 & 0.0097 & 0.0079 \\
\frac{\sqrt{\text{var}(\varepsilon_t)}}{\text{E}(\varepsilon_t)} & 0.63 & 0.31 & 0.26 & 1.23 & 1.09 & 1.19 & 0.63 & 0.83 & 0.76 \\
E(\varepsilon) & 0.1519 & 0.0838 & 0.0270 & 0.0838 & 0.0270 & 0.0838 & 0.0270 & 0.0838 & 0.0270 \\
\end{array}
\]

Table 1: One-month ahead predictive regressions. The term structure of equity is taken from Binsbergen-Brandt-Koijen data; the term structure of interest rates is taken from Fama-Bliss data. Excess returns are in excess over the 1-month Tbill rate. The regressors are the respective dividend yields; \( pd_t = (pd_t + pd_{t-1} + pd_{t-2})/3 \) is the average dividend yield in the previous quarter. Newey-West standard errors.

3.2. Term structures in some consumption-based asset pricing models

The evidence in section 3.1 is consistent with a downward-sloping term structure. Since the available sample allows for estimating only the first two components of the term structure of welfare costs at yearly frequency, I now turn to a model-based approach to capture and quantify the rest of the term structure. The evidence in section 3.1 helps to identify a suitable model. The asset pricing literature offers many models, already calibrated to match several stylized asset pricing facts, that have implications for the term structure of welfare costs. Unfortunately, from a structural perspective, replicating a downward-sloping term structure of equity and an upward-sloping term structure of interest rates is problematic (Lettau and Wachter, 2007; Binsbergen, Brandt and Koijen, 2012a; Croce, Lettau and Ludvigson, 2012). I therefore have to discard a structural explanation to answer the question I am interested in and be satisfied with a reduced-form model. I thus turn to the model of Lettau and Wachter (2011), which is a parsimonious framework able to capture the term structures of equity and interest rates.

3.2.1. Structural approach

Table 2 and figure 3 show the implications of some of today’s leading consumption-based asset pricing models for the three term structures and for the welfare cost of uncertainty and the equity premium. I consider the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the long-run risk model under limited information of Croce, Lettau and Ludvigson (2012), the recursive preferences of Tallarini (2000) and Barillas, Hansen and Sargent (2009), the rare disasters model of Gabaix (2012), and the reduced-form model of Lettau and Wachter (2011). In studying the term structures in the different asset pricing models, I consider
(a) Cumulated excess returns on dividend strips and the market index (S&P500).

(b) Cumulated excess returns on zero-coupon bonds.

(c) Average term structures of welfare costs, equity, and interest rates (bootstrapped 95% confidence interval to test that the welfare costs are zero).

(d) Average term structures of welfare costs, equity, and interest rates (bootstrapped 95% confidence interval to test that the welfare costs equal the equity premium).

Figure 2: Evidence on the term structures of equity and interest rates. The term structure of equity is taken from Binsbergen-Brandt-Koijen data; the term structure of interest rates is taken from Fama-Bliss data. Excess returns are in excess over the 1-month Tbill rate.
the original calibrations, which the authors choose to match some asset pricing facts. Note that alternative calibrations and refinements of the models are possible and some of the model-based predictions about the term structures of equity and interest rates might change accordingly. The online appendix works out the details of each model. I refer to the original writings for a list of the stylized asset pricing facts each model is matched to.

The habit formation model of Campbell and Cochrane (1999) predicts a flat term structure of interest rates and an upward-sloping term structure of equity. The term structure of interest rates is driven by a particular calibration of the time-varying risk aversion which produces a constant risk-free rate. The term structure of equity is instead driven by the positive correlation between the pricing factor—shocks to consumption growth—and dividend growth, and by the perfectly negative correlation between the pricing factor and the shocks to the price of risk, which decreases as consumption grows away from the external habit. Since dividend strips load negatively on shocks to the price of risk, and the more so the longer the maturity, people command a greater risk premium to bear long-run dividend strip risk. Under the baseline calibration, the model of Campbell and Cochrane predicts a marginal cost of all fluctuations of 3.7% and an equity premium of about 7%.

The long-run risk model of Bansal and Yaron (2004) generates an upward-sloping term structure of equity and a downward-sloping term structure of interest rates. Bansal and Yaron introduce rich dynamics in consumption growth, which is driven both by shocks to expected consumption growth and by stochastic volatility. The Epstein-Zin preferences then make all shocks to the consumption opportunity set show up as pricing factors. In the calibration of Bansal and Yaron, long-run dividend strips load more heavily on the shocks to the consumption opportunity set and therefore are more risky, as long as the elasticity of intertemporal substitution is larger than one. In the model the risk-free rate is driven by shocks to the predictable component of consumption, which is positively priced; since long-run zero-coupon bonds load less on this state than the risk-free rate, they provide long-run insurance. This property explains the downward-sloping term structure of interest rates (see also Koijen, Lustig, Nieuwerburgh and Verdelhan, 2010). The quantitative implications of the long-run risk model is a marginal cost of all fluctuations of 5.1% and an equity premium of about the same size.

Croce, Lettau and Ludvigson (2012) consider the long-run risk model of Bansal and Yaron (2004) and change the information structure. Under limited information, not all shocks to the cashflow opportunity set are observable. The shocks that are priced are therefore a linear combination of both short-run cashflow shocks and long-run cashflow shocks. Then, since long-run shocks have a relatively small volatility, long-run dividend strips load less on the shocks that are priced under limited information than short-run dividend strips. This strategy allows for generating a downward-sloping term structure of equity. Notably, Croce, Lettau and Ludvigson (2012) is the only state-of-the-art structural model that can generate a downward-sloping term structure of equity; however, the curvature is not enough quantitatively, at least under the baseline calibration, it still predicts a downward-sloping term structure of interest rates, and it works in a world in which risk premia are not time-varying. The model predicts a marginal cost of all fluctuations of 7.9%, against a predicted equity premium of 6.9%.

The ambiguity averse multiplier preferences in Barillas, Hansen and Sargent (2009), the recursive preferences of Tallarini (2000), and the rare disasters model of Gabaix (2012) yield two flat term structures which, consistent with proposition 4, imply the equality between the equity
premium and the cost of fluctuations. The unitary elasticity of intertemporal substitution that characterizes the recursive preferences of Tallarini (2000) and the robust control literature implies constant dividend yields, as discount-rate effects exactly offset cashflow effects. The intuition behind the flat term structure of the rare disasters model is that different dividend strips have the same exposure to the disaster event, whose probability is independent of the cashflow shocks that are priced. Both models imply, disregarding second-order terms, a marginal cost of all fluctuations equal to the equity premium, which is of 1.9% under the multiplier preferences of Barillas, Hansen and Sargent (2009) and the observationally equivalent model of Tallarini (2000), and 6.9% in the model of Gabaix (2012).

\[ \sum_{n=1}^{\infty} \omega_n E(l_t^{(n)}) \quad \sum_{n=1}^{\infty} \frac{\omega_n}{2n} V(\exp n l_t^{(n)}) \quad L^N \quad \ln E(R_{em}) \]

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Campbell and Cochrane</td>
<td>3.41</td>
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<td>3.73</td>
<td>7.06</td>
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<td>Bansal and Yaron</td>
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<td>0.01</td>
<td>5.14</td>
<td>5.12</td>
</tr>
<tr>
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<td>0</td>
<td>7.94</td>
<td>6.95</td>
</tr>
<tr>
<td>Barillas, Hansen and Sargent</td>
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<td>0</td>
<td>1.84</td>
<td>1.94</td>
</tr>
<tr>
<td>Gabaix</td>
<td>6.54</td>
<td>0.04</td>
<td>6.58</td>
<td>6.86</td>
</tr>
<tr>
<td>Lettau and Wachter</td>
<td>2.31</td>
<td>0.08</td>
<td>2.39</td>
<td>7.04</td>
</tr>
</tbody>
</table>

Table 2: Unconditional cost of uncertainty and unconditional equity premium (in percent per year) for different consumption-based asset pricing models. I consider the habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), the long-run risk model under limited information of Croce, Lettau and Ludvigson (2012), the ambiguity averse multiplier preferences of Barillas, Hansen and Sargent (2009), the rare disasters model of Gabaix (2012), and the reduced-form model of Lettau and Wachter (2011).

3.2.2. Reduced-form approach

I next turn to the reduced-form model of Lettau and Wachter (2011), which is designed to capture a downward-sloping term structure of equity and an upward-sloping term structure of interest rates. They combine lognormal pricing formulas and a loglinear state-space system with a price of risk, \( x_t \), linear in the states of the economy. Their exponential-Gaussian model is a particularly tractable setting to study term structures, as their equilibrium values have closed-form solutions (Ang and Piazzesi, 2003; Lettau and Wachter, 2007, 2011; Lustig, Nieuwerburgh and Verdelhan, 2012).

Without micro-founding it, Lettau and Wachter (2011) directly specify a stochastic discount factor, whose existence is guaranteed by the no-arbitrage theorems (Harrison and Kreps, 1979; Hansen and Richard, 1987). To keep low the number of degrees of freedom in the model, they assume a single conditional pricing factor perfectly related to short-run cashflow shocks and a single state driving the price of risk. They then assume a zero correlation between cashflow and discount-rate shocks and show how that property seems crucial to generate a downward-sloping term structure of equity (see also Lettau and Wachter, 2007). To match the downward-sloping term structure of equity, Lettau and Wachter (2011) assume that the predictable component of cashflows is negatively related to priced shocks. Long-run dividend strips thus contain a component that provides long-run insurance. The independence of discount-rate shocks and cashflows shocks then
Figure 3: The term structures of equity, interest rates and welfare costs of uncertainty in some consumption-based asset pricing models.
avoids that the negative load of long-run dividend strips on the state that drives the price of risk offsets the long-run insurance effect.

Finally, since only short-run cashflow shocks are priced, Lettau and Wachter manage to replicate the upward-sloping term structure of interest rates by assuming that shocks to the state driving the risk-free rate are negatively correlated with priced shocks. Since long-run zero-coupon bonds are less exposed to this state than short-run bonds, the assumption generates a positive bond risk premium as the maturity increases.

The model of Lettau and Wachter predicts a marginal cost of total uncertainty of 2.4% and an equity premium of 7%.

3.2.3. Robustness across models

Even though I can only turn to the reduced-form model of Lettau and Wachter (2011) to capture the term structure features I am interested in, I can draw some lessons that are robust across all asset pricing models considered.

Table 2 shows how the marginal cost of uncertainty in the entire consumption process is well above 1% in each one of today’s leading asset pricing models. This robustness across models is in line with the result by Alvarez and Jermann (2004) that a model that is consistent with stylized asset pricing facts must increase the original estimates of Lucas (1987) by two orders of magnitude. Moreover, in line with equation (6), table 2 and figure 3 decompose the marginal cost of uncertainty in a component due to the average term structure of welfare cost and another due to the entropy surrounding the time-variation of the term structure. State-of-the-art asset pricing models suggest that the uncertainty in the term structure of welfare costs has a non-trivial yet limited contribution to the total cost of aggregate uncertainty.

4. Policy implications

On average, according to the marginal cost of fluctuations, economic stabilization should be a primary focus of attention of policymakers, especially over short horizons. People fear more short-horizon than long-horizon cashflow volatility and they are willing to trade a substantial amount of growth in exchange for a marginal reduction in volatility. Moreover, over time, these macroeconomic priorities vary substantially. Against this background, a policymaker that wishes to use the concept of cost of fluctuations to assess the macroeconomic priorities can observe the state that drives the term structure of welfare costs over time. In fact, since the term structure components are risk premia, return predictors reveal the state that drives the time-variation in the term structure of welfare costs and thereby forecast future developments in the macroeconomic priorities.

Finally, from a welfare perspective, I explore whether the fact that the term structure of welfare costs is made of risk premia can make the case for a risk premia targeting regime as a welfare-enhancing policy. In fact, the cost of fluctuations decreases as the mean and the volatility of the individual components of its term structure decrease. However, smaller welfare costs are not

\[16\] Note how the models of Croce, Lettau and Ludvigson (2012) and Barillas, Hansen and Sargent (2009) do not allow for time-varying risk premia and therefore predict a zero entropy in equation (6).
necessarily desirable. Risk premia targeting is unambiguously desirable if the policy regime is neutral on the mean growth rate of cashflows and on the level of any other additional factors that affect utility.

4.1. Macroeconomic priorities

The negative slope of the term structure of the cost of fluctuations is a robust feature over the available sample, driven both by the upward-sloping term structure of interest rates and by the downward-sloping term structure of equity. The immediate consequence for policy-makers is that, on average, short-run economic stabilization should be a primary focus of attention. On the one hand, short-run aggregate consumption stabilization is a greater priority than long-run stabilization. On the other, the high level of the cost of fluctuations reveals that people are willing to trade a large amount of growth against short-run consumption stabilization.

Table 3 reports the cost of short- and long-run fluctuations over different stabilization sets as captured by the reduced-form model of Lettau and Wachter (2011). An increase in consumption uncertainty by a fraction $\theta$ over a 10-year period has a marginal cost of more than $10\theta$ percentage points of growth per year during the decade. These numbers compare to smaller yet non-trivial marginal benefits of long-run stabilization, which tend to zero as the stabilization becomes asymptotic.

<table>
<thead>
<tr>
<th>Periodicity</th>
<th>Marginal Cost of Consumption Uncertainty (10 years)</th>
<th>Marginal Cost of Growth (10 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>15.63</td>
<td>2.42</td>
</tr>
<tr>
<td>1-5 years</td>
<td>13.76</td>
<td>2.25</td>
</tr>
<tr>
<td>1-8 years</td>
<td>12.17</td>
<td>2.10</td>
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<td>1-10 years</td>
<td>11.22</td>
<td>2.01</td>
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<tr>
<td>1-20 years</td>
<td>7.89</td>
<td>1.62</td>
</tr>
<tr>
<td>$\infty$</td>
<td>2.39</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Marginal cost of fluctuations at all periodicities $n \in \mathcal{N}$. Lettau and Wachter (2011) model-based estimates

Furthermore, the volatility of the cost of fluctuations at different frequencies is large. The evidence in Table 4 shows how the volatility of the first two term structure components is more than half the level. The cost of fluctuations is huge at some junctures of the business cycle. The volatility of the term structure as captured by the model of Lettau and Wachter (2011), reported in Figure 4a, confirms the direct evidence. The standard deviation of the cost of fluctuations at short periodicities is large and decays over long horizons.

The variation of the term structure of welfare costs over the business cycle is driven by the time-varying components of $\{l_t^{(n)}\}$. By the closed-form solution for the term structure components in the model of Lettau and Wachter, the online appendix shows how this component is driven entirely by movements in the market price of risk $x_t$ as, up to a constant term of second-order importance,

$$l_t^{(n)} = \frac{1}{n!} \frac{1 - \phi_x^n}{1 - \phi_x} \sigma_d + \left( \frac{1 - \phi_x^n}{1 - \phi_x} - \frac{\phi_z^n - \phi_x^n}{\phi_x - \phi_z} \right) \frac{\sigma_z}{\|\sigma_d\|} \frac{\sigma_d'}{\|\sigma_d\|} x_t,$$

where the price of risk follows the autoregressive process $x_{t+1} = (1 - \phi_x) x_t + \phi_x x_t + \sigma_z \varepsilon_{t+1}$, with $x$ the average discount rate. Vectors $\sigma_d'$, $\sigma_z'$ and $\sigma_x'$ are the loadings of short-run and long-run cashflows.
and of the price of risk, respectively, on the reduced-form shocks that drive the system, and $\phi_z$ is the persistence of the predictable component of cashflows.

The term structure of the costs of uncertainty starts from a level of $||\sigma_d||x$, which is about 17% per year in the calibration of Lettau and Wachter (2011), to then decay with a slope determined by the persistence coefficients $\phi_z$ and $\phi_x$.

![Graph](image)

(a) Model-based estimates of volatilities in the model of Lettau and Wachter (2011).

(b) Impulse response $\partial l_{t+k}(n)/\partial \varepsilon_t^x$ to a one-standard deviation discount-rate shock.

Figure 4: Time-variation in the term structure of welfare costs in the model of Lettau and Wachter (2011).

Figure 4b shows what happens to the term structure of the cost of uncertainty after a one standard deviation discount-rate shock. News to the price of risk signal a transitory change in the state of the economy that makes consumers temporarily change their aversion to uncertainty, hence the cost of uncertainty and the risk premium they require to hold stocks. The positive initial effect then decays across maturities and over time.

The effect in figure 4b compares to a zero effect on the term structure after short-run cashflow news, which is a consequence of the assumed orthogonality between discount-rate shocks and the shocks to the conditional pricing factor. Long-run cashflow news do not have a direct effect on the term structure but signal something about the price of risk, because on average long-run cashflow news tend to occur together with discount-rate news.

A policy-maker interested in following the developments of the term structure of welfare costs over the business cycle would therefore find useful some information about the state that drives the term structure of welfare costs. To observe the state that drives the cost of fluctuations, ICAPM logic tells us that return predictors must reveal the state of the economy that is priced. News to return predictors signal therefore the tradeoff between growth and macroeconomic stabilization at each juncture of the business cycle, and they reveal what periodicities are the priority.

I turn once more to the model of Lettau and Wachter (2011) to put the point formally. The model suggests that a sufficient information set to reveal news to the market price of risk is made by the dividend yield of the one-period strip $pd(1)$, the consumption-dividend ratio $cd$—which reveals the predictable component of dividend growth, $z$ (Letttau and Ludvigson, 2005)—and the risk-free rate $r_f$. For example, using the one-period welfare cost, for which we have direct evidence over a
longer sample,

\[
\begin{align*}
    \hat{r}_t^{(1)} &= ||\sigma_d||x_t = \ln E_t R_{d,t+1}^{e(1)} \\
    pd_t^{(1)} &= z_t - ||\sigma_d||x_t - r_f^t \\
    cd_t &= \lambda z_t
\end{align*}
\]

for some scalar \( \lambda \). Thus, since \( r_{d,t+1}^{e(1)} \) and \( y_t = \{pd_t^{(1)}; cd_t; r_f^t\} \) are all observable, the regression

\[
    r_{d,t+1}^{e(1)} = \beta_0 + \beta_1 pd_t^{(1)} + \beta_2 cd_t + \beta_3 r_f^t + e_{t+1}
\]

has in population OLS coefficients \( \beta_1 = -1 \), \( \beta_2 = \frac{1}{\lambda} \), \( \beta_3 = -1 \). Therefore, the projection of the return on the information set is \( E_t[r_{d,t+1}^{e(1)}|y_t] = ||\sigma_d||x_t \). The residuals of the Wold representation of the information set \( y_t \) then reveal the discount-rate news as

\[
\beta_1(E_{t+1} - E_t)pd_t^{(1)} + \beta_2(E_{t+1} - E_t)cd_{t+1} + \beta_3(E_{t+1} - E_t)r_f^{t+1} = ||\sigma_d||e_{t+1}^t
\]

Thus, given news extracted from predictive regressions and the impulse responses reported in figure 4b, a policy-maker is able to assess the current macroeconomic priorities and to forecast the developments of the term structure of welfare costs.

Outside the framework of Lettau and Wachter (2011) the general lesson remains true, provided the policy-maker has some belief about a sufficient set of variables to predict the cumulated excess returns, \( r_{d,t+n}^{e(1)} \), that form the basis of the term structure of welfare costs.

### 4.2. Risk premia targeting

The result the aggregate stability is a macroeconomic priority suggests that policy-makers should mind the quantity of aggregate cashflow uncertainty, especially in the short-run. Moreover, the greater priority of short-run stability suggests that a policy-maker should consider trading short-run stability against long-run uncertainty if presented with that choice. A natural subsequent question is whether policy-makers have traction on the aggregate quantity of uncertainty. Unfortunately, even though it is generally accepted that greater policy forecastability reduces aggregate uncertainty (e.g., Clarida, Gali and Gertler, 1999), the quantitative implications on the economy of a change in policy uncertainty are not yet well-understood.\(^1\)

Alternatively, we can think of directly targeting the cost of uncertainty, which is a slightly different prescription from targeting the quantity of uncertainty. Such a policy regime would not just target the amount of uncertainty but also the market price of uncertainty. By equation (6), a policy that reduces either the mean or the time-variation of the term structure components reduces the cost of fluctuations. Therefore, the question is whether it is desirable, from a welfare viewpoint, to target the risk premia that form the term structure of welfare costs. The advantage of this prescription is that the effect of policy on equity premia is more documented than the effect of policy on the amount of uncertainty. For example, Rigobon and Sack (2004) and Bernanke and Kuttner (2005) report

---

\(^1\) Baker, Bloom and Davis (2011) and Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2011) make progress in answering the question.
direct evidence about the effect of monetary policy shocks on a broad array of asset market data, although the theoretical mechanism explaining the documented effects is still an open question.

Against this background, proposition 5 provides a basis for using the cost of uncertainty as a welfare criterion. It shows how people prefer a smaller cost of fluctuations for a given expected path of consumption growth and of factor $X$. The argument is one of second-order stochastic dominance.

**Proposition 5.** For two lognormally distributed lotteries $C^{(1)}$ and $C^{(2)}$ that have the same expectation $E(C)$, and for a given level of the factor $X$, one has that

$$L^{(1)} < L^{(2)} \Rightarrow E[U(C^{(1)}, X)] > E[U(C^{(2)}, X)]$$

There is an important caveat in proposition 5 and thereby a qualifier for the risk premia targeting regime that is desirable. An intervention that reduces the mean cost of fluctuations and that affects the unconditional expectation of consumption or the level of the other variables in the utility function is not necessarily welfare-enhancing. Therefore, the desirability of a regime that aims at reducing the cost of fluctuations at any frequency is not to be taken for granted.

The careful design of desirable policy rules requires a full-fledged structural model able to account for, on the one hand, the shape of the term structures of equity and interest rates and, on the other, the stock market’s reaction to a policy shock documented in the literature.

5. Conclusion

The result that the welfare cost of uncertainty is a linear combination of risk premia makes one of the main tasks of macroeconomics—that of assessing the macroeconomic priorities (Lucas, 2003)—inextricably linked to finance. I used recent evidence extracted from index option markets and found a high, downward-sloping and volatile term structure. Therefore, asset markets suggest that cashflow stability is on average a macroeconomic priority, especially in the short-run. However, the macroeconomic priorities can vary substantially across the business cycle. Against this background, I showed how a policy-maker can assess the current position and evolution of the term structure of welfare costs and forecast its movements by looking at innovations in the information set made by excess return predictors.

Among the implications of these findings is that a structural model able to explain the stylized evidence about the term structures of equity and interest rates, and therefore of welfare costs, should be high on the research agenda. Such a project promises to deliver important new insights in terms of welfare analysis and in the policy transmission mechanism.

Appendix

A. Relationship with the definitions of Lucas (1987) and Alvarez and Jermann (2004)

Definition (1) is more general and slightly different from to the one studied by Lucas (1987) and Alvarez and Jermann (2004). First, they measure the cost of fluctuations by the uniform compensation $\Omega_t$ in

$$E_t U((1 + \Omega_t(\theta))C_{t+n})_{n=1}^{\infty}, [X_{t+n}]_{n=1}^{\infty} = E_t U((\theta E_t C_{t+n} + (1 - \theta)C_{t+n})_{n=1}^{\infty}, [X_{t+n}]_{n=1}^{\infty})$$
whereas the definition I consider measures the cost of fluctuations by a compounded compensation. The alternative definition I study can be interpreted as the tradeoff between growth and macroeconomic stabilization and thereby has an arguably more intuitive appeal.

Second, I allow for considering the stabilization of only some coordinates of consumption—the set $\mathcal{N}$ in definition (1)—rather than of the whole stochastic process. This flexibility allows for a direct focus on the relevant periodicity of economic fluctuations.

B. Proof of proposition 1

I can rewrite equation (2) as

$$L_t^N = \sum_{n \in \mathcal{N}} n \frac{E_t(M_{t,t+n}C_{t+n})}{E_t(M_{t,t+n}C_{t+n})} \times \frac{1}{n} \left( \frac{E_t(M_{t,t+n})E_t(C_{t+n})}{E_t(M_{t,t+n}C_{t+n})} - 1 \right)$$

$$= \sum_{n \in \mathcal{N}} \omega_{n,t} r_t^{(n)}$$

C. Proof of proposition 2

The $n$th component of the term structure of welfare costs is a risk premium. Note in fact that

$$R_{t\rightarrow t+n}^{c,(n)} = \frac{E_t(M_{t,t+n})}{\prod_{j=1}^{n} E_{t+j-1}(M_{t+j})} \prod_{j=1}^{n} \frac{E_t(M_{t+j})E_{t+j}(M_{t+j+n}C_{t+n})}{E_{t+j-1}(M_{t+j-1,n}C_{t+n})}$$

$$= \prod_{j=1}^{n} \frac{R_{t\rightarrow t+n}^{c,(j)}}{R_{t\rightarrow t+n}^{c,(j)}}$$

which implies

$$r_t^{(n)} = \frac{1}{n} \left( E_t(r_{t\rightarrow t+n}^{c,(n)} + \mathcal{V}_t(R_{t\rightarrow t+n}^{c,(n)})) - 1 \right)$$

$$= \frac{1}{n} \left( E_t(r_{t\rightarrow t+n}^{c,(n)} + \mathcal{V}_t(R_{t\rightarrow t+n}^{c,(n)})) - 1 \right)$$

$$= \frac{1}{n} \sum_{j=1}^{n} \left( E_t(r_{d,t+n-j}^{c,(n-j)} - r_{b,t+n-j}^{c,(n-j)} + \mathcal{V}_t(R_{t\rightarrow t+n}^{c,(n)}) \right)$$

where $\mathcal{V}(\cdot)$ is a particular measure of uncertainty in a random variable that Theil (1967) calls the second entropy measure.

Then, equation (3) becomes

$$L_t = \sum_{n=1}^{\infty} \omega_{n,t} r_t^{(n)} = \sum_{n=1}^{\infty} \omega_{n,t} E_t(r_{t\rightarrow t+n}^{c,(n)}) + O(||\varepsilon||^2)$$

$$= \sum_{n=1}^{\infty} \omega_{n,t} \sum_{j=1}^{n} E_t(r_{d,t+j}^{c,(n-j+1)} - r_{b,t+j}^{c,(n-j+1)}) + O(||\varepsilon||^2)$$

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where \( O(\|\varepsilon\|^k) \) denotes a term of order \( k \).

You can repeat all calculations unconditionally\footnote{\( E(C_{t+n}) \) stands for the nonstationary part of \( C_{t+n} \) conditional at time 0 plus the unconditional expectation of the stationary part. E.g., if \( \Delta c_{t+1} = \mu + z_t + \varepsilon_{t+1} \), with \( \varepsilon_t \sim WN(0, \sigma^2_\varepsilon) \) and \( z_t \sim I(0) \) with \( E(z_t) = 0 \), then \( E(C_{t+n}) = e^{(\varepsilon_{t+n}\varepsilon_0)}. \)} and find \( L = \sum_{n=1}^{\infty} \omega_n I^{(n)} \), where \( \omega_n = \frac{D^{(n)}}{\sum D^{(n)}} \). Note that, unconditionally,

\[
I^{(n)} = \frac{1}{n} \left( \frac{E(M_{t+n}E(C_{t+n})}{E(M_{t+n}C_{t+n})} - 1 \right)
\]

\[
= \frac{1}{n} \left( E(\varepsilon_{t+i+n}^{(n)}) + \frac{1}{2n} \varepsilon^2_{t+i+n} \right) + O(\|\varepsilon\|^4)
\]

\[= E(\varepsilon_{t+i+n}) \frac{1}{2n} \varepsilon^2_{t+i+n} + \frac{1}{2n} \varepsilon^2_{t+i+n} + O(\|\varepsilon\|^4)
\]

\[= E(\varepsilon_{t+i+n}) + \frac{1}{2n} \varepsilon^2_{t+i+n} + O(\|\varepsilon\|^4)
\]

where the first equality is by the definition of the unconditional term structure components, the third equality uses equation \( (C.1) \) and the law of iterated expectations, and the last equality uses the law of total entropy.

**D. Proof of proposition [3]**

**Lemma 1 (Lognormal no-arbitrage pricing).** The absence of arbitrage opportunities in the financial market implies the fundamental asset pricing representation

\[E_t M_{t+1} \frac{dF_t}{F_t} = 1\] (Harrison and Kreps, 1979; Hansen and Richard, 1987). Under lognormality, the no-arbitrage pricing formula becomes

\[E_t m_{t+1} + E_t r^i_{t+1} + \frac{1}{2} \text{var}_i (m_{t+1} + r^i_{t+1}) = 0\] (D.2)

This result also holds for the risk-free rate as

\[E_t m_{t+1} + r^f_{t+1} + \frac{1}{2} \text{var}_i (m_{t+1}) = 0\]

Combining the formulas for the ex-ante return of the \( i \)th security and of the risk-free rate we have the lognormal pricing formula for risk premia

\[E_t r^e_{t+1} + \frac{1}{2} \text{var}_i (r^e_{t+1}) = -\text{cov}_i (m_{t+1}, r^e_{t+1})\]

I define the market portfolio as the portfolio that pays off the entire consumption stream. The approximate Campbell and Shiller (1988) identity for the market portfolio then is

\[r^m_{t+1} = k + dp_t - \beta dp_{t+1} + \Delta c_{t+1}\]

for some constant intercept \( k \).
Then, by no-arbitrage pricing under lognormality (lemma 1) and by the definition of the one-period welfare cost \( l_t^{(1)} \), the equity premium is

\[
E_t r_t^m + \frac{1}{2} \text{var}_t(r_t^m) = -\text{cov}_t(m_{t+1}, r_t^m)
\]

\[
= -\text{cov}_t(m_{t+1}, \Delta c_{t+1}) + \beta \text{cov}_t(m_{t+1}, d p_{t+1})
\]

\[
= l_t^{(1)} + \beta \text{cov}_t(m_{t+1}, d p_{t+1}) \quad \text{(D.3)}
\]

The residual \( \beta \text{cov}_t(m_{t+1}, d p_{t+1}) \), the systematic risk in the price-dividend ratio, measures the distance between the cost of uncertainty and the equity premium.

E. Proof of proposition 4

Lemma 2. Let assumptions 1 and 2. Suppose that the first dividend strip premium, \( E_t r_{d,t+1}^{(1)} \), equals the equity premium, \( E_t r_t^m \). Then the cost of uncertainty \( L_t^N \) equals the equity premium, for any \( N \subset \mathbb{N} \).

Proof of lemma 2. By no-arbitrage, \( E_t r_{t+1}^m = \sum_{n=1}^{\infty} w_n E_t r_{d,t+1}^{(n)} \), where \( w_n \equiv D_t^{(n)}/\sum_{n=1}^{\infty} D_t^{(n)} \). Now suppose \( E_t r_{t+1}^m = E_t r_{d,t+1}^{(1)} \). Then, under assumption 1, \( E_t r_{d,t+1}^{(n)} \geq E_t r_{d,t+1}^{(n-1)} \), or vice-versa. Suppose that the inequality is strict for some \( n \). Then \( E_t r_{d,t+1}^{(1)} = \sum_{n=1}^{\infty} w_n E_t r_{d,t+1}^{(n)} > E_t r_{d,t+1}^{(1)} \sum_{n=1}^{\infty} w_n = E_t r_{d,t+1}^{(1)} \), a contradiction. Therefore, the relation must hold with equality for all \( n \).

Under assumption 3, equation (D.3) becomes

\[
m_{t+1} = -\rho_t - \gamma_t \sum_{j=0}^{\infty} \delta_j (E_{t+1} - E_t) \Delta c_{t+j+1}
\]

and therefore

\[
E_t r_{t+1}^m + \frac{1}{2} \text{var}_t(r_t^m) = l_t^{(1)} - \gamma_t \sum_{j=0}^{\infty} \beta \delta_j \text{cov}_t((E_{t+1} - E_t) \Delta c_{t+j+1}, d p_{t+1}) \quad \text{(E.4)}
\]

which proves the first part of proposition 4.

Moreover, if consumption is a random walk, expression (E.4) collapses to

\[
E_t r_{t+1}^m + \frac{1}{2} \text{var}_t(r_{t+1}^m) = l_t^{(1)} - \gamma_t \text{cov}_t(\Delta c_{t+1}, \beta d p_{t+1})
\]

If, additionally, news to consumption growth and to the price-dividend ratio are orthogonal, then

\[
E_t r_{t+1}^m + \frac{1}{2} \text{var}_t(r_{t+1}^m) = l_t^{(1)}
\]

i.e., the one-period welfare cost equals the equity premium. This result combined with assumptions 1 and 2 yields the result, by lemma 2.
F. Equity or consumption equity?

The definition of cost of fluctuations is given in terms of consumption, not dividends. The evidence I consider is about dividend strips, which may pose a problem. Indeed, along with the choice of the moving-average filter, the other main empirical challenge of Alvarez and Jermann (2004) was in fact to find a proxy for the price of a consumption claim.

Now, in an endowment economy, $C_t = D_t$, so that from a theoretical viewpoint the critique does not bite. However, in a production economy, the equality breaks down. Fortunately, under mild general equilibrium assumptions we can save the link between the cost of fluctuations and market equity.

A desirable approach in a production economy is to consider preferences that depend also on labor effort, $N_t$. Then, consider the definition

$$E_t U \{ ((1 + \hat{L}^N_t(\theta))^n C_{t+n}, [N_{t+n}], [X_{t+n}]) \} =$$

$$= E_t U \{ \theta E_t C_{t+n} + (1 - \theta) C_{t+n}, [\theta \overline{N}_{t+n} + (1 - \theta) N_{t+n}], [X_{t+n}] \} \quad (F.5)$$

where stable hours worked are defined as $\overline{N}_{t+n} \equiv \frac{p^n_{t+n}}{W_{t+n}} E_t \{ \frac{W_{t+n}}{P_{t+n}} N_{t+n} \}$.[19] In a production economy, $\hat{L}_t$ measures the cost of aggregate uncertainty around consumption and labor income. Therefore, differentiating with respect to $\theta$ and evaluating in $\theta = 0$,

$$L^N_t = \frac{\sum_i E_i (U_{1,i+n}) E_i (C_{t+n}) + E_i (U_{2,i+n}) \frac{P_{t+n}}{W_{t+n}} E_i (\frac{W_{t+n}}{P_{t+n}} N_{t+n}) - E_i (U_{1,i+n} C_{t+n} + U_{2,i+n} \frac{W_{t+n}}{P_{t+n}} N_{t+n})}{\sum n E_i (U_{1,i+n} C_{t+n})} \quad (F.6)$$

**Assumption 5.** Assume the first-best optimality condition $W_t = -\frac{U_t}{U_c}$ (the marginal rate of substitution between labor and consumption equals the relative price). Assume that the aggregate firm pays off all profits as dividends and therefore $D_t = Y_t - \frac{W_t}{P_t} N_t - I_t$, where $Y_t$ is total output and $I_t$ are gross investments made by the firm. Assume the market-clearing condition $Y_t = C_t + I_t$.

The definition of dividends as current aggregate profits is standard in the Q literature (Hayashi, 1982). The optimality condition implies that there are no distortions in the consumption-side of the economy that generate so-called labor wedges (such as, for example, the aggregate wage markup considered by Gall, Gertler and López-Salido, 2007). Note how assumption 5 imposes $D_t = C_t - \frac{W_t}{P_t} N_t$.

Under assumption 5, expression (F.6) becomes

$$L^N_t = \frac{\sum_i E_i (M_{1,i+n}) E_i (D_{t+n}) - E_i (M_{1,i+n} D_{t+n})}{\sum n E_i (M_{1,i+n} C_{t+n})}$$

$$= \sum \omega_{n,i} l^{(n)}$$

[19] Stable hours are the hours that provide a stable labor income $\frac{W_{t+n}}{P_t} = E_i (\frac{W_{t+n}}{P_{t+n}} N_{t+n})$. 

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\[
\omega_{n,t} = \frac{n \ E_i(M_{t+1}D_{t+1})}{\sum_{n \in N} n \ E_i(M_{t+1}C_{t+1})}  \\
\ell_t^{(n)} = \frac{1}{n} \left( \frac{E_i(M_{t+1})E_i(D_{t+1})}{E_i(M_{t+1}D_{t+1})} - 1 \right)  \\
= \frac{1}{n} \ln E_i\epsilon_t^{(n)}
\]

In this production economy, as we distinguish between equity and consumption equity, only the weights \(\{\omega_{n,t}\}\) depend on consumption equity. Most importantly, the term structure components \(\{\ell_t^{(n)}\}\) remain functions of equity risk premia rather than of consumption-equity risk premia.

G. Proof of proposition 5

First note that

\[
E[U((1 + L^{(1)})^n C_{t+n}), X] = E[U((1 + L^{(2)})^n C_{t+n}), X] > E[U((1 + L^{(1)})^n C_{t+n}), X] \tag{G.7}
\]

because utility is strictly increasing in consumption and the scalars \(L^{(1)}, L^{(2)}\) are such that \(L^{(1)} < L^{(2)}\). Since the two lotteries have the same mean and are conditionally lognormal, they must differ in variance.

Suppose \(C^{(1)}\) has a larger variance, i.e., by the projection theorem in Hilbert spaces, \(C^{(1)} = C^{(2)} + \epsilon\), with \(E[\epsilon|C^{(2)}] = 0\). This implies \(E[U(C^{(1)}, X)] = E[E[U(C^{(1)}, X)|C^{(2)}]] < E[U(E[C^{(2)} + \epsilon|C^{(2)}], X)] = E[U(C^{(2)}, X)]\), where the first and last equalities use the projection theorem, and the inequality is by Jensen’s theorem.

However, the assumption also implies that the variance of \((1 + L^{(1)})^n C_{t+n}\) is larger than the variance of \((1 + L^{(1)})^n C_{t+n}\). Therefore, by the projection theorem and Jensen’s inequality, \(E[U((1 + L^{(1)})^n C_{t+n}), X] < E[U((1 + L^{(1)})^n C_{t+n}), X]\), which contradicts expression \(G.7\).

Therefore, it must be that \(C^{(2)} = C^{(1)} + \epsilon\), with \(E[\epsilon|C^{(1)}] = 0\), which implies \(E[U(C^{(2)}, X)] < E[U(C^{(1)}, X)]\).

References


