A dynamic approach to short run economic fluctuations. The DAD/DAS model.

Part 2.
The demand side of the model – the dynamic aggregate demand (DAD)
Inflation and dynamics in the short run

• So far, to analyze the short run we have used
  – the IS-LM model (shows just the demand side) and
  – Static AS/AD model
• Both theories are silent about
  – Inflation, and
  – Dynamics
• Last week, we started to develop a dynamic aggregate demand and dynamic aggregate supply (DAD-DAS)
• The DAD-DAS model presents a dynamic short-run theory of output, inflation, and interest rates.
Introduction

- The dynamic model of aggregate demand and aggregate supply (DAD-DAS) determines both
  - real GDP ($Y$), and
  - the inflation rate ($\pi$)

- This theory is *dynamic* in the sense that the outcome in one period affects the outcome in the next period
Introduction

• Instead of representing monetary policy by an exogenous money supply, the central bank will now be seen as following a **monetary policy rule**
  – The central bank’s monetary policy rule adjusts interest rates automatically when output or inflation are not where they should be.
Introduction

• The DAD-DAS model is built on the following concepts:
  – The Phillips Curve
  – Adaptive expectations

• Which together built the supply side of the model — we have done that last week
  – The IS curve (a negative relationship between the real interest rate and aggregate demand)
  – The Fisher effect
  – The monetary policy rule of the central bank

• Which together built the demand side of the model (our job for today)
Keeping track of time

• The subscript “$t$” denotes a time period, e.g.
  – $Y_t = \text{real GDP in period } t$
  – $Y_{t-1} = \text{real GDP in period } t - 1$
  – $Y_{t+1} = \text{real GDP in period } t + 1$

• We can think of time periods as years. 
  E.g., if $t = 2008$, then
  – $Y_t = Y_{2008} = \text{real GDP in 2008}$
  – $Y_{t-1} = Y_{2007} = \text{real GDP in 2007}$
  – $Y_{t+1} = Y_{2009} = \text{real GDP in 2009}$
The model’s elements

• The DAD/DAS model has five equations and five endogenous variables:
  – output, inflation, the real interest rate, the nominal interest rate, and expected inflation.
• We already know the supply side equation – the Phillips curve
Inflation: The Phillips Curve

• Recall the Phillips Curve:

\[ \pi_t = E_{t-1} \pi_t - \beta (u_t - u_N) + \nu_t \]

• Use the Okun’s Law to get the relationship between inflation and output:

\[ \pi_t = E_{t-1} \pi_t + \phi (Y_t - \bar{Y}_t) + \nu_t \]
Inflation: The Phillips Curve

\[ \pi_t = E_{t-1} \pi_t + \phi(Y_t - \bar{Y}_t) + \nu_t \]

- **Current inflation**
- **Previously expected inflation**
- **Supply shock, random and zero on average**

\( \phi > 0 \) indicates how much inflation responds when output fluctuates around its natural level.
Expected Inflation: Adaptive Expectations

\[ E_t \pi_{t+1} = \pi_t \]

Assumption: people expect prices to continue rising at the current inflation rate.

Examples: \( E_{2000} \pi_{2001} = \pi_{2000}; \) \( E_{2013} \pi_{2014} = \pi_{2013}; \) etc.
Phillips Curve

\[ \pi_t = E_{t-1} \pi_t + \phi \cdot (Y_t - \bar{Y}_t) + \nu_t \]

- At any particular time, inflation would be high if
  - people in the past were expecting it to be high
  - current demand is high (relative to natural GDP)
  - there is a high inflation shock. That is, if prices are rising rapidly for some exogenous reason such as scarcity of imported oil or drought-caused scarcity of food
Dynamic Aggregate Supply

\[
\pi_t = E_{t-1}\pi_t + \phi \cdot \left(Y_t - \bar{Y}_t\right) + \nu_t
\]

Phillips Curve

\[
E_t \pi_{t+1} = \pi_t
\]

Adaptive Expectations

\[
\Rightarrow E_{t-1}\pi_t = \pi_{t-1}
\]

\[
\pi_t = \pi_{t-1} + \phi \cdot \left(Y_t - \bar{Y}_t\right) + \nu_t
\]

DAS Curve
The Dynamic Aggregate Supply Curve

\[ \pi_t = \pi_{t-1} + \phi \cdot \left( Y_t - \bar{Y}_t \right) + \nu_t \]

- DAS slopes upward: high levels of output are associated with high inflation. This is because of demand-pull inflation.
The Dynamic Aggregate Supply Curve

\[ \pi_t = \pi_{t-1} + \phi \cdot ( Y_t - \bar{Y}_t ) + \nu_t \]

Assume \( \nu_t = 0 \)

- A change in previous period's inflation shifts the DAS curve
The Dynamic Aggregate Supply Curve

\[ \pi_t = \pi_{t-1} + \phi \cdot (Y_t - \bar{Y}_t) + \nu_t \]

Assume that \( \nu = 0 \)

- An change of the natural level of output will shift the DAS curve
The Dynamic Aggregate Supply Curve

$$\pi_t = \pi_{t-1} + \phi \cdot (Y_t - \bar{Y}_t) + \nu_t$$

Assume that $$\pi_{t=2} = \pi_{t=1} and \nu_{t=3} > 0$$

A supply shock will shift the DAS curve
The *DAS* Curve: Summary

- The *DAS* curve is upward sloping
- When the economy is at full employment, the height of the *DAS* curve equals inherited inflation plus the current supply shock
- When either the previous period’s inflation or the current period’s inflation shock increases (decreases), the *DAS* curve shifts up (down) by the same amount
- When natural GDP increases (decreases), the *DAS* curve shifts right (left) by the same amount
DAD-DAS: 5 Equations

• **The supply side** - **DONE 😊**
  – 1. Phillips Curve
  – 2. Adaptive Expectations

• **The Demand Side**
  – 3. **The Demand Equation**
  – 4. Fisher Equation
  – 5. Monetary Policy Rule
**IS Curve = Demand Equation**

\[ Y_t = \bar{Y}_t - \alpha \cdot (r_t - \rho) + \varepsilon_t \]

- The **IS curve** can simply be renamed the **Demand Equation curve**
The Demand Equation

\[ Y_t = \bar{Y}_t - \alpha \cdot (r_t - \rho) + \varepsilon_t \]

- **Natural (or long-run or potential) Real GDP**
- **Real interest rate**
- **Natural (or long-run) Real interest rate**

- **Real GDP**
- **Parameter representing the response of demand to the real interest rates**
- **Demand shock, represents changes in \( G, T, C_0, \) and \( I_0 \)**
The Demand Equation

Assumption: $\rho > 0$; although the real interest rate can be negative, in the long run people will not lend their resources to others without a positive return.

\[ Y_t = \bar{Y}_t - \alpha \cdot (r_t - \rho) + \varepsilon_t \]

Assumption: There is a negative relation between output ($Y_t$) and interest rate ($r_t$). The justification is the same as for the IS curve.

Positive when $C_0$, $I_0$, or $G$ is higher than usual or $T$ is lower than usual.
DAD-DAS: 5 Equations

- Phillips Curve
- Adaptive Expectations
- Demand Equation
- Fisher Equation
- Monetary Policy Rule
The Real Interest Rate: The Fisher Equation

The real interest rate is the inflation-adjusted interest rate. To adjust the nominal interest rate for inflation, one must simply subtract the expected inflation rate during the duration of the loan (the true future inflation is not known).

\[ r_t = i_t - E_t \pi_{t+1} \]

where:
- \( r_t \) is the real interest rate
- \( i_t \) is the nominal interest rate
- \( E_t \) is the expected inflation rate
- \( \pi_{t+1} \) is the expected inflation rate

\[ \pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \]
The Real Interest Rate: The Fisher Equation

• Using adaptive expectations:

\[ E_t \pi_{t+1} = \pi_t \]

• We get the following relationship:

\[ r_t = i_t - \pi_t \]
DAD-DAS: 5 Equations

- Phillips Curve
- Adaptive Expectations
- Demand Equation
- Fisher Equation
- Monetary Policy Rule
Monetary Policy Rule

• The fifth and final main assumption of the DAD-DAS theory is that
  – The central bank sets the nominal interest rate
  – and, in setting the nominal interest rate, the central bank is guided by a very specific formula called the monetary policy rule
The Nominal Interest Rate: The Monetary-Policy Rule

\[ i_t = \pi_t + \rho + \theta \pi \cdot (\pi_t - \pi_t^*) \]

- **Nominal interest rate rate, set each period by the central bank**
- **Current inflation rate**
- **Natural real interest rate**
- **Parameter that measures how strongly the central bank responds to the inflation gap**
- **Inflation Gap: The excess of current inflation over the central bank’s inflation target**
The Nominal Interest Rate: The Monetary-Policy Rule

- The Central Bank has a desired (target) inflation rate
- It adjusts the interest rate in such a way, as to reach this target
- When current inflation is equal to the target inflation, nominal interest rate is equal to:

\[ i_t = \pi^*_t + \rho \]
The Dynamic Aggregate Demand Curve

Start with the Demand Equation:

\[ Y_t = \bar{Y} - \alpha(r_t - \rho) + \varepsilon_t \]

\[ r_t = i_t - \pi_t \]

Fisher equation with adaptive expectations

\[ Y_t = \bar{Y} - \alpha(i_t - \pi_t - \rho) + \varepsilon_t \]

\[ i_t = \pi_t + \rho + \theta_{\pi} \cdot (\pi_t - \pi^*_t) \]

monetary policy rule

\[ Y_t = \bar{Y} - \alpha[\pi_t + \rho + \theta_{\pi} \cdot (\pi_t - \pi^*_t) - \pi_t - \rho] + \varepsilon_t \]
The Dynamic Aggregate Demand Curve

\[ Y_t = \bar{Y} - \alpha \left[ \theta \pi \left( \pi_t - \pi^* \right) \right] + \varepsilon_t \]

\[ Y_t = \bar{Y} - A \left( \pi_t - \pi^* \right) + \varepsilon_t \]
The Dynamic Aggregate Demand Curve

\[ Y_t = \bar{Y} - A(\pi_t - \pi^*) + \varepsilon_t \]

- **DAD slopes downward:**
  When inflation rises, the central bank raises the real interest rate, reducing the demand for goods and services.

Note that the DAD equation has no dynamics in it: it only shows how simultaneously measured variables are related to each other.
The Dynamic Aggregate Demand Curve

\[ Y_t = \bar{Y} - A(\pi_t - \pi^*) + \varepsilon_t \]

- When the central bank’s target inflation rate increases (decreases) the DAD curve moves up (down) by the exact same amount.

Assume: \( \varepsilon = 0 \)

Note how monetary policy is described in terms of the target inflation rate in the DAD-DAS model.
The Dynamic Aggregate Demand Curve

\[ Y_t = \bar{Y} - A(\pi_t - \pi^*) + \varepsilon_t \]

- When the natural rate of output increases (decreases) the DAD curve moves right (left) by the exact same amount.

- When there is a positive (negative) demand shock the DAD curve moves right (left).

A positive demand shock could be an increase in \( C_0 \), \( I_0 \), or \( G \), or a decrease in \( T \).
The Dynamic Aggregate Demand Curve

\[ Y_t = \bar{Y} - A(\pi_t - \pi^*) + \varepsilon_t \]

- When there is a positive (negative) demand shock the DAD curve moves right (left).

A positive demand shock could be an increase in \( C_0 \), \( I_0 \), or \( G \), or a decrease in \( T \).
The Dynamic Aggregate Demand Curve

\[ Y_t = Y - A(\pi_t - \pi^*) + \varepsilon_t \]

The DAD curve shifts right (or up) if:
1. the central bank’s target inflation rate goes up,
2. there is a positive demand shock, or
3. the natural rate of output increases.
The *DAD* Curve: Summary

- The *DAD* curve is downward sloping
- When the central bank’s target inflation rate increases (decreases), the *DAD* curve shifts up (down) by the same amount
- When natural GDP increases (decreases), the *DAD* curve shifts right (left) by the same amount
- When the demand shock increases (decreases), the *DAD* curve shifts right (left)
**DAS and DAD Equations**

**DAS**

\[
\pi_t = \pi_{t-1} + \phi \cdot (Y_t - \bar{Y}_t) + \nu_t
\]

**DAD**

\[
Y_t = \bar{Y} - A(\pi_t - \pi^*) + \varepsilon_t
\]

- Note that there are *two* endogenous variables—\(Y_t\) and \(\pi_t\)—in these *two* equations
- Therefore, we can solve for the equilibrium values of \(Y_t\) and \(\pi_t\)
Solution

\[ Y_t = \bar{Y} - A(\pi_{t-1} + \phi \cdot (Y_t - \bar{Y}) + \nu_t - \pi^*) + \varepsilon_t \]

\[ Y_t - \bar{Y} = -A\pi_{t-1} - A\phi \cdot (Y_t - \bar{Y}) - A\nu_t + A\pi^* + \varepsilon_t \]

\[ (1 - A\phi)(Y_t - \bar{Y}) = -A(\pi_{t-1} + \nu_t - \pi^*) + \varepsilon_t \]

\[ (Y_t - \bar{Y}) = \frac{-A}{(1 + A\phi)}(\pi_{t-1} + \nu_t - \pi^*) + \frac{\varepsilon_t}{(1 + A\phi)} \]
The model’s variables and parameters

• Endogenous variables:

\[ Y_t = \text{Output} \]
\[ \pi_t = \text{Inflation} \]
\[ r_t = \text{Real interest rate} \]
\[ i_t = \text{Nominal interest rate} \]
\[ E_t \pi_{t+1} = \text{Expected inflation} \]
The model’s variables and parameters

- **Exogenous variables:**
  \[
  \bar{Y}_t = \text{Natural level of output}
  \]
  \[
  \pi_t^* = \text{Central bank’s target inflation rate}
  \]
  \[
  \varepsilon_t = \text{Demand shock}
  \]
  \[
  \nu_t = \text{Supply shock}
  \]
- **Predetermined variable:**
  \[
  \pi_{t-1} = \text{Previous period’s inflation}
  \]
The model’s variables and parameters

- Parameters:
  - $\alpha = \text{Responsiveness of demand to the real interest rate}$
  - $\rho = \text{Natural rate of interest}$
  - $\phi = \text{Responsiveness of inflation to output in the Phillips Curve}$
  - $\theta_{\pi} = \text{Responsiveness of } i \text{ to inflation in the monetary-policy rule}$
Long-Run Equilibrium

\[ \pi^* \]

\[ \bar{Y} \]

\[ DAD \]

\[ DAS \]
The *DAD-DAS* model’s long-run equilibrium

- The long-run equilibrium values in the *DAD-DAS* theory:

\[ Y_t = \bar{Y}_t \]

\[ r_t = \rho \]

\[ \pi_t = \pi_t^* \]

\[ E_t \pi_{t+1} = \pi_t^* \]

\[ i_t = \rho + \pi_t^* \]

*In the short-run, the values of the various variables fluctuate around the long-run equilibrium values.*