Human capital in growth theory & empirics

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Human capital in neoclassical growth theory – the MRW model


\[ Y_t = K_t^\alpha H_t^\beta (AN)^{1-\alpha-\beta} \]

Production function is still neoclassical:
  • Constant returns to scale in K, H, N
  • Marginal product of K, H, N is positive but diminishing
  • Inada conditions hold.

• To make the exposition simpler, we assume A=1
Human capital in neoclassical growth theory – the MRW model

• Production function:

\[ Y_t = K_t^\alpha H_t^\beta N^{1-\alpha-\beta} \]

• ... and the same function in per capita form:

\[ y_t = k_t^\alpha h_t^{1-\alpha} \]

, where:

\[ k = \frac{K_t}{N_t}, h_t = \frac{H_t}{N_t}, y_t = \frac{Y_t}{N_t} \]

The original MRW paper is here: http://eml.berkeley.edu/~dromer/papers/MRW_QJE1992.pdf
Human capital in neoclassical growth theory – the MRW model

- MRW assumed that the accumulation of K and H are governed by very similar processes:
  - $\dot{K} = s_k Y - dK$
  - $\dot{H} = s_h Y - dH$
- where:
  - $s_k$ – rate of saving in physical capital
  - $s_h$ – rate of saving in human capital

The dynamics of both capital and human capital are like in the Solow growth model.

\[
\dot{k} = s_k y - (d + n)k \\
\dot{h} = s_h y - (d + n)h
\]
Like in the Solow model, there is a steady-state:
\[ \dot{k} = s_K y_t - (d + n)k = 0 \]
\[ \dot{h} = s_H y - (d + n)h = 0 \]

The steady-state values are:
\[ \dot{k} = \left( \frac{s_k^{1-\beta} s_h^\beta}{n+d} \right)^{1/(1-\alpha-\beta)} \]
\[ \dot{h} = \left( \frac{s_k^\alpha s_h^{1-\alpha}}{n+d} \right)^{1/(1-\alpha-\beta)} \]
Human capital in neoclassical growth theory – the MRW model

• MRW test empirically the proposition that a modified Solow model „fits” the data; see:

• They also supply other evidence that „Solow-type” growth models are broadly consistent with the actual growth processes.
• The economy reaches a steady-state, where per capita production depends, among others, on the rates of both types of savings: $s_K$ and $s_H$.

• The rate of growth of per capita production in the steady-state is zero.

• The only way to "add" sustained growth to the model is to assume that technology A is growing at a constant rate $g$; then the rate of per capita output growth is given by $g$. 

Human capital in neoclassical growth theory – the MRW model
Human capital in endogenous models

• *Simplified Lucas model (1988)*

• Neoclassical production function:

\[ Y = A(K)^\alpha (H)^{1-\alpha} \]

• Accumulation of K and H is described by a very similar process; however there is no exogenous saving rates, like in MRW

\[ \dot{H} = I_H - dH \quad \text{oraz} \quad \dot{K} = I_K - dK \]

Investing means resigning from current consumption

• \[ Y = C + I_H + I_K \]

• Household maximise utility like in the Ramsey model
Human capital in endogenous growth models

- In equilibrium the rate of return from investing in K has to be the same as the rate of return from investing in H:
  \[ A\alpha \left(\frac{H}{K}\right)^{1-\alpha} - d = A(1 - \alpha) \left(\frac{K}{H}\right)^{\alpha} - d \]
  \[ \frac{\alpha}{1-\alpha} = \frac{K}{H} \]
  - Hence, the K to H ratio is constant and : \( H = \frac{1-\alpha}{\alpha} K \)
  - This implies that the production function:
    \[ Y = A(K)^{\alpha} \left(\frac{1-\alpha}{\alpha} K\right)^{1-\alpha} = A\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} K \]
  - ..is linear ("AK") style
Kapitał ludzki w endogenicznej teorii wzrostu

- We know what this means - at a given saving rate - which is derived from household optimization problem – growth rate is constant & endogenous, given by:

  \[ \gamma = \frac{r - \rho}{\theta}, \]

  where \( r \) is equal to the rate of return from investment in \( K \) and \( H \):

  \[ r = \frac{A(\frac{1}{\alpha})^{1-\alpha}}{\alpha} - d \]
Human capital - data

• Human capital is measure by years of schooling or/and by the scores of standardised school tests

Empirical research

• It is not difficult to find correlation between long run growth and education.

Empirical research

Table 4: From Schooling Institutions to Cognitive Skills to Economic Growth: Instrumental Variable Estimates

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tbody>
<tr>
<td>Second stage:</td>
<td></td>
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<tr>
<td>2SLS:</td>
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<tr>
<td>Cognitive skills</td>
<td>2.151</td>
<td>2.023</td>
<td>3.050</td>
<td>4.091</td>
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<td></td>
<td>(2.73)</td>
<td>(5.81)</td>
<td>(5.32)</td>
<td>(3.20)</td>
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<td>Fuller (1) modification of LIML:</td>
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<tr>
<td>Cognitive skills</td>
<td>2.121</td>
<td>2.022</td>
<td>3.036</td>
<td>3.871</td>
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<tr>
<td></td>
<td>(3.01)</td>
<td>(5.94)</td>
<td>(3.42)</td>
<td>(3.21)</td>
</tr>
<tr>
<td>Moreira 95% confidence band:</td>
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<td></td>
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<tr>
<td>Cognitive skills</td>
<td>[-3.888, 19.871]</td>
<td>[1.190, 2.868]</td>
<td>[1.601, 4.689]</td>
<td>[1.626, 9.499]</td>
</tr>
<tr>
<td>p-value</td>
<td>(0.100)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.001)</td>
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<td>First stage (dependent variable: cognitive skills):</td>
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<td></td>
<td></td>
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<tr>
<td>External exit exam system</td>
<td>0.286</td>
<td>0.286</td>
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<tr>
<td></td>
<td>(2.01)</td>
<td>(2.01)</td>
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<tr>
<td>Initial years of schooling</td>
<td>0.176</td>
<td>0.132</td>
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<td></td>
<td>(4.11)</td>
<td>(3.83)</td>
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<td>(2.00)</td>
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<tr>
<td>Private enrollment share</td>
<td>0.493</td>
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<td></td>
<td>(2.15)</td>
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<tr>
<td>Centralization (share) of decisions on organization of instruction</td>
<td>-0.993</td>
<td>-2.49</td>
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</tr>
</tbody>
</table>

Notes: Dependent variable (of the second stage): average annual growth rate in GDP per capita, 1960-2000. Control variables: Initial GDP per capita, initial years of schooling, and a constant. Test scores are average of math and science, primary through end of secondary school, all years. t-statistics in parentheses unless otherwise noted.

Source: Hanushek & Woessmann, 2009
**Empirical research**

Table 7: Rocket Scientists or Basic Education for All?

<table>
<thead>
<tr>
<th>(1)</th>
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<th>(5)²</th>
<th>(6)²</th>
<th>(7)²</th>
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</thead>
<tbody>
<tr>
<td>Share of students reaching basic literacy</td>
<td>4.717</td>
<td>2.732</td>
<td>1.002</td>
<td>3.460</td>
<td>5.159</td>
<td>5.869</td>
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<td>(6.64)</td>
<td>(3.61)</td>
<td>(1.33)</td>
<td>(3.81)</td>
<td>(2.87)</td>
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<td>(2.653)</td>
<td>(4.35)</td>
<td>(4.18)</td>
<td>(2.37)</td>
<td>(0.65)</td>
<td>(0.22)</td>
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<tr>
<td>Share of students reaching basic literacy x Initial GDP per capita</td>
<td>0.376</td>
<td>1.25</td>
<td>-2.148</td>
<td>-1.649</td>
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<td></td>
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<tr>
<td>(2.11)</td>
<td>(0.97)</td>
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</tr>
<tr>
<td>Share of top-performing students x Initial GDP per capita</td>
<td>42.357</td>
<td>53.538</td>
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</tr>
<tr>
<td>(1.49)</td>
<td>(1.91)</td>
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</tr>
</tbody>
</table>

**Notes:** Dependent variable: average annual growth rate in GDP per capita, 1960-2000. Control variables: GDP per capita 1960, years of schooling 1960, and a constant. Shares are based on average test scores in math and science, primary through end of secondary school, all years. t-statistics in parentheses.

a. Specification includes additional controls for openness, property rights, fertility, and tropical location.

b. All interacted variables are centered on zero.

Source: Hanushek & Woessmann, 2009
Education & inequality

Andersen, 2015;

Glaeser et al., 2007
Summary

• Human capital is an important factor of production
• The simultaneous accumulation of physical and human capital allows to „escape” from the decreasing marginal product of capital
• Empirical research indicates that human capital is correlated with long run growth rate
• Policies aiming at an equal access to high-quality education are one of the most important policies that support long-term growth, equality and democracy