University of Warsaw
Faculty of Economic Sciences

Endogenous Growth (second generation)

Advanced Macroeconomics IE: Lecture 15

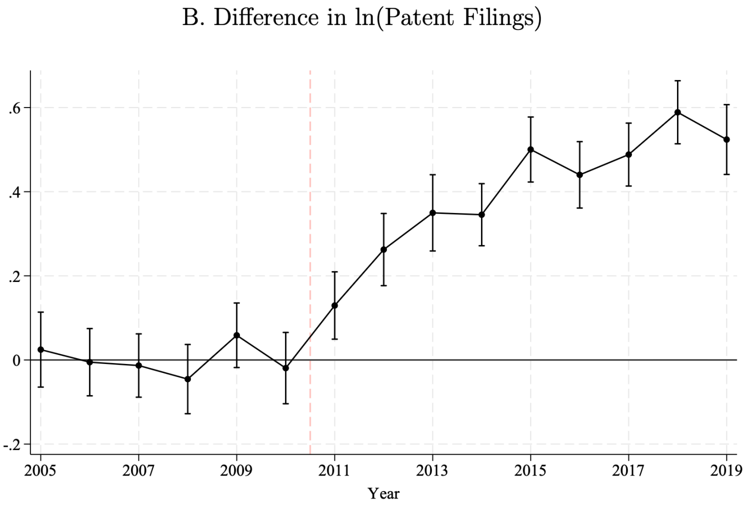
Marcin Bielecki

Spring 2026

University of Warsaw

Increasing product variety (horizontal innovation)

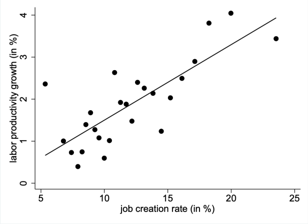
Effects of increasing patent duration on certain antibiotic types



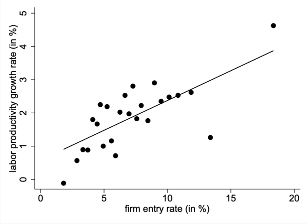
Kong, Zhao (2025)

Increasing product quality (vertical innovation)

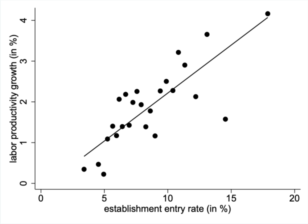
Productivity growth and measures of creation (individual sectors in USA)



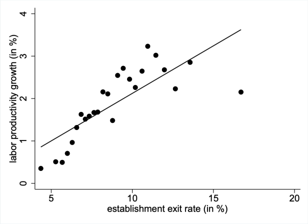
(a) Job creation rate



(b) Firm entry rate

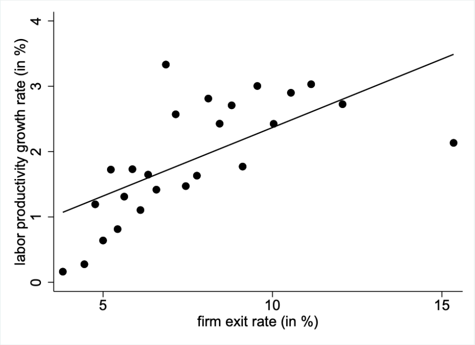


(c) Establishment entry rate

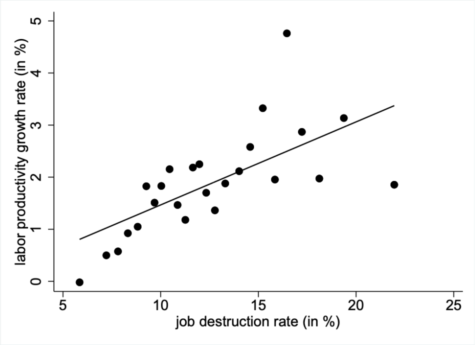


(d) Establishment exit rate

Productivity growth and measures of destruction (individual sectors in USA)



(a) Firm exit rate

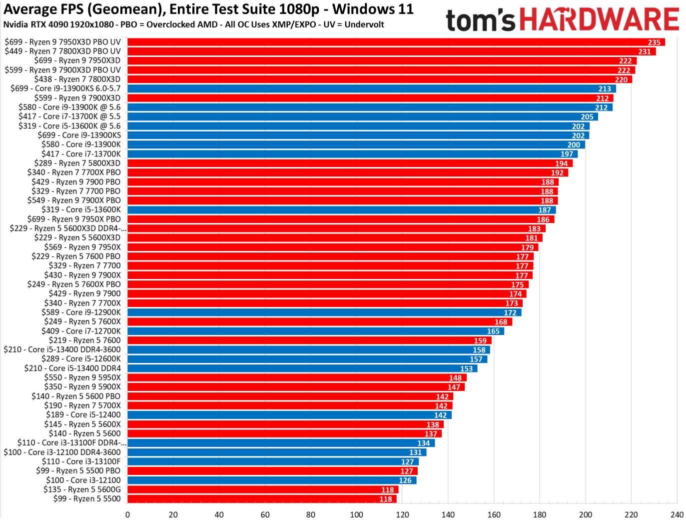


(b) Job destruction rate

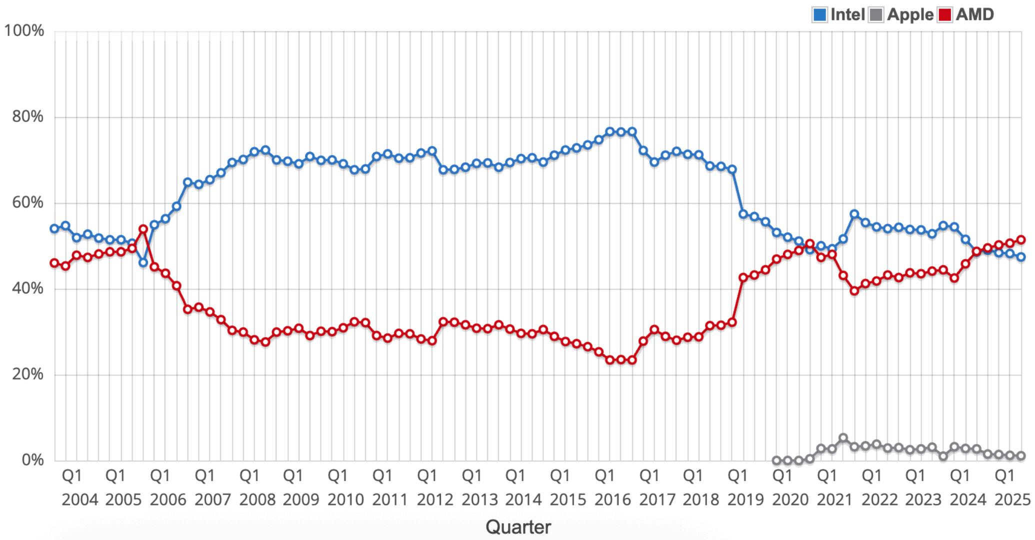
2025 Nobel Prize: Mokyr, Aghion, Howitt

Innovation and market segmentation

PC CPU performance ranking (2025)



PC CPU market shares



Passmark

Capital accumulation and innovation

Balanced Growth Path General Equilibrium

BGP growth rate g^* depends positively on $\hat{k}^*$ (**GG** curve)

$$g^* = z^* q = q\left[\eta d(\hat{k}^*)\ell^* - r(\hat{k}^*) - \epsilon\right] = q\left[\eta\,(1-\alpha)\,\alpha(\hat{k}^*)^{\alpha}\frac{\epsilon+n}{\psi} - \left(\alpha^2(\hat{k}^*)^{\alpha-1} - \delta\right) - \epsilon\right]$$

Higher capital per effective labor $\hat{k}^*$ means higher profits and lower interest rates

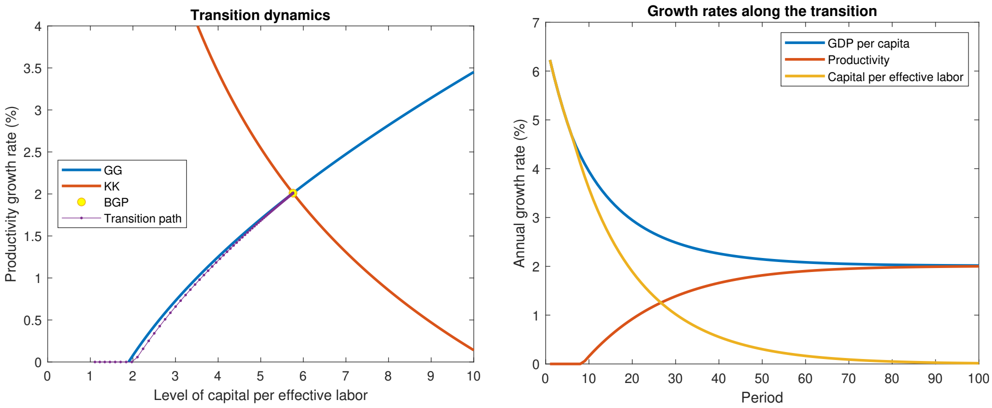
In turn $\hat{k}^*$ depends negatively on g^*: from the Euler equation we get

$$g_c^* = g^* = \frac{r-\rho-n}{\sigma} = \frac{\alpha^2(\hat{k}^*)^{\alpha-1} - \delta - \rho - n}{\sigma}$$

KK curve

$$\hat{k}^* = \left(\frac{\alpha^2}{\rho+\delta+n+\sigma g^*}\right)^{1/(1-\alpha)}$$

Model dynamics

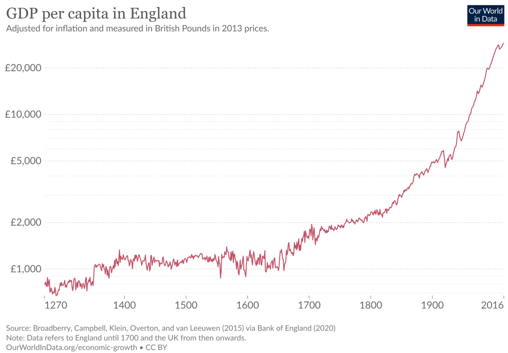


Whenever $\hat{k} < \hat{k}^*$, the rate of growth of productivity is $0 \leq g < g^*$

Initially capital accumulation is the main driver of growth

Improvements in productivity become increasingly more important and in the long run are the sole driver of GDP per worker growth

Economic growth in the very long run



Adding the "minimal" consumption (Stone-Geary) makes convergence to BGP slow:

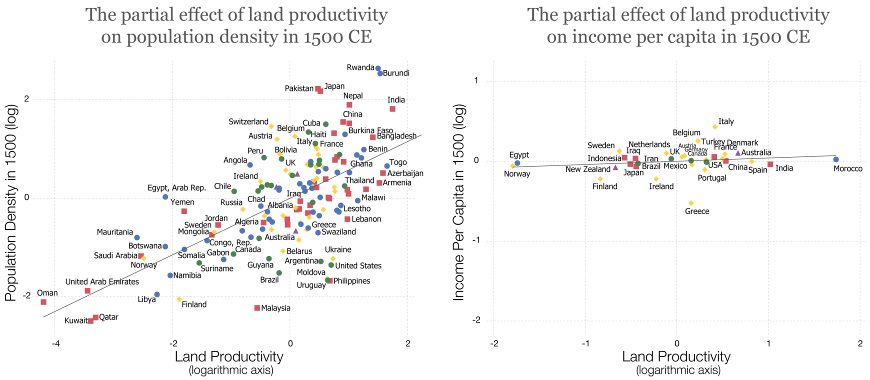
- slow capital accumulation
- slow increase in productivity (even stagnation), growing number of "brains"
- initial innovation "accidental" (scientific revolution: $A \uparrow$ and $\eta \uparrow$),
  only later we get industrial innovation (industrial revolution), 20th century is BGP

Malthusian trap

In the Malthusian Economy productivity produces people not prosperity

Our World in Data

This figure depicts the partial regression line for the effect of land productivity on income per capita in the year 1500 CE, while controlling for the influence of land productivity, absolute latitude, access to waterways, and continental fixed effects.
The x- and y-axes plot the residuals obtained from regressing land productivity and income per capita, respectively, on the aforementioned set of covariates.
The color represents the continent of the country: Africa Europe Asia Oceania Americas



Data source: Quamrul Ashraf and Oded Galor (2011) – *Dynamics and Stagnation in the Malthusian Epoch.* American Economic Review, 101(5): 2003-41.
This is a data visualization from OurWorldinData.org. There you find more visualizations and research on how the world is changing.

James Watt's steam engine patent from 1769

A.D. 1769 . . . . N° 913.

Steam Engines, &c.

WATT'S SPECIFICATION.

TO ALL TO WHOM THESE PRESENTS SHALL COME, I, James Watt, of Glasgow, in Scotland, Merchant, send greeting.

WHEREAS His most Excellent Majesty King George the Third, by His Letters Patent under the Great Seal of Great Britain, bearing date the Fifth day of January, in the ninth year of His said Majesty's reign, did give and grant unto me, the said James Watt, His special licence, full power, sole priviledge and authority, that I, the said James Watt, my exõrs, adm̃ors, and assigns, should and lawfully might, during the term of years therein expressed, use, exercise, and vend, throughout that part of His Majesty's Kingdom of Great Britain called England, the Dominion of Wales, and Town of Berwick upon Tweed, and also in His Majesty's Colonies and Plantations abroad, my "**New Invented Method of Lessening the Consumption of Steam and Fuel in Fire Engines**;" in which said recited Letters Patent is contained a proviso obliging me, the said James Watt, by writing under my hand and seal, to cause a particular description of the nature of the said Invention to be inrolled in His Majesties High Court of Chancery within four calendar months after the date of the said recited Letters Patent, as in and by the said Letters Patent, and the Statute in that behalf made, relation being thereunto respectively had, may more at large appear.

NOW KNOW YE, that in compliance with the said provisoe, and in pursuance of the said Statute, I, the said James Watt, do hereby declare that the

following is a particular description of the nature of my said Invention, and of the manner in which the same is to be performed (that is to say):—

My method of lessening the consumption of steam, and consequently fuel, in fire engines consists of the following principles:—

First, that vessell in which the powers of steam are to be employed to work the engine, which is called the cylinder in common fire engines, and which I call the steam vessell, must during the whole time the engine is at work be kept as hot as the steam that enters it, first, by enclosing it in a case of wood or any other materials that transmit heat slowly; secondly, by surrounding it with steam or other heated bodies; and, thirdly, by suffering neither water or any other substance colder than the steam to enter or touch it during that time.

Secondly, in engines that are to be worked wholly or partially by condensation of steam, the steam is to be condensed in vessells distinct from the steam vessells or cylinders, although occasionally communicating with them. These vessells I call condensers, and whilst the engines are working, these condensers ought at least to be kept as cold as the air in the neighbourhood of the engines by application of water or other cold bodies.

Thirdly, whatever air or other elastic vapour is not condensed by the cold of the condenser, and may impede the working of the engine, is to be drawn out of the steam vessells or condensers, by means of pumps wrought by the engines themselves, or otherwise.

Fourthly, I intend in many cases to employ the expansive force of steam to press on the pistons, or whatever may be used instead of them, in the same manner as the pressure of the atmosphere is now employed in common fire engines. In cases where cold water cannot be had in plenty, the engines may be wrought by this force of steam only, by discharging the steam into the open air after it has done its office.

Fifthly, where motions round an axis are required, I make the steam vessells in form of hollow rings or circular channels, with proper inletts and outletts for the steam, mounted on horizontal axles like the wheels of a water mill; within them are placed a number of valves that suffer any body to go round the channell in one direction only. In these steam vessells are placed weights, so fitted to them as intirely to fill up a part or portion of their channels, yet rendered capable of moving freely in them by the means herein-after mentioned or specified. When the steam is admitted in these engines between these weights and the valves, it acts equally on both, so as to raise the weight to one side of the wheel, and by the reaction on the valves successively to give a circular motion to the wheel, the valves opening in the direction in which the

weights are pressed, but not in the contrary. As the steam vessel moves round it is supplied with steam from the boiler, and that which has performed its office may either be discharged by means of condensers, or into the open air.

Sixthly, I intend in some cases to apply a degree of cold not capable of reducing the steam to water, but of contracting it considerably, so that the engines shall be worked by the alternate expansion and constraction of the steam.

Lastly, instead of using water to render the piston or other parts of the engines air and steam tight, I employ oils, wax, rosinous bodies, fat of animals, quicksilver and other metalls, in their fluid state.

In witness whereof, I have hereunto set my hand and seal, this Twenty-fifth day of April, in the year of our Lord One thousand seven hundred and sixty-nine.

JAMES WATT. (L.S.)

Sealed and delivered in the presence of

Coll. Wilkie.
Geo. Jardine.
John Roebuck.

Be it remembered, that the said James Watt doth not intend that any thing in the fourth article shall be understood to extend to any engine where the water to be raised enters the steam vessell itself, or any vessell having an open communication with it.

James Watt.

Witnesses,

Coll. Wilkie.
Geo. Jardine.

AND BE IT REMEMBERED, that on the Twenty-fifth day of April, in the year of our Lord 1769, the aforesaid James Watt came before our said Lord the King in His Chancery, and acknowledged the Specification aforesaid, and all and every thing therein contained and specified, in form above written. And also the Specification aforesaid was stampt according to the tenor of the Statute made in the sixth year of the reign of the late King and Queen William and Mary of England, and so forth.

Wilkie, Extr.

Inrolled the Twenty-ninth day of April, in the year of our Lord One thousand seven hundred and sixty-nine.

LONDON:
Printed by George Edward Eyre and William Spottiswoode,
Printers to the Queen's most Excellent Majesty. 1855.

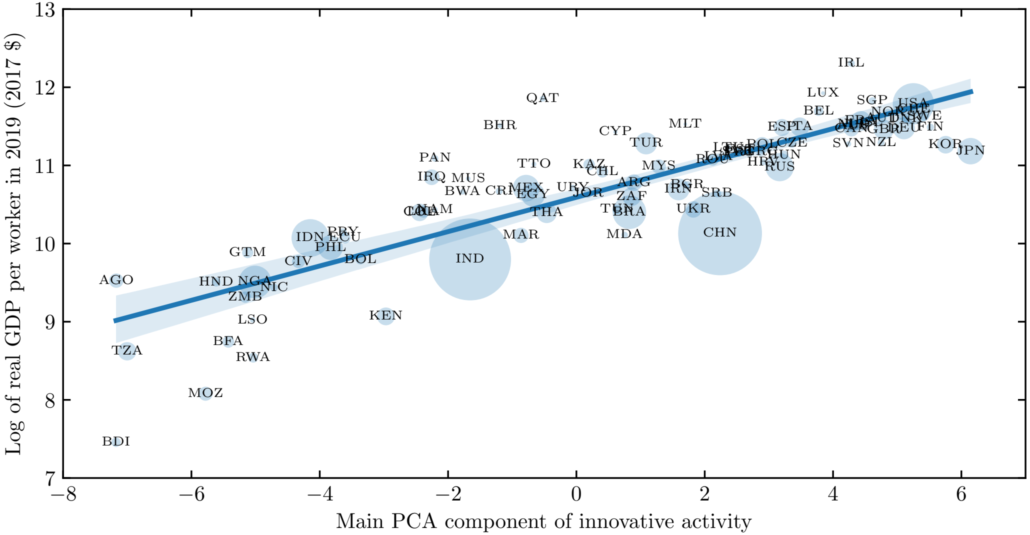
Wikimedia Commons

Patents awarded during the industrial revolution

Our World in Data

International technology transfer

Advanced economies are innovation leaders



International technology transfer

Based on Aghion and Howitt (1999) The Economics of Growth, chapter 7.2

Assume two groups of countries: technology leaders and technology followers

- Technology leaders invent new technologies: their rate of growth is explained by the previous model and is denoted with $\bar{g}$
- Technology followers adopt / imitate the leading technologies

Probability of successful adoption / imitation of leading technology $\bar{A}$ is z

$$A_{i,t+1} = \begin{cases} \bar{A}_{it} & \text{with probability} \quad z \\ A_{it} & \text{with probability} \quad 1-z \end{cases}$$

Higher z translates to higher productivity growth

$$A_{t+1} = \frac{1}{M}\sum_{i=1}^{M} A_{i,t+1} = \frac{1}{M}\sum_{i=1}^{M}\left[z\bar{A}_{it} + (1-z)\,A_{it}\right] = z\frac{1}{M}\sum_{i=1}^{M}\bar{A}_{it} + (1-z)\,\frac{1}{M}\sum_{i=1}^{M}A_{it}$$

$$A_{t+1} = z\bar{A}_t + (1-z)\,A_t$$

Proximity to technology frontier

Proximity to technology frontier $a_t \equiv A_t/\bar{A}_t$

Dynamics of proximity

$$A_{t+1} = z\bar{A}_t + (1-z)\,A_t \quad \Big| \quad : \bar{A}_t$$

$$\frac{A_{t+1}}{\bar{A}_t} = \frac{A_{t+1}}{\bar{A}_{t+1}}\frac{\bar{A}_{t+1}}{\bar{A}_t} = z + (1-z)\,\frac{A_t}{\bar{A}_t} = z + (1-z)\,a_t$$

$$a_{t+1}\,(1+\bar{g}) = z + (1-z)\,a_t$$

$$a_{t+1} = \frac{z + (1-z)\,a_t}{1+\bar{g}}$$

BGP proximity

$$a^*\,(1+\bar{g}) = z + (1-z)\,a^*$$

$$a^*\,(\bar{g}+z) = z$$

$$a^* = \frac{z}{z+\bar{g}} < 1$$

Proximity to technology frontier and growth

BGP growth rate of technology followers is also $\bar{g}$

$$g = \frac{A_{t+1} - A_t}{A_t} - 1 = \frac{z\bar{A}_t + (1 - z)\,A_t}{A_t} - 1 = \frac{z}{a^*} + (1 - z) - 1 = \bar{g} + z - z = \bar{g}$$

Before they converge to BGP, tech followers grow faster (“advantage of backwardness”)

$$q_t \equiv \frac{\bar{A}_t - A_t}{A_t} = \frac{1}{a_t} - 1 \geq \frac{1}{a^*} - 1$$

$$g_t = zq_t = z\left(\frac{1}{a_t} - 1\right) \geq \bar{g}$$

Domestic $z = \eta d(\hat{k})\ell - r(\hat{k}) - \epsilon$ does not determine growth rate,
but instead determines **relative GDP per worker**

Determinants of country “rank” in world GDP per worker distribution

- R&D productivity η (quantity and quality of human capital, top universities)
- Firm profitability d (efficient bureaucratic and legal system, no corruption)
- Financing conditions r (efficient equity markets, access to venture capital funds)

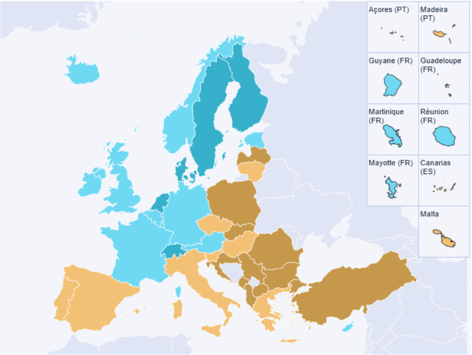
Innovation activity in Europe

Year: 2024

Comparison year: Change vs 2023

Country scope: EU and neighbouring countries

Zoom using the mouse wheel and pan by dragging.



Administrative boundaries: © EuroGeographics © UN–FAO © Turkstat
The boundaries and names shown and the designations used on this map do not imply official endorsement or acceptance by the European Union.

Innovation Leaders

Switzerland	152.2	▲ 1.9
Denmark	149.3	▲ 0.5
Sweden	146.2	▲ 0.2
Finland	140.6	▼ 0.7
Netherlands	138.3	▼ 0.5

Strong Innovators

Belgium	136.0	▼ 0.2
Norway	128.7	▲ 0.9
Austria	127.9	▼ 1.2
United Kingdom	126.3	▼ 2.1
Ireland	124.5	▲ 2.1
Luxembourg	123.3	▼ 2.5
Germany	122.8	▼ 3.4
Cyprus	116.9	▲ 3.3
Estonia	115.3	▼ 2.9
France	114.4	▲ 0.3
Iceland	110.6	▼ 1.0

Moderate Innovators

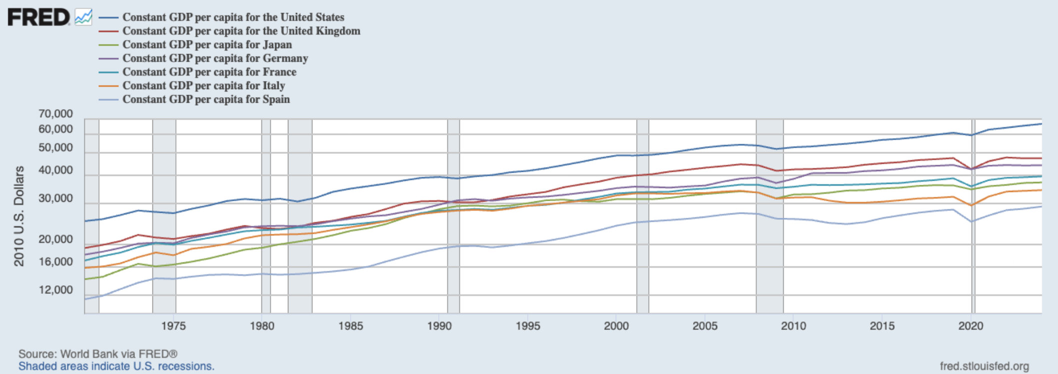
Slovenia	100.1	▲ 1.5
Spain	98.9	▲ 2.0
Czechia	98.7	▲ 2.3
Italy	98.6	▲ 0.8
Malta	96.8	▼ 1.9
Lithuania	92.0	▲ 3.7
Portugal	91.8	▲ 0.5
Greece	85.3	▼ 1.0
Hungary	77.6	▲ 2.1

Emerging Innovators

Croatia	76.6	▼ 0.1
Poland	72.5	▲ 3.3
Slovakia	71.6	▲ 1.5
Serbia	69.1	▲ 4.4
Latvia	59.0	▼ 0.2
Turkey	56.9	▲ 5.7
Montenegro	52.3	▲ 0.8
Bulgaria	50.6	▲ 1.9

European Innovation Scoreboard (2024)

Advanced economies grow together, but persistently differ in GDP levels



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