Marcin Bielecki, Advanced Macroeconomics IE, Spring 2026
Homework 5, deadline: May 5, 4:45 PM

Problem 1 (Problem 5 from Homework 4)

Consider an "endowment" version of the two-period OLG model. N_t^y agents are born in time t, where $N_t^y = (1+n)^t N_0^y$. Normalize $N_0^y = 1$ and let $n > 0$. Preferences of a young agent born in time t are:

$$U_t = \ln c_t^y + \beta \ln c_{t+1}^o$$

The initial old agents want to consume as much as possible. Each young agent is endowed with y units of the consumption good. The old have no endowment whatsoever. There is a storage technology that allows to convert one unit of period t goods into $1+r$ units of period $t+1$ goods. There is a social security system that is "pay-as-you-go". In each period t the government taxes the young and uses the receipts to make transfers to the old. We consider a per capita tax on the young that is constant over time, i.e. $\tau_t = \tau$ for all $t = 0, 1, 2, \ldots$

(a) What is the government budget constraint in period t? Write it both in aggregate and in per person terms.

(b) Solve for the competitive equilibrium consumption levels. Find the savings of the representative young agent a_{t+1}.

(c) What is the effect of an increase in τ on savings of the representative young agent?

(d) What is the optimal tax rate τ if $n < r$? Explain why.

Problem 2

Consider the standard OLG model. For simplicity assume no technological progress: $g = 0$, $A = 1$ and $\delta = 1$.

(a) Solve the following utility maximization problem of the household for the CRRA utility function:

$$\begin{aligned} \max \quad & U = \frac{\left(c_t^y\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(c_{t+1}^o\right)^{1-\sigma}}{1-\sigma} \\ \text{subject to} \quad & c_t^y + a_{t+1} = w_t \\ & c_{t+1}^o = (1 + r_{t+1})\, a_{t+1} \end{aligned}$$

(b) Verify that for $\sigma = 1$ the solution from (a) reduces to expressions obtained in the lectures. How the consumption of young and their accumulated assets prior to retirement depend on the expected rate of return on assets (r) for different σ?

(c) Obtain equilibrium factor prices by solving the profit maximization problem of the firms:

$$\max_{K,L} \quad D_t = K_t^\alpha L_t^{1-\alpha} - (r_t + \delta)\, K_t - w_t L_t$$

(d) Show that in the OLG model the goods market is in equilibrium if the labor and asset markets are in equilibrium.

(e) Assume $\sigma = 1$ for simplicity and derive the steady state level of capital per worker k^*.

Problem 3

The so-called social planner solves the following utility maximization problem:

$$\begin{aligned} \max_{\{c_t,\, k_{t+1}\}_{t=0}^{\infty}} \quad & U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ \text{subject to} \quad & c_t + k_{t+1} = A k_t^{\alpha} + (1-\delta)\, k_t \\ & k_0 > 0 \end{aligned}$$

(a) Find the first order conditions for the choice of c_t and k_{t+1}. Obtain the Euler equation. Is it any different to the Euler equation obtained in class for the decentralized case?

(b) Compute the steady state values of k and c.

(c) How do they change in response to changes in A, δ and β?

(d) Use the following assumptions $\sigma = 1$ and $\delta = 1$. Assuming that household behavior can be expressed as $c_t = (1-s)\, y_t$, where s is a constant, find the value of s. *Hint: use the Euler equation first.*

(e) Find the expression for k_{t+1} as a function of k_t and model parameters.

Problem 4

Consider a Ramsey-Cass-Koopmans economy where for simplicity we assume $n = g = 0$ and $N = A = 1$. The representative households solve the following utility maximization problem:

$$\begin{aligned} \max_{\{c_t,\, a_{t+1}\}_{t=0}^{\infty}} \quad & U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ \text{subject to} \quad & c_t + a_{t+1} = w_t + (1+r_t)\, a_t + d_t + v_t \end{aligned}$$

where v is the lump-sum transfer from the government to households.

The representative firm solves the following profit maximization problem:

$$\begin{aligned} \max_{Y_t,\, K_t,\, L_t} \quad & D_t = (1-\tau^y)\, Y_t - (r_t + \delta)\, K_t - w_t L_t \\ \text{subject to} \quad & Y_t = K_t^{\alpha} L_t^{1-\alpha} \end{aligned}$$

where τ^y is the firm revenue tax (equivalent to taxing all households' income regardless of its source).

(a) Derive the first order conditions of the household.

(b) Recast the problem of the firm in per worker terms. Derive the first order conditions of the firm.

(c) Write down the government budget constraint. Using the assumptions of closed economy and balanced government budget, find the conditions for general equilibrium in this economy.

(d) Find the steady state level of capital per worker k^* and consumption per worker c^* in this economy. Discuss how they depend on the tax rate τ^y.

Problem 5

Consider the effects of increasing government spending in the Ramsey model with endogenous labor supply. Households maximize their utility subject to the standard budget constraint:

$$\begin{aligned}\max_{\{c_t,\, h_t,\, a_{t+1}\}_{t=0}^{\infty}} \quad & U = \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \chi \ln \gamma_t + \psi \ln (1 - h_t)\right] \\ \text{subject to} \quad & c_t + a_{t+1} = w_t h_t + (1 + r_t)\, a_t + d_t - \tau_t\end{aligned}$$

The problem of the firms is the same as in Problem 4, with $\tau^y = 0$. The government taxes households via the lump-sum tax τ and maintains a balanced budget at all times:

$$\gamma_t = \tau_t$$

And the government spending is a constant fraction ω of GDP:

$$\gamma_t = \omega y_t$$

(a) Derive the first order conditions of the households.

(b) Find the conditions for general equilibrium in this economy.

(c) Find the steady state of the system for given ω. You will need to utilize variables per hour worked, e.g. k/h, y/h, etc.

(d) Suppose now that the government increases its expenditure to $\omega' > \omega$. What is the effect on this higher government spending on GDP?

(e) What is the effect of increased government spending from (d) on consumption and hours worked? If $\chi = 0$, what would be the impact of higher government spending on welfare (utility)? What if $\chi > 0$?