Chapter 2

Neoclassical Labor Market

In the previous chapter we learned how an optimizing household chooses to optimally allocate their consumption across multiple time periods given an exogenous stream of income. In this chapter we take a first look at the households decisions that influence their income, namely how much to work given the current market wage and other circumstances.

2.1 Single period labor supply choice problem

An agent enjoys consumption c and leisure, defined as the difference between total time endowment in hours T and hours worked (labor supply) h. The agent solves the following utility maximization problem:

$$\max_{c,\,h} \quad U = \ln c + \psi \ln (T - h)$$
$$\text{subject to} \quad c = wh + x$$

where $\psi \geq 0$ describes the relative preference toward leisure, w is hourly wage (taken as given), and x is exogenous non-labor income.

Lagrangian:

$$\mathcal{L} = \ln c + \psi \ln (T - h) + \lambda [wh + x - c]$$

First order conditions:

$$c: \quad \frac{1}{c} - \lambda = 0 \qquad \rightarrow \quad \lambda = \frac{1}{c}$$
$$h: \quad -\frac{\psi}{T-h} + \lambda w = 0 \qquad \rightarrow \quad \lambda w = \frac{\psi}{T-h}$$

Optimality condition:

$$\frac{w}{c} = \frac{\psi}{T-h} \quad \rightarrow \quad c = \frac{w(T-h)}{\psi} \quad \rightarrow \quad h = T - \frac{\psi c}{w}$$

We will call this particular optimality condition the labor-consumption choice condition.

Let us plug in the labor-consumption choice condition into the budget constraint:

$$\begin{aligned}
c &= wh + x \\
c &= w\left(T - \frac{\psi c}{w}\right) + x \\
c &= wT - \psi c + x \\
c(1+\psi) &= wT + x \\
c &= \frac{1}{1+\psi}(wT + x) \\
h &= T - \frac{\psi}{w}\left[\frac{1}{1+\psi}(wT + x)\right] \\
h &= T - \frac{\psi}{1+\psi}\left(T + \frac{x}{w}\right) \\
h &= \frac{1}{1+\psi}T - \frac{\psi}{1+\psi}\frac{x}{w}
\end{aligned}$$

Optimal choice of consumption and hours worked (labor supply):

$$c = \frac{1}{1+\psi}(wT + x)$$
$$h = \frac{1}{1+\psi}T - \frac{\psi}{1+\psi}\frac{x}{w}$$

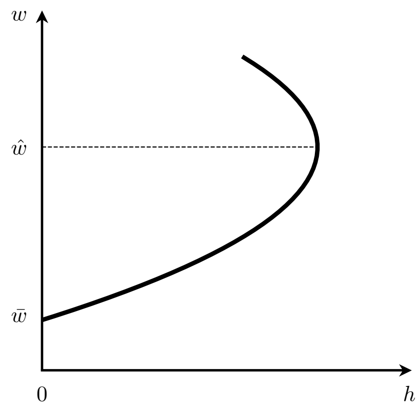
Optimal choice properties:

$$\frac{\partial c}{\partial w} = \frac{T}{1+\psi} > 0 \qquad \frac{\partial c}{\partial T} = \frac{w}{1+\psi} > 0 \qquad \frac{\partial c}{\partial x} = \frac{1}{1+\psi} > 0$$
$$\frac{\partial h}{\partial w} = \frac{\psi}{1+\psi}\frac{x}{w^2} \geq 0 \qquad \frac{\partial h}{\partial T} = \frac{1}{1+\psi} > 0 \qquad \frac{\partial h}{\partial x} = -\frac{\psi}{1+\psi}\frac{1}{w} < 0$$

Obviously, everything that boosts income exerts a positive effect on consumption. Labor supply increases with total available time and decreases with non-labor income. The effect of an increase in hourly wage for this utility function is positive, except for the case $x = 0$, when the labor supply becomes perfectly inelastic. While easy to work with, the logarithmic utility yields very strong conclusions that are not supported by data.

In the general case, the sign of $\partial h/\partial w$ depends on whether the substitution or income effect dominates. On one hand, higher wages mean that leisure is more "costly" – the opportunity cost of not working is higher. This incentivizes the agent to substitute away from leisure to working. On the other hand, higher wages exert positive income effect, so the agent can work less and still earn the same amount as before. This incentivizes the agent to decrease her labor supply. As usual, the sign of $\partial h/\partial w$ is ambiguous, and is positive (negative) when the substitution effect is stronger (weaker) than the income effect.

Microeconomic literature admits the possibility of a backwards-bending labor supply curve. Basically, there can exist a level of wage $\hat{w}$, where if $w < \hat{w}$ then $\partial h/\partial w > 0$ and if $w > \hat{w}$ then $\partial h/\partial w < 0$:



However, usually in macroeconomics we assume utility functions that imply $\partial h/\partial w > 0$ for all $w > 0$.

Reservation wage

An important matter to discuss is the existence of a reservation wage, which is the wage level for which the optimal labor supply of the agent is exactly 0. That is, the agent is willing to work if the prevailing market wage is above the reservation wage level and remains out of labor force if the prevailing market wage is below the reservation wage level. To find this level we equate the optimal labor supply to zero:

$$
\begin{aligned}
0 &= \frac{1}{1+\psi}T - \frac{\psi}{1+\psi}\frac{x}{w} \\
\frac{1}{1+\psi}T &= \frac{\psi}{1+\psi}\frac{x}{w} \\
Tw &= \psi x \\
\bar{w} &= \psi \frac{x}{T}
\end{aligned}
$$

It is apparent that the reservation wage $\bar{w}$ increases with higher non-labor income x and with stronger preference towards leisure ψ. It decreases with the total time endowment T, as the marginal utility of leisure becomes then lower.

2.2 Two-period problem

The agent solves the following utility maximization problem:

$$\begin{aligned} \max_{c_t,\, h_t,\, c_{t+1},\, h_{t+1},\, a_{t+1}} \quad & U = \ln c_t + \psi \ln (1 - h_t) + \beta \left[\ln c_{t+1} + \psi \ln (1 - h_{t+1}) \right] \\ \text{subject to} \quad & c_t + a_{t+1} = w_t h_t \\ & c_{t+1} = w_{t+1} h_{t+1} + (1 + r)\, a_{t+1} \end{aligned}$$

where the total time endowment T has been normalized to 1, so that h has now the interpretation of a fraction of total time available that is spent working.

Lifetime budget constraint:

$$c_t + \frac{c_{t+1}}{1+r} = w_t h_t + \frac{w_{t+1} h_{t+1}}{1+r}$$

Lagrangian:

$$\mathcal{L} = \ln c_t + \psi \ln (1 - h_t) + \beta \left[\ln c_{t+1} + \psi \ln (1 - h_{t+1}) \right] + \lambda \left[w_t h_t + \frac{w_{t+1} h_{t+1}}{1+r} - c_t - \frac{c_{t+1}}{1+r} \right]$$

First order conditions:

$$\begin{aligned} c_t &: \quad \frac{1}{c_t} - \lambda = 0 & \rightarrow \quad & \lambda = \frac{1}{c_t} \\ h_t &: \quad -\frac{\psi}{1 - h_t} + \lambda w_t = 0 & \rightarrow \quad & \lambda = \frac{\psi}{1 - h_t} \frac{1}{w_t} \\ c_{t+1} &: \quad \frac{\beta}{c_{t+1}} - \frac{\lambda}{1+r} = 0 & \rightarrow \quad & \lambda = \beta (1 + r) \frac{1}{c_{t+1}} \\ h_{t+1} &: \quad -\frac{\beta \psi}{1 - h_{t+1}} + \frac{\lambda w_{t+1}}{1+r} = 0 & \rightarrow \quad & \lambda = \beta (1 + r) \frac{\psi}{1 - h_{t+1}} \frac{1}{w_{t+1}} \end{aligned}$$

Euler equation:

$$\frac{1}{c_t} = \beta (1 + r) \frac{1}{c_{t+1}} \quad \rightarrow \quad c_{t+1} = \beta (1 + r)\, c_t$$

Labor supply choice in period t:

$$\begin{aligned} \frac{1}{c_t} &= \frac{\psi}{1 - h_t} \frac{1}{w_t} \\ 1 - h_t &= \frac{\psi}{w_t} c_t \\ h_t &= 1 - \frac{\psi}{w_t} c_t \end{aligned}$$

Labor supply choice in period $t + 1$:

$$\begin{aligned} \frac{1}{c_t} &= \beta (1 + r) \frac{\psi}{1 - h_{t+1}} \frac{1}{w_{t+1}} \\ 1 - h_{t+1} &= \beta (1 + r) \frac{\psi}{w_{t+1}} c_t \\ h_{t+1} &= 1 - \beta (1 + r) \frac{\psi}{w_{t+1}} c_t \end{aligned}$$

We have expressed all choice variables that appear in the lifetime budget constraint as functions of c_t.

Plug all optimality conditions into the budget constraint:

$$\begin{aligned}
c_t + \frac{c_{t+1}}{1+r} &= w_t h_t + \frac{w_{t+1} h_{t+1}}{1+r} \\
c_t + \frac{\beta(1+r)c_t}{1+r} &= w_t\left(1 - \frac{\psi}{w_t}c_t\right) + \frac{w_{t+1}\left(1-\beta(1+r)\frac{\psi}{w_{t+1}}c_t\right)}{1+r} \\
c_t + \beta c_t &= w_t - \psi c_t + \frac{w_{t+1}}{1+r} - \beta\psi c_t \\
c_t(1+\beta+\psi+\beta\psi) &= w_t + \frac{w_{t+1}}{1+r}
\end{aligned}$$

Optimal levels of consumption:

$$\begin{aligned}
c_t &= \frac{1}{(1+\beta)(1+\psi)}\left[w_t + \frac{w_{t+1}}{1+r}\right] \\
c_{t+1} &= \frac{\beta}{(1+\beta)(1+\psi)}\left[(1+r)w_t + w_{t+1}\right]
\end{aligned}$$

Optimal levels of hours worked:

$$\begin{aligned}
h_t &= 1 - \frac{\psi}{w_t}c_t = 1 - \frac{\psi}{(1+\beta)(1+\psi)}\left[1 + \frac{w_{t+1}/w_t}{1+r}\right] \\
h_{t+1} &= 1 - \beta(1+r)\frac{\psi}{w_{t+1}}c_t = 1 - \frac{\beta\psi}{(1+\beta)(1+\psi)}\left[(1+r)\frac{w_t}{w_{t+1}} + 1\right]
\end{aligned}$$

Reaction of variables to an increase in r:

$$\frac{\partial c_t}{\partial r} = -\frac{1}{(1+\beta)(1+\psi)}\frac{w_{t+1}}{(1+r)^2} < 0 \qquad \frac{\partial c_{t+1}}{\partial r} = \frac{\beta}{(1+\beta)(1+\psi)}w_t > 0$$

$$\frac{\partial h_t}{\partial r} = \frac{\psi}{(1+\beta)(1+\psi)}\frac{w_{t+1}/w_t}{(1+r)^2} > 0 \qquad \frac{\partial h_{t+1}}{\partial r} = -\frac{\beta\psi}{(1+\beta)(1+\psi)}\frac{w_t}{w_{t+1}} < 0$$

Just as in the case of exogenous incomes, for logarithmic utility function the substitution effect is stronger than income effect, so first period consumption decreases, while second period consumption increases.

With higher interest rates, it is worth working harder in the first period to generate more savings and reap benefits of higher interest rate. In the second period the gross asset return constitutes non-labor income (which increases in r) and thus the second period labor supply decreases. This is one example of intertemporal substitution of labor.

Let us now look at the reaction of labor supply to an increase in wages in different periods:

$$\frac{\partial h_t}{\partial w_t} = \frac{\psi}{(1+\beta)(1+\psi)}\frac{w_{t+1}/w_t^2}{1+r} > 0 \qquad \frac{\partial h_{t+1}}{\partial w_t} = -\frac{\beta\psi}{(1+\beta)(1+\psi)}(1+r)\frac{1}{w_{t+1}} < 0$$

$$\frac{\partial h_t}{\partial w_{t+1}} = -\frac{\psi}{(1+\beta)(1+\psi)}\frac{1/w_t}{1+r} < 0 \qquad \frac{\partial h_{t+1}}{\partial w_{t+1}} = \frac{\beta\psi}{(1+\beta)(1+\psi)}(1+r)\frac{w_t}{w_{t+1}^2} > 0$$

If wages increase in the first (second) period, first period labor supply increases (decreases) while second period labor supply decreases (increases). This is another example of intertemporal substitution of labor – the agent wants to supply more labor in times when wages are high and less labor in times when wages are low. This behavioral reaction, together with consumption smoothing behavior, underlies the basic real business cycles model which we will get to know later in the course.

2.3 Frisch elasticity

The Frisch elasticity of labor supply captures the elasticity of hours worked to the wage rate, given a constant marginal utility of wealth. Linking this definition to already introduced concepts, the Frisch elasticity measures the substitution effect of a change in the wage rate on labor supply. The reason we care about Frisch elasticity however, is that it is closely related to the intertemporal elasticity of substitution for labor supply.

The general formula for Frisch elasticity is:

$$\nu = \frac{U_h}{h\left(U_{hh} - U_{ch}^2/U_{cc}\right)}$$

Logarithmic utility function

$$U = \ln c + \psi \ln(1-h)$$

Derivatives:

$$U_c = c^{-1} \qquad U_h = -\psi(1-h)^{-1}$$
$$U_{ch} = 0 \qquad U_{hh} = -\psi(1-h)^{-2}$$
$$U_{cc} = -c^{-2}$$

Frisch elasticity:

$$\nu = \frac{-\psi(1-h)^{-1}}{h\left[-\psi(1-h)^{-2} - 0\right]} = \frac{1-h}{h}$$

Usually, we assume that workers work for about 1/3 of available time. The usual 40 hours per week can be compared to either (24 hours/day x 5 days/week = 120 hours/week) or (16 hours/day x 7 days/week = 112 hours/week) to yield about a third. Then the Frisch elasticity for logarithmic utility equals:

$$\nu = \frac{1-\frac{1}{3}}{\frac{1}{3}} = 2$$

However, the above estimate of 2 is at odds with most microeconometric studies that find the Frisch elasticity of labor supply between 0 and 0.5. To correct for this, we can use an alternative type of utility function.

Alternative utility function

In business cycles research an often-used utility function is:

$$U = \frac{c^{1-\sigma}}{1-\sigma} - \psi\frac{h^{1+\varphi}}{1+\varphi}$$

Derivatives:

$$U_c = c^{-\sigma} \qquad U_h = -\psi h^{\varphi}$$
$$U_{ch} = 0 \qquad U_{hh} = -\psi\varphi h^{\varphi-1}$$
$$U_{cc} = -\sigma c^{-\sigma-1}$$

Frisch elasticity:

$$\nu = \frac{-\psi h^{\varphi}}{h\left[-\psi\varphi h^{\varphi-1} - 0\right]} = \frac{1}{\varphi}$$

The φ parameter now can be freely adjusted to yield the desired value for Frisch elasticity. For example, to get the Frisch elasticity of 0.5, we should set $\varphi = 2$.

2.4 Indivisible labor and labor lotteries

Until now, we have assumed that agents can choose any number of hours worked between 0 and T. However, this assumption is at odds with the observed distribution of hours worked, which typically concentrates around full-time and a few select part-time values.

A compelling theoretical alternative to continuous hours choice is the approach of indivisible labor with labor lotteries. In its simplest form this approach assumes that the agent is faced with either working 0 or some fixed number of $\bar{h}$ hours per week. Given that the optimal choice would lie between 0 and $\bar{h}$, the agent would then choose the probability of being employed $p \in [0, 1]$. We assume here that workers belong to a big family that insures them to receive exactly the same levels of consumption regardless of their employment status (equivalent to "full" unemployment insurance).

Let us return to the simple logarithmic utility case. The agent solves the following utility maximization problem:

$$\begin{aligned} \max_{c,\, p} \quad & U = \ln c + \mathrm{E}\left[\psi \ln\left(1-h\right)|p\right] \\ \text{subject to} \quad & c = wp\bar{h} \end{aligned}$$

where $\mathrm{E}\left[\psi \ln\left(1-h\right)|p\right]$ denotes the expected utility from leisure given the agent's choice of probability of working p.

We can rewrite the problem to make it easier to handle. Since p denotes the probability of working $\bar{h}$ hours and $(1-p)$ is the probability of working 0 hours, we can expand the expected value expression:

$$\mathrm{E}\left[\psi \ln\left(1-h\right)|p\right] = p \cdot \psi \ln\left(1-\bar{h}\right) + (1-p) \cdot \psi \ln\left(1-0\right) = p\psi \ln\left(1-\bar{h}\right)$$

If all agents behave symmetrically, then the observed average hours worked per agent will be $h = p\bar{h} \quad \rightarrow$ $p = h/\bar{h}$ and the utility function can be expressed as:

$$\mathrm{E}\left[\psi \ln\left(1-h\right)|p\right] = p\psi \ln\left(1-\bar{h}\right) = h \cdot \frac{\psi \ln\left(1-\bar{h}\right)}{\bar{h}}$$

Note now that since $\bar{h}$ is a fixed number between 0 and 1, then $\psi \ln\left(1-\bar{h}\right)/\bar{h} < 0$ is a certain constant, let us call it $-B$. The utility function becomes linear, and the problem can be rewritten as:

$$\begin{aligned} \max_{c,\, p} \quad & U = \ln c - Bh \\ \text{subject to} \quad & c = wh \end{aligned}$$

Derivatives:

$$\begin{aligned} U_h &= -B \\ U_{hh} &= (-)\,0 \end{aligned}$$

Frisch elasticity:

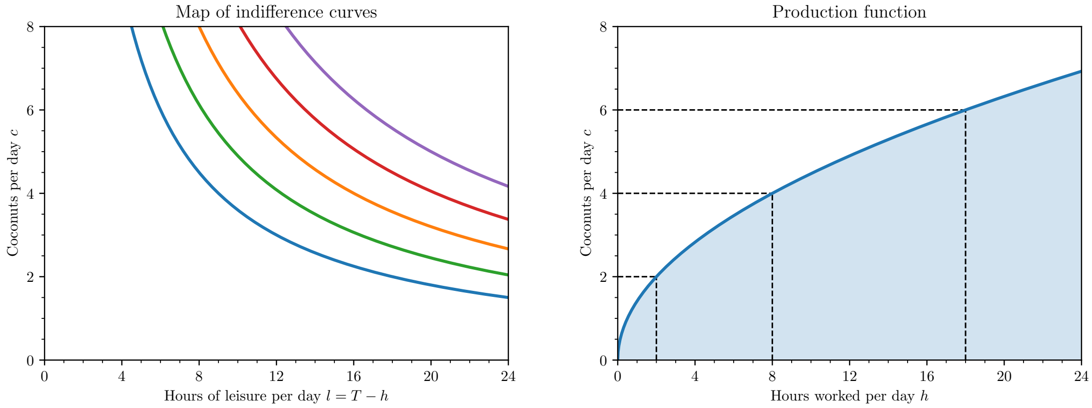
$$\nu = \frac{-B}{h \cdot (-)\,0} = +\infty$$

The Frisch elasticity at the macroeconomic level becomes infinite, since the second derivative of U with respect to h twice equals 0. In that way the Frisch elasticity at the microeconomic level does not influence the Frisch elasticity at the macroeconomic level.

2.5 General Equilibrium: Robinson Crusoe's economy

We now examine our first general equilibrium model with two interacting markets, for goods and labor. At first, assume that all decisions in the economy are decided by a single decision maker, typically nicknamed as a famous literary castaway, Robinson Crusoe. Robinson chooses levels of consumption c and hours worked h to maximize his utility, subject to the production function that maps inputs (in this case hours ow work) into outputs of consumption goods[1]:

$$\max_{c,\,h} \quad U = \ln c + \ln (T-h)$$
$$\text{subject to} \quad c = Ah^{1-\alpha}$$



Lagrangian:

$$\mathcal{L} = \ln c + \ln (T-h) + \lambda \left[Ah^{1-\alpha} - c\right]$$

First order conditions:

$$c: \quad \frac{1}{c} - \lambda = 0 \qquad \rightarrow \qquad \lambda = \frac{1}{c}$$
$$h: \quad \frac{-1}{T-h} + \lambda (1-\alpha) Ah^{-\alpha} = 0 \qquad \rightarrow \qquad \lambda = \frac{1}{T-h}\frac{1}{(1-\alpha)Ah^{-\alpha}}$$

Resulting optimality conditions:

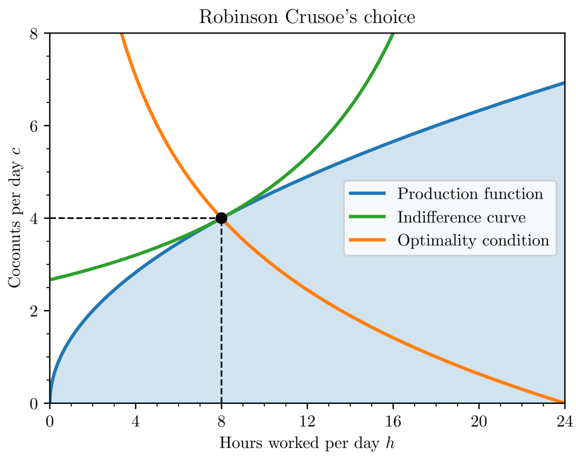
$$\frac{1}{c} = \frac{1}{T-h}\frac{1}{(1-\alpha)Ah^{-\alpha}}$$
$$c = (T-h)(1-\alpha)Ah^{-\alpha}$$

Use the production function constraint:

$$c = (T-h)(1-\alpha)Ah^{-\alpha}$$
$$Ah^{1-\alpha} = (T-h)(1-\alpha)Ah^{-\alpha}$$
$$h = (T-h)(1-\alpha)$$
$$(2-\alpha)h = (1-\alpha)T$$

Optimal decisions:

$$h = \frac{1-\alpha}{2-\alpha}T$$
$$c = A\left(\frac{1-\alpha}{2-\alpha}T\right)^{1-\alpha}$$



[1] Parameter values used for graphs: $T = 24$, $A = \sqrt{2}$, $\alpha = 1/2$.

2.6 General Equilibrium: Crusoe Inc.

Now let us examine what happens if the decisions are taken by two independent decision makers: consumers and firm owners. We establish here the conditions under which the resulting equilibrium perfectly coincides with the single decision maker equilibrium.

Firm

The firm maximizes its profit (which owners receive as dividend d), conditional on market wage w:

$$\max_{y,\,h} \quad d = y - wh$$
$$\text{subject to} \quad y = Ah^{1-\alpha}$$

Plug in the production function into the profit equation to simplify the problem:

$$\max_{h} \quad d = Ah^{1-\alpha} - wh$$

First order condition:

$$h: \quad (1-\alpha)\,Ah^{-\alpha} - w = 0 \qquad \rightarrow \qquad w = (1-\alpha)\,Ah^{-\alpha}$$

Labor demand h^d as a function of market wage w:

$$h^d(w) = \left[\frac{(1-\alpha)\,A}{w}\right]^{1/\alpha}$$

Maximum dividend as a function of market wage w:

$$d(w) = y\left(h^d(w)\right) - w \cdot h^d(w) = A\left(h^d(w)\right)^{1-\alpha} - w \cdot h^d(w)$$

Household

Since households are (in aggregate) ultimately the firm owners, dividends become their non-labor income:

$$\max_{c,\,h} \quad U = \ln c + \ln(T-h)$$
$$\text{subject to} \quad c = wh + d(w)$$

Lagrangian:

$$\mathcal{L} = \ln c + \ln(T-h) + \lambda\left[wh + d(w) - c\right]$$

First order conditions:

$$c: \quad \frac{1}{c} - \lambda = 0 \qquad \rightarrow \qquad \lambda = \frac{1}{c}$$
$$h: \quad -\frac{1}{T-h} + \lambda w = 0 \qquad \rightarrow \qquad \lambda w = \frac{1}{T-h}$$

Resulting optimality condition (OC):

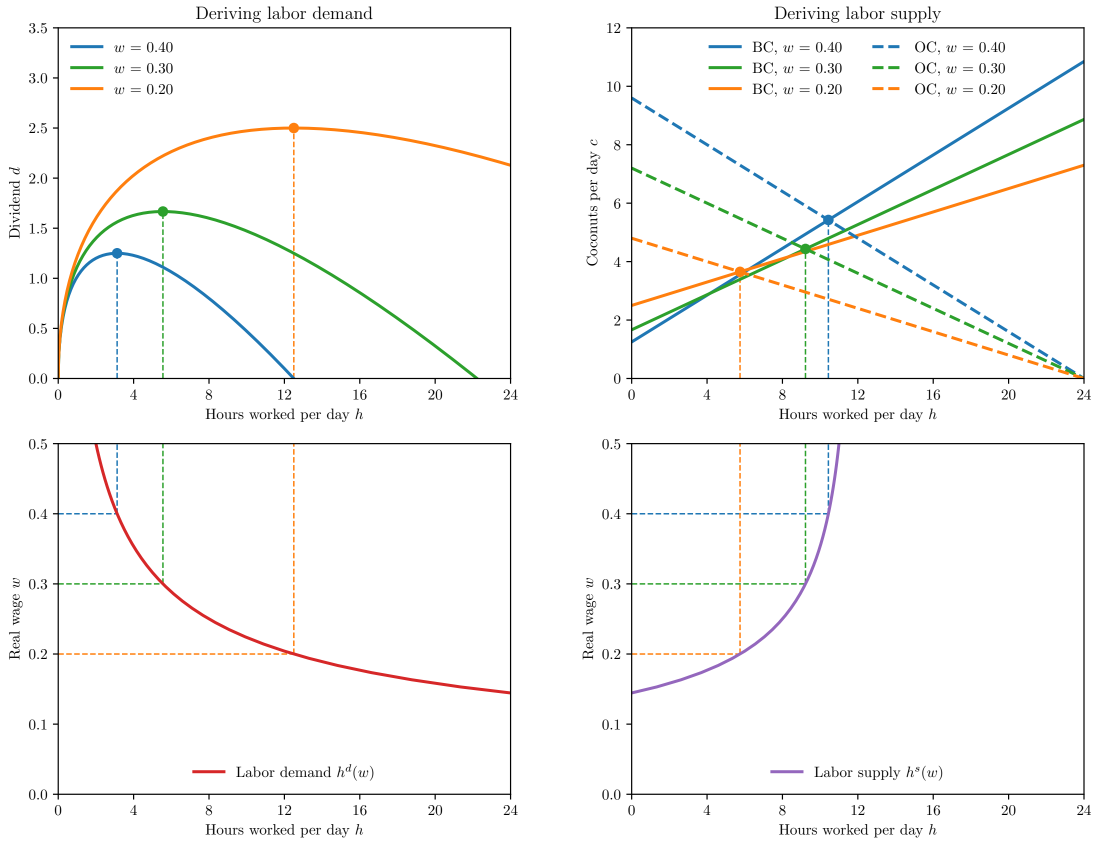
$$\frac{w}{c} = \frac{1}{T-h}$$
$$c = (T-h) \cdot w$$

Plug into the budget constraint (BC):

$$c = wh + d(w)$$
$$(T-h) \cdot w = wh + d(w)$$
$$wT - d(w) = 2 \cdot wh$$

Labor supply as a function of market wage w (and dividend $d(w)$):

$$h^s(w) = \frac{1}{2}\left(T - \frac{d(w)}{w}\right)$$



Note that since the dividend d depends on market wage w, the intercept of the budget constraint changes.

General Equilibrium

In this economy there are two markets: for goods and for labor. They are in equilibrium if:

$$c = y \quad \text{and} \quad h^s(w) = h^d(w)$$

Thanks to **Walras' law**, if there are N markets and $N-1$ are in equilibrium, then the N-th market is also in equilibrium. In our context, if the goods market is in equilibrium, so is the labor market, and vice versa.

Let us find the labor market equilibrium:

$$\begin{aligned}
h^d(w) = h^s(w) &= h \\
h &= \frac{1}{2}\left(T - \frac{d(w)}{w}\right) \\
2h &= T - \frac{y(h) - w \cdot h}{w} \\
2h &= T - \frac{y(h)}{w} + h \\
h &= T - \frac{Ah^{1-\alpha}}{(1-\alpha)Ah^{-\alpha}} \\
h &= T - \frac{h}{(1-\alpha)} \\
\left(1 + \frac{1}{1-\alpha}\right)h &= T \\
h &= \frac{1-\alpha}{2-\alpha}T
\end{aligned}$$

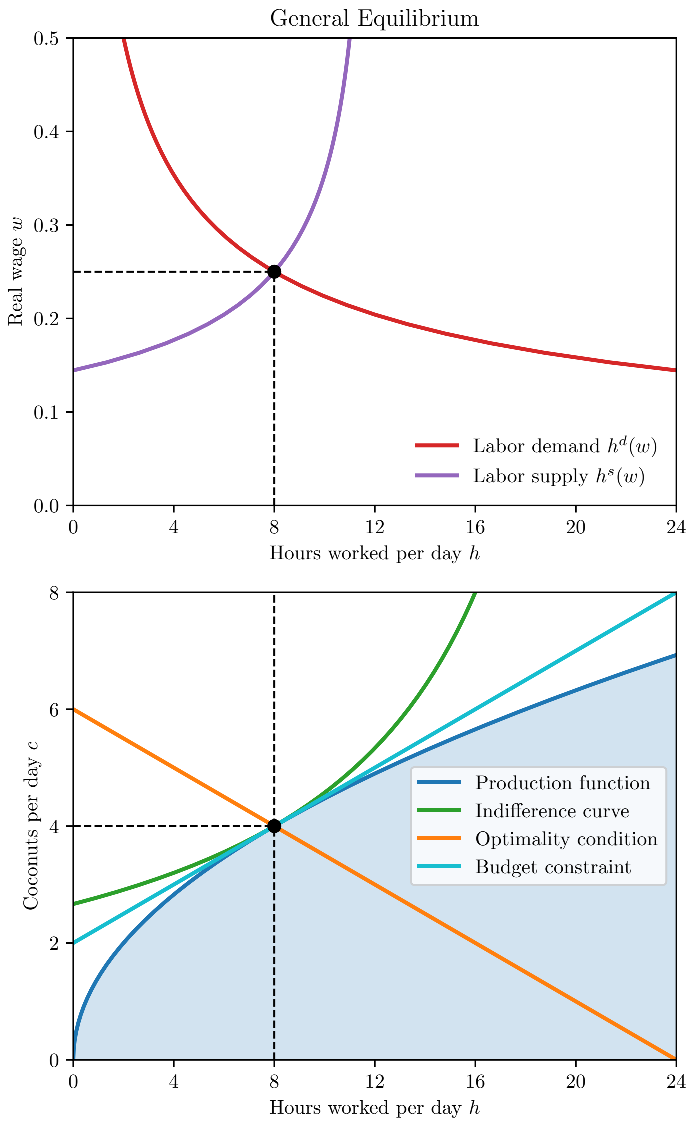
The wage associated with the equilibrium is equal to:

$$w = (1 - \alpha) A \left(\frac{1 - \alpha}{2 - \alpha} T \right)^{-\alpha}$$

Verify that the goods market is in equilibrium:

$$\begin{aligned} c &= wh^s + d(w) \\ &= wh^s + y\left(h^d\right) - wh^d \\ &= y\left(h^d\right) + w\left(h^s - h^d\right) \end{aligned}$$

If the labor market is in equilibrium ($h^s = h^d$), so is the goods market ($c = y$).



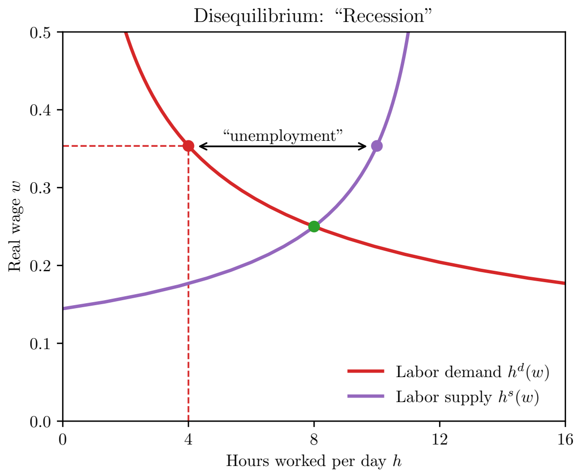
When both households and firms are price-takers, there are no externalities or information assymetry and the labor market clears, the levels of hours worked and produced goods in the market economy are identical to those that would have been chosen by Robinson himself, who maximizes his own utility. This emphasizes the information role of the price system. Households do not need to be aware of the production function of the firms and firms do not need to know the utility function of the households in order to reproduce the "first best" allocation that maximizes households' utility, as long as the prices (in this case: real wage w) can freely adjust to equate supply and demand.

2.7 Disequilibrium: "Recession"

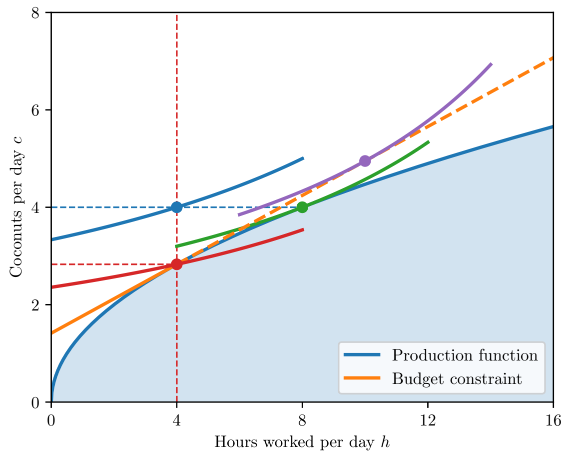
We have seen in the previous section that the market economy can replicate the allocation that maximizes households' utility, as long as markets are functioning properly. In this simple setup, the "proper" functioning of labor markets is ensured when the market wage is equal to the marginal productivity of labor, which is equivalent to assuming that neither the firms nor the workers are able to exert market power to influence the level of wage and that there are no impediments to the price adjustment.

We might be however interested in the case where the labor market fails to clear. An easy, although not very elegant way to create such a scenario is to impose a binding wage floor. In this situation, labor demand is lower than labor supply, which normally would put a downward pressure on the level of wage, but we artificially prevent the wage from decreasing. Such situation can (in a very crude way) approximate a recession, during which the unemployment rate is higher than usual and the economy produces less goods and services than desired.

Although we do not have a well-defined notion of unemployment in this model (we will get to know models of unemployment later), let us treat the difference between the labor supply and demand as "unemployment". In the graph to the right, we fix the level of wage above the level that would clear the labor market. This introduces a wedge between firms' and households' optimal choices: the firms demand "red" level of hours, while households would like to supply "violet" level of hours, but there are not enough "buyers" for their supply.



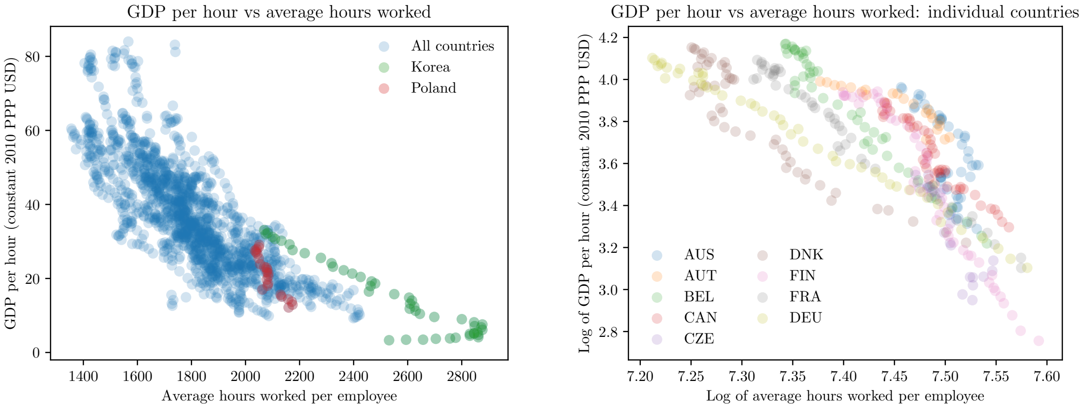
The economy then produces the "red" level of goods, which is not consistent with the desires of households, who at the current level of wages would like to consume the "violet" level of goods. However, the "violet" point is impossible to be attained for two reasons. First, the points to the right of the "red" point are not actually in the budget set, as the households are only able to sell the "red" level of hours. Second, it lies beyond the production possibilities frontier, and the economy would not be able to produce at the "violet" point anyway.



An important takeaway from the above exercise is that recessions decrease welfare not because of unemployment *per se*: people actually enjoy their free time and would like to work less if only they could maintain the usual level of consumption ("blue" point). The "red" point lies at a lower indifference curve than the "green" point because it involves lower level of goods produced, and lower consumption.

2.8 Long-run labor supply behavior

In the data, in the countries where the hourly labor productivity is high, employees usually work fewer hours.



To capture this effect, let us assume the following Robinson economy:

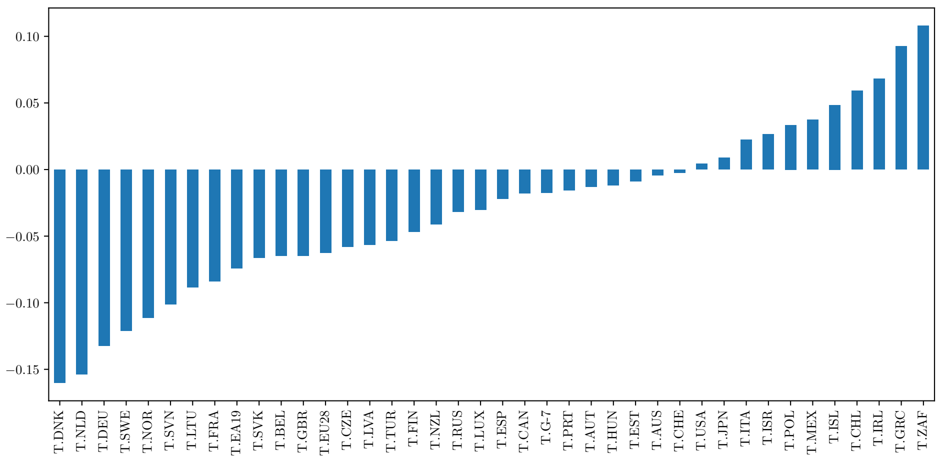
$$\begin{aligned} \max_{c,\,h} \quad & U = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\varphi}}{1+\varphi} \\ \text{subject to} \quad & y = A h^{1-\alpha} \\ & c = y \end{aligned}$$

The derivation of the dependence of hours worked on hourly labor productivity is quite lengthy. Here I present only the end result:

$$\ln h = \frac{1-\sigma}{\varphi+\sigma} \ln\left(y/h\right) + \frac{1}{\varphi+\sigma} \ln\left(\frac{1-\alpha}{\psi}\right)$$

$$\frac{\partial \ln h}{\partial \ln\left(y/h\right)} = \frac{1-\sigma}{\varphi+\sigma}$$

I then run the regression on country-level data on labor productivity and hours worked. Since I allow for the parameters α and ψ to vary across countries, I employ the fixed effects panel OLS regression and get that $\left(1-\sigma\right)/\left(\varphi+\sigma\right) = -0.1678 \pm 0.003$ while country effects relative to OECD average are below:



If I assume that $\sigma \approx 2$, as is usually assumed in the literature, this implies that $\varphi \approx 4$. This gives the estimate of the long-run Frisch elasticity of labor supply of 0.25, which is well in range of typical estimates obtained by the microeconometric studies, see e.g. **Chetty, Guren, Manoli and Weber (2013)** or **Peterman (2016)**.

2.9 Ben-Porath (1967) model of human capital accumulation

Lastly, let us to take a look at a relatively simple, yet powerful, model of human capital accumulation over the lifetime. The model assumes (for simplicity) that individuals do not value leisure, and can devote their time either to schooling s which then increases their level of human capital H, or they can use their remaining time $1-s$ to supply labor. For simplicity, the person's hourly wage is equal to their level of human capital. Finally, the person is alive for $T+1$ periods, with a period representing one year of life.

Since the individuals do not value leisure, maximizing utility can be redefined as maximizing lifetime income. At each time period, the individual chooses the fraction of time spent schooling s_t and next-period level of human capital H_{t+1}, subject to the constraint on human capital accumulation process, where $A > 0$ is the productivity of schooling, $\alpha \in (0,1)$ is a parameter of the schooling function, and $\delta \in (0,1)$ is the rate of human capital depreciation:

$$\begin{aligned} \max_{\{s_t,\, H_{t+1}\}_{t=0}^{T}} \quad & \sum_{t=0}^{T} \frac{1}{(1+r)^t} H_t (1-s_t) \\ \text{subject to} \quad & H_{t+1} = A (s_t H_t)^\alpha + (1-\delta) H_t \\ & s_t \in [0,\, 1] \\ & H_0 > 0 \end{aligned}$$

Set up the Lagrangian:

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^{T} \frac{1}{(1+r)^t} H_t (1-s_t) + \sum_{t=0}^{T} \frac{1}{(1+r)^t} \mu_t \left[A (s_t H_t)^\alpha + (1-\delta) H_t - H_{t+1} \right] \\ &= \ldots + H_t (1-s_t) + \mu_t \left[A (s_t H_t)^\alpha + (1-\delta) H_t - H_{t+1} \right] \\ &\quad + \frac{1}{1+r} \left[H_{t+1} (1-s_{t+1}) + \mu_t \left[A (s_t H_t)^\alpha + (1-\delta) H_{t+1} - H_{t+2} \right] \right] + \ldots \end{aligned}$$

First order conditions:

$$\begin{aligned} s_t \quad &: \quad -H_t + \mu_t \alpha A s_t^{\alpha-1} H_t^\alpha = 0 \quad \to \quad s_t = (\mu_t \alpha A)^{1/(1-\alpha)} / H_t \\ H_{t+1} \quad &: \quad -\mu_t + \frac{1}{1+r} \left[1 - s_{t+1} + \mu_{t+1} \left[\alpha A s_{t+1}^\alpha H_{t+1}^{\alpha-1} + 1 - \delta \right] \right] = 0 \\ &\quad \hookrightarrow \quad \mu_t = \frac{1}{1+r} \left[1 + (1-\delta) \mu_{t+1} - s_{t+1} \left(1 - \mu_{t+1} \alpha A s_{t+1}^{\alpha-1} H_{t+1}^{\alpha-1} \right) \right] \end{aligned}$$

After substituting the expression for s into the FOC for human capital:

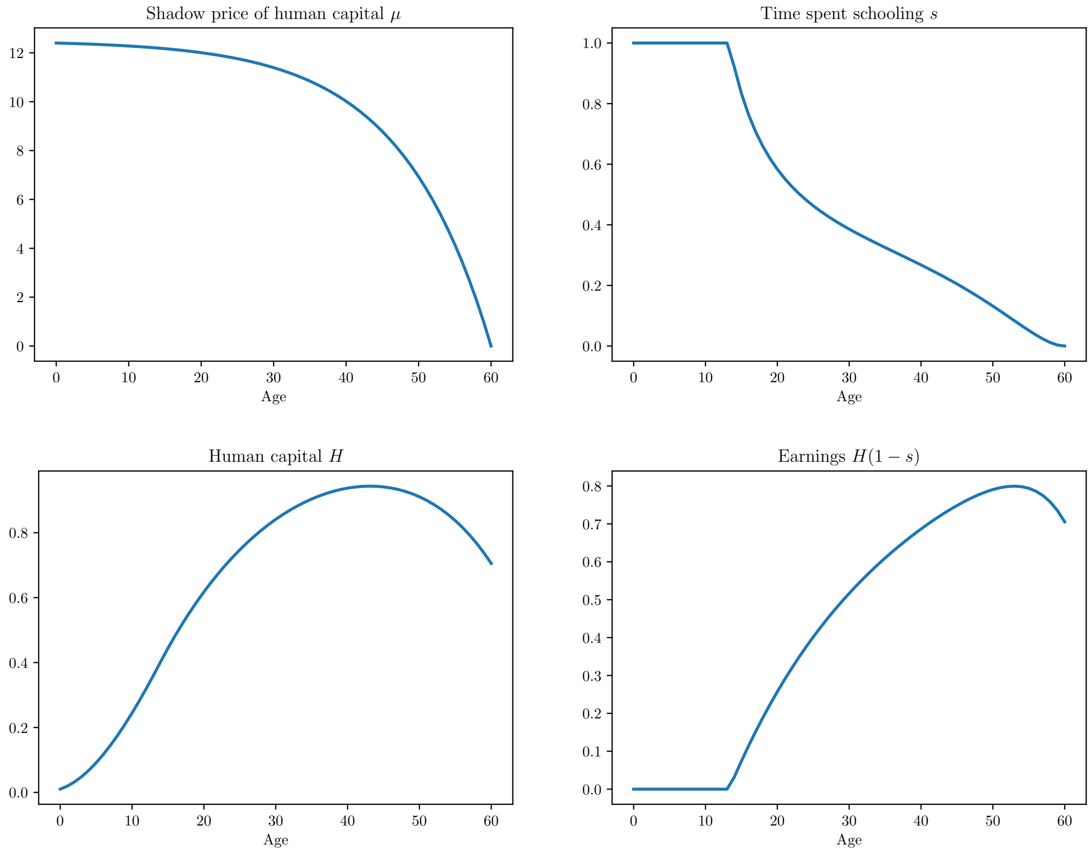
$$\mu_t = \frac{1}{1+r} \left[1 + (1-\delta) \mu_{t+1} - s_{t+1} \left(1 - \frac{\mu_{t+1} \alpha A H_{t+1}^{\alpha-1}}{\mu_{t+1} \alpha A H_{t+1}^{\alpha-1}} \right) \right] = \frac{1}{1+r} \left[1 + (1-\delta) \mu_{t+1} \right]$$

Multiplier μ_t represents the marginal increase in PDV of lifetime income when human capital in period t is marginally increased. Since human capital is worthless beyond the planning horizon, it implies that $\mu_T = 0$. This allows us to solve for μ "backwards" and then calculate optimal s and H:

$$\begin{aligned} \mu_t &= \left[1 + (1-\delta) \mu_{t+1} \right] / (1+r) \\ s_t &= \min \left\{ 1,\, (\mu_t \alpha A)^{1/(1-\alpha)} / H_t \right\} \\ H_{t+1} &= A (s_t H_t)^\alpha + (1-\delta) H_t \end{aligned}$$

The graphs on the next page display the model solution[2] where an individual chooses to spend their entire time on schooling for the first 16 years, and then gradually decreases investment in human capital until the final age. The level of human capital peaks in the forties, while earnings peak in the early fifties, roughly consistent with patterns observed in the developed economies.

[2]Parameters are set to: $T = 60$, $A = 0.1$, $H_0 = 0.001$, $\alpha = 0.5$, $\delta = 0.05$, $r = 0.03$. Note that T here represents the last year in the labor force, and not literally the last period of life.



Note that the above model does not feature a retirement period, as the individual that does not value leisure would be happy to supply labor until death. To generate the period of retirement, we need to reintroduce preference for leisure. Unfortunately, this greatly complicates the problem of the individual. To the right I reproduce the qualitative, graphical solution found in **Blinder and Weiss (1976)**, where ℓ represents leisure, h time devoted to both working and schooling, x the fraction of non-leisure time spent on schooling, and K the level of human capital. What is the most interesting is that the model generates 4 distinct lifetime phases. In the first individual does not work, but goes to school. In the second individual works but still accumulates human capital. In the third the individual does not invest in human capital at all, but still works. Finally, the period of retirement arises endogenously.

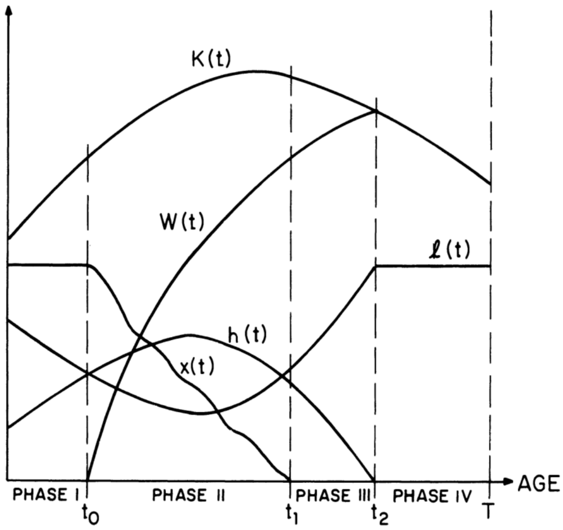


Fig. 5—The age profiles of human capital, wages, leisure, work, and investment in a "normal" life cycle.

The model of lifetime human capital accumulation can also be used to explain other observed patterns. Suppose that after leaving school, human capital can be only accumulated on-the-job, while the person is employed. **Davis and von Wachter (2011)** show that persons who become unemployed experience a permanent loss in their earnings, and this effect is stronger for those persons who lose a job during a recession (as their unemployment spell lasts longer). **Kleven, Landais and Søgaard (2018)** show that Danish women who become mothers experience a permanent earnings loss following childbirth (there is no effect for fathers), which in 2013 accounted for around 80% of gender pay inequality.