University of Warsaw
Faculty of Economic Sciences

New Keynesian Model

Advanced Macroeconomics QF: Lecture 11

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Nominal rigidities and sticky prices

Monopolistic competition: setup

Under perfect competition firms are selling homogeneous goods and are price-takers

In reality many firms sell differentiated products and are able to set their own prices

Convenient framework: **monopolistic competition**

- Households enjoy consuming many different goods ("love for variety")
- Firms' market power depends on elasticity of substitution $\varepsilon > 1$
- Perfect competition: infinitely high elasticity of substitution
- One sector: all firms "compete" with each other
- (Can easily extend to a multisector setup: higher elasticity of substitution within industries, lower elasticity of substitution across industries)
- Resulting demand and inverse demand functions for the i-th producer where P is the aggregate price index and Y is aggregate output

$$Y_i = (P/P_i)^{\varepsilon} Y \quad \text{and} \quad P_i = PY^{1/\varepsilon} Y_i^{-1/\varepsilon}$$

Monopolistic competition: firms' profit maximization problem

For Constant Returns to Scale production function the marginal cost MC_i is constant

$$\max_{P_i, Y_i} \quad D_i = P_i Y_i - MC_i Y_i$$
$$\text{subject to} \quad P_i = P Y^{1/\varepsilon} Y_i^{-1/\varepsilon}$$

Plug in the inverse demand function into the profit function

$$\max_{Y_i} \quad D_i = P Y^{1/\varepsilon} Y_i^{1-\frac{1}{\varepsilon}} - MC_i Y_i$$

First order condition with respect to Y_i ($MR_i = MC_i$)

$$\left(1 - \frac{1}{\varepsilon}\right) \underbrace{P Y^{1/\varepsilon} Y_i^{-1/\varepsilon}}_{P_i} - MC_i = 0 \quad \rightarrow \quad \frac{\varepsilon - 1}{\varepsilon} P_i = MC_i$$

"Markup pricing" is the profit-maximizing strategy (where $\mu \geq 0$)

$$P_i^* = \frac{\varepsilon}{\varepsilon - 1} MC_i \equiv (1 + \mu)\, MC_i$$

Empirical evidence on markups in US and EA

Perfect competition can be rejected for almost all sectors in all countries
Markups are generally higher in services than manufacturing

Table 1. Weighted average markup, 1981-2004

Country	Manufacturing & Construction		Market Services		All (Manufacturing, Construction & Market Services)	
Germany	1.16	(0.01)*	1.54	(0.03)*	1.33	(0.01)*
France	1.15	(0.01)*	1.26	(0.02)*	1.21	(0.01)*
Italy	1.23	(0.01)*	1.87	(0.02)*	1.61	(0.01)*
Spain	1.18	(0.00)*	1.37	(0.01)*	1.26	(0.01)*
Netherlands	1.13	(0.01)*	1.31	(0.02)*	1.22	(0.01)*
Belgium	1.14	(0.00)*	1.29	(0.01)*	1.22	(0.01)*
Austria	1.20	(0.02)*	1.45	(0.03)*	1.31	(0.02)*
Finland	1.22	(0.01)*	1.39	(0.02)*	1.28	(0.01)*
Euro Area	1.18	(0.01)*	1.56	(0.01)*	1.37	(0.01)*
USA	1.28	(0.02)*	1.36	(0.03)*	1.32	(0.02)*

Christopoulou and Vermeulen (2008)

Prices do not change every period

Survey about price setting practices carried out by the Banco de Portugal

Firms in the sample are generally quicker to react to cost shocks,
in particular when they are positive, than to demand shocks

TABLE 1

Distribution of the price responses to demand and cost shocks

	Cost shocks		*Demand shocks*	
Price adjustment lag	*Positive*	*Negative*	*Positive*	*Negative*
1 – less than one week	4.7	3.5	2.8	4.8
2 – from one week to one month	16.8	15.2	12.2	16.8
3 – from one month to three months	25.0	25.7	19.3	23.4
4 – from three to six months	17.6	14.9	13.4	13.6
5 – from six months to one year	26.3	21.2	17.7	14.0
6 – more than one year	9.6	19.5	34.6	27.4
Total	100.0	100.0	100.0	100.0

Dias et al. (2014)

Stylized facts on price stickiness

Benefits of price stickiness: no need to survey all prices everytime we go to a store, easy to plan expenditures ahead

Average price duration

- US: average time between price changes is 2-4 quarters
  Blinder et al. (1998), Klenow and Kryvstov (2008), Nakamura and Steinsson (2008)
- EA: average time between price changes is 4-5 quarters
  Dhyne et al. (2005), Altissimo et al. (2006)
- PL: average time between price changes is 4 quarters
  Macias and Makarski (2013)

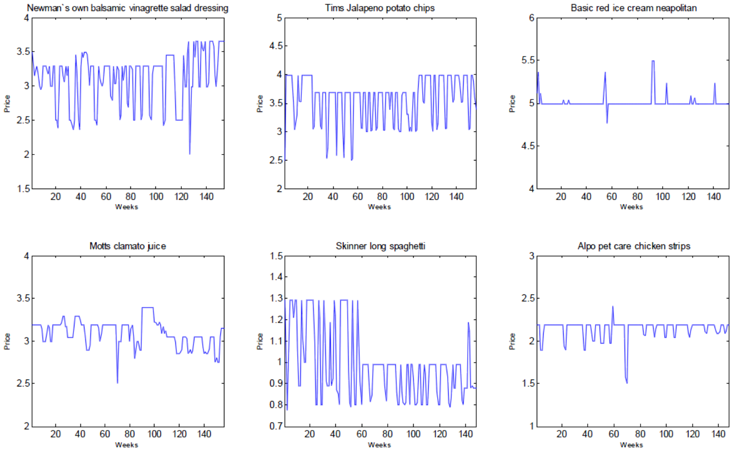
Cross-industry heterogeneity

- Prices of tradables less sticky than those of nontradables
- Retail prices usually more sticky than producer prices

Gagnon (2009): for inflation above 10-15% prices change more frequently with higher inflation

Example retail prices behavior

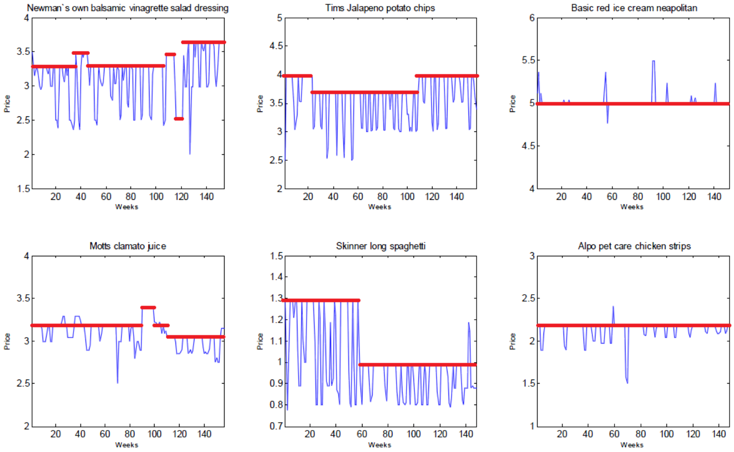
Raw retail scanner data



Koning (2015)

Example retail prices behavior

After “controlling” for short-lived sales prices: reference prices



Koning (2015)

Price stickiness depends on the sector

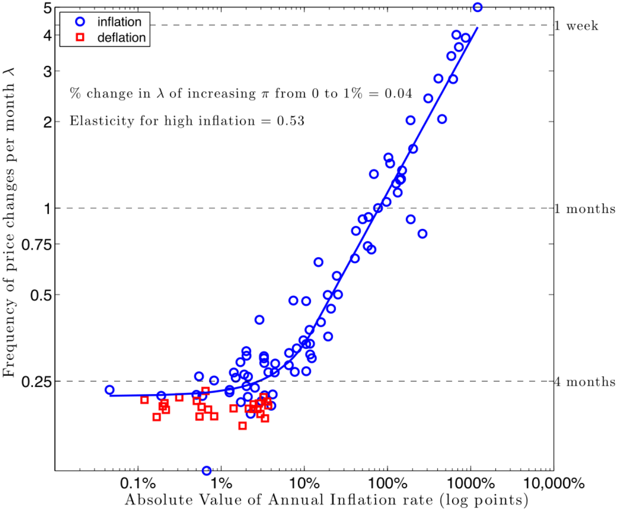
Table 4.1 Frequency of consumer price changes by product type, in %

Country	Unprocessed food	Processed food	Energy (oil products)	Non-energy industrial goods	Services	Total, country weights	Total, Euro area weights
Belgium	31.5	19.1	81.6	5.9	3.0	17.6	15.6
Germany	25.2	8.9	91.4	5.4	4.3	13.5	15.0
Spain	50.9	17.7	n.a.	6.1	4.6	13.3	11.5
France	24.7	20.3	76.9	18.0	7.4	20.9	20.4
Italy	19.3	9.4	61.6	5.8	4.6	10.0	12.0
Luxembourg	54.6	10.5	73.9	14.5	4.8	23.0	19.2
The Netherlands	30.8	17.3	72.6	14.2	7.9	16.2	19.0
Austria	37.5	15.5	72.3	8.4	7.1	15.4	17.1
Portugal	55.3	24.5	15.9	14.3	13.6	21.1	18.7
Finland	52.7	12.8	89.3	18.1	11.6	20.3	-
Euro Area	28.3	13.7	78.0	9.2	5.6	15.1	15.8

Source: Dhyne et al. (2005). Figures presented in this table are computed on the basis of the 50 product sample, with the only exception of Finland for which figures based on the entire CPI are presented. The total with country weights is calculated using country-specific weights for each item, the total with euro area weights using common euro area weights for each sub-index. No figures are provided for Finland because of a lack of comparability of the sample of products used in this country.

Altissimo, Ehrmann and Smets (2006)

Frequency of price changes when inflation is low vs when inflation is high



Alvarez et al. (2018)

New Keynesian Phillips Curve

Convex costs of price changes

Based on Rotemberg (1982)

Assumes that bigger price changes are more costly, e.g. due to losses in customer loyalty

For simplicity we assume full symmetry across firms ($p_1 = p_2 = p$)

The dynamic profit maximizing problem can be recast as a simpler problem of minimizing a loss function (where p is the logarithm of firm's price and $\phi > 0$)

$$L = \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t \left[\left(p_{t+j} - p^*_{t+j}\right)^2 + \phi \left(p_{t+j} - p^e_{t+j}\right)^2 \right]$$

- $\mathrm{E}_t[(p_{t+j} - p^*_{t+j})^2]$ is the loss of profit by setting price other than $(1+\mu)\,MC$
- $\mathrm{E}_t[\phi(p_{t+j} - p^e_{t+j})^2]$ is the convex cost of price changes
- p^e_t is the price expected by the customers, assume $p^e_t = p_{t-1}$

Rotemberg model: solution

Expand the loss function for convenience

$$L = (p_t - p_t^*)^2 + \phi (p_t - p_{t-1})^2 + \beta \mathrm{E}_t \left[\left(p_{t+1} - p_{t+1}^*\right)^2 + \phi (p_{t+1} - p_t)^2 \right] + \dots$$

First order condition with respect to p_t

$$2(p_t - p_t^*) + 2\phi (p_t - p_{t-1}) + \beta \mathrm{E}_t [2\phi (p_{t+1} - p_t)(-1)] = 0$$

$$\frac{1}{\phi}(p_t - p_t^*) + (p_t - p_{t-1}) - \beta \mathrm{E}_t [p_{t+1} - p_t] = 0$$

$$(p_t - p_{t-1}) = \beta \mathrm{E}_t [p_{t+1} - p_t] - \frac{1}{\phi}(p_t - p_t^*)$$

Since firms set prices symmetrically, the inflation rate is $\pi_t \equiv p_t - p_{t-1}$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{\phi}(p_t^* - p_t)$$

Current inflation depends on inflation expecations!

Staggered price adjustment

Based on Calvo (1983)

In Rotemberg firms make many small price changes

In Calvo firms are not "allowed" to do so

- Firms can change their price only if they receive a "signal"
- Price remains unchanged with probability θ
- If a firm set the price in period t, then the price remains unchanged in period $t + j$ with probability θ^j
- Average price duration is $1/(1-\theta)$ periods
- Denote price set in period t with $\tilde{p}_{i,t}$ ("reset" price)

Firm's loss function

$$L = \sum_{j=0}^{\infty} (\beta\theta)^j \, \mathrm{E}_t \left[\left(\tilde{p}_{i,t} - p^*_{i,t+j} \right)^2 \right]$$

Calvo model: solution

First order condition

$$\sum_{j=0}^{\infty} (\beta\theta)^j \, \mathrm{E}_t \left[2 \left(\tilde{p}_{i,t} - p^*_{i,t+j} \right) \right] = 0$$

$$\tilde{p}_{i,t} \sum_{j=0}^{\infty} (\beta\theta)^j = \sum_{j=0}^{\infty} (\beta\theta)^j \, \mathrm{E}_t p^*_{i,t+j} = p^*_{i,t} + \sum_{j=1}^{\infty} (\beta\theta)^j \, \mathrm{E}_t p^*_{i,t+j}$$

$$\tilde{p}_{i,t} \frac{1}{1-\beta\theta} = p^*_{i,t} + \beta\theta \mathrm{E}_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \mathrm{E}_{t+1} p^*_{i,t+1+j}$$

$$\frac{1}{1-\beta\theta} \tilde{p}_{i,t} = p^*_{i,t} + \beta\theta \frac{1}{1-\beta\theta} \mathrm{E}_t \tilde{p}_{i,t+1}$$

Reset price $\tilde{p}_{j,t}$ is the weighted average of today's and future prices that would be optimal in a frictionless setting, which can be expressed as

$$\boldsymbol{\tilde{p}_{i,t}} = (1-\beta\theta) \, p^*_{i,t} + \beta\theta \mathrm{E}_t \tilde{p}_{i,t+1}$$

Dynamics of inflation in the Calvo scheme

Within each period a random fraction θ of firms keeps prices unchanged at average p_{t-1}, the remaining $1-\theta$ fraction resets prices fully symmetrically to $\tilde{p}_t$

$$p_t = \theta p_{t-1} + (1-\theta)\,\tilde{p}_t$$

After a series of algebraic manipulations (next slide for the curious) we get that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\,(p_t^* - p_t)$$

Compare to Rotemberg's outcome

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{\phi}\,(p_t^* - p_t)$$

In both settings expectations on future inflation affect inflation today!

Dynamics of inflation in the Calvo scheme: algebra

$$p_t = (1-\theta)\,\tilde{p}_t + \theta p_{t-1}$$

$$\tilde{p}_t = \frac{p_t - \theta p_{t-1}}{1-\theta} = \frac{p_t - p_{t-1} + (1-\theta)\,p_{t-1}}{1-\theta} = \frac{\pi_t}{1-\theta} + p_{t-1}$$

$$\tilde{p}_t = (1-\beta\theta)\,p_t^* + \beta\theta \mathrm{E}_t \tilde{p}_{t+1} = (1-\beta\theta)\,p_t^* + \beta\theta\left[\frac{1}{1-\theta}\mathrm{E}_t\pi_{t+1} + p_t\right]$$

$$(1-\theta)\,\tilde{p}_t = p_t - \theta p_{t-1} = \theta\,(p_t - p_{t-1}) + (1-\theta)\,p_t$$

$$\theta\,(p_t - p_{t-1}) = (1-\theta)\,(\tilde{p}_t - p_t)$$

$$\theta\pi_t = (1-\theta)\left[(1-\beta\theta)\,p_t^* + \frac{\beta\theta}{1-\theta}\mathrm{E}_t\pi_{t+1} + \beta\theta p_t - p_t\right]$$

$$\theta\pi_t = \beta\theta\mathrm{E}_t\pi_{t+1} + (1-\theta)\left[(1-\beta\theta)\,p_t^* + (\beta\theta - 1)\,p_t\right]$$

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \frac{(1-\theta)\,(1-\beta\theta)}{\theta}\,(p_t^* - p_t)$$

Comparing Rotemberg and Calvo

Identical (up to the first-order approximation) functional form, different economic conclusions: see Lombardo and Vestin (2007) and Ascari and Rossi (2012)

- Rotemberg: inflation is "costly" due to costs of changing prices, relative prices across firms are unaffected
- Calvo: inflation is "costly" since not every firm adjusts prices within each period, price dispersion arises
- Price dispersion introduces inefficiencies into the economy ($P_1/P_2 \neq MC_1/MC_2$)
- Welfare costs of inflation are higher in the Calvo scheme
- Calvo scheme fits data well under single-digit inflation (constant price change frequency), for higher inflation rates price adjustment models perform better (frequent price changes, "feeling out" the market in an unstable environment)

New Keynesian Phillips Curve

Under both schemes we have that (where $\chi > 0$ with $\partial\chi/\partial\phi < 0$ or $\partial\chi/\partial\theta < 0$)

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \left(p_t^* - p_t\right) = \beta \mathrm{E}_t \pi_{t+1} + \chi \ln \left(P_t^* / P_t\right)$$

Relate it to the expression for "optimal" price

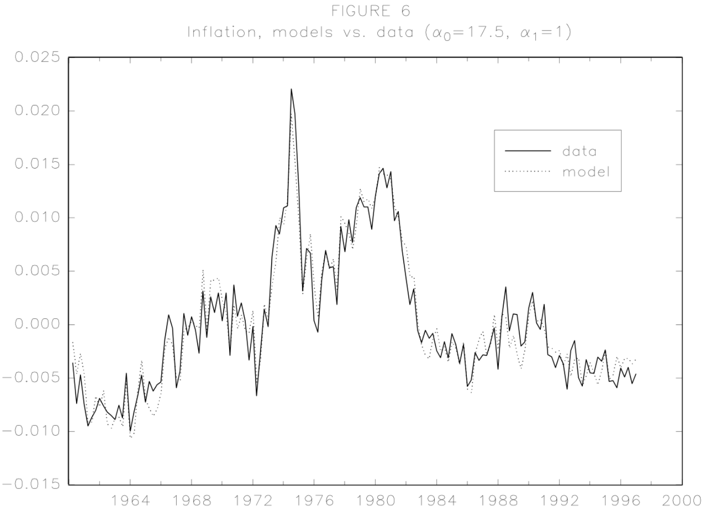
$$P_t^* = (1+\mu)\, MC_t \quad \rightarrow \quad P_{ss} = (1+\mu)\, MC_{ss} \quad \rightarrow \quad MC_{ss}^R \equiv MC_{ss}/P_{ss} = \frac{1}{1+\mu}$$

$$\frac{P_t^*}{P_t} = (1+\mu)\,\frac{MC_t}{P_t} = \frac{MC_t^R}{MC_{ss}^R}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \ln \left(MC_t^R / MC_{ss}^R\right) \equiv \beta \mathrm{E}_t \pi_{t+1} + \chi \hat{mc}_t$$

where $\hat{mc}_t$ is the percentage deviation of the real marginal cost from the steady state, dependent i.a. on the ratio of real wages to labor productivity

Inflation vs its one period ahead forecast from the NKPC



Sbordone (2002)

Estimating the NKPC slope

Gali and Gertler (1999) construct a proxy for the real marginal cost gap

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \quad \rightarrow \quad \hat{mc}_t \equiv \frac{W_t/P_t}{\partial Y_t/\partial L_t} = \frac{W_t/P_t}{(1-\alpha)\, Y_t/L_t} = \frac{1}{1-\alpha}\frac{W_t L_t}{P_t Y_t}$$

And estimate the following NKPC

$$\pi_t = 0.942\, \mathrm{E}_t \pi_{t+1} + 0.023\, \hat{mc}_t$$

Nowadays we can directly estimate the “marginal cost” NKPC using micro-level data

Estimating the NKPC slope using micro-level data

Table 4: Structural estimates and slope of the sectoral Phillips curves: Sectoral estimates.

Sector:	θ_s	Ω_s	λ_s	$\bar{s}_s$
Transport equipment	0.582	0.077	0.281	0.033
	(0.009)	(0.015)	(0.019)	(0.012)
Electrical equipment	0.632	0.191	0.177	0.020
	(0.005)	(0.021)	(0.007)	(0.007)
Machinery equipment	0.668	0.087	0.154	0.019
	(0.021)	(0.041)	(0.029)	(0.012)
Wood, paper and printing	0.703	0.202	0.102	0.106
	(0.019)	(0.082)	(0.020)	(0.034)
Metals	0.728	0.127	0.091	0.099
	(0.016)	(0.056)	(0.013)	(0.016)
Rubber and plastic	0.714	0.367	0.075	0.106
	(0.018)	(0.047)	(0.012)	(0.011)
Textiles, apparel and leather	0.737	0.509	0.047	0.030
	(0.023)	(0.144)	(0.019)	(0.024)
Chemicals	0.779	0.330	0.043	0.275
	(0.011)	(0.031)	(0.005)	(0.066)
Food, beverages and tobacco	0.760	0.483	0.041	0.289
	(0.018)	(0.048)	(0.010)	(0.031)

Notes. This table presents the estimates of the structural parameters (θ and Ω), the implied slope of the NKPC (λ), and the sector-specific Törnqvist weight ($\bar{s}$) for different manufacturing sectors. The estimates are obtained using Model C. For each sector, observations are weighted in the regression using Törnqvist weights. Standard errors (in parenthesis) are robust to heteroskedasticity and autocorrelation at the firm level.

Gagliardone et al. (2023), $\lambda \equiv \chi$

New Keynesian IS curve

Households' problem

Here households have access to **nominal** bonds B that yield the **nominal** interest rate i and can also trade stocks

$$\max_{\{C_t, L_t, A_{t+1}\}_{t=0}^{\infty}} \quad U_0 = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \right]$$

$$\text{subject to} \quad P_t C_t + B_t + P_t^S S_{t+1} = W_t L_t + (1 + i_{t-1}) B_{t-1} + \left(P_t^S + D_t^S \right) S_t$$

If we don't want to price assets, we can just consider equilibrium with $S_t = S_{t+1} \equiv 1$

$$\max_{\{C_t, L_t, A_{t+1}\}_{t=0}^{\infty}} \quad U_0 = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \right]$$

$$\text{subject to} \quad P_t C_t + B_t = W_t L_t + (1 + i_{t-1}) B_{t-1} + D_t$$

Lagrangian

$$\mathcal{L} = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{L_t^{1+\varphi}}{1+\varphi} \right) + \sum_{t=0}^{\infty} \beta^t \lambda_t \left[W_t L_t + (1 + i_{t-1}) B_{t-1} + D_t - P_t C_t - B_t \right] \right]$$

Households' problem

Expanded Lagrangian (adjusted notation for t denoting "today" and $t+1$ "tomorrow")

$$\mathcal{L} = \frac{\boldsymbol{C_t}^{1-\sigma}}{1-\sigma} - \psi \frac{\boldsymbol{L_t}^{1+\varphi}}{1+\varphi} + \ldots + \lambda_t \left[W_t \boldsymbol{L_t} + (1+i_{t-1}) B_{t-1} + D_t - P_t \boldsymbol{C_t} - \boldsymbol{B_t} \right]$$
$$+ \beta \mathrm{E}_t \left[\lambda_{t+1} \left[W_{t+1} L_{t+1} + (1+i_t) \boldsymbol{B_t} + D_{t+1} - P_{t+1} C_{t+1} - B_{t+1} \right] \right] + \ldots$$

First Order Conditions

$$\boldsymbol{C_t}: \quad C_t^{-\sigma} - P_t \lambda_t = 0 \quad \rightarrow \quad \lambda_t = C_t^{-\sigma} / P_t$$
$$\boldsymbol{L_t}: \quad -\psi L_t^{\varphi} + \lambda_t W_t = 0 \quad \rightarrow \quad \lambda_t = \psi L_t^{\varphi} / W_t$$
$$\boldsymbol{B_t}: \quad -\lambda_t + \beta \mathrm{E}_t \left[\lambda_{t+1} (1+i_t) \right] = 0 \quad \rightarrow \quad \lambda_t = \beta \mathrm{E}_t \left[\lambda_{t+1} (1+i_t) \right]$$

Euler equation and consumption-hours choice (labor supply)

$$C_t^{-\sigma} / P_t = \beta \mathrm{E}_t \left[C_{t+1}^{-\sigma} / P_{t+1} (1+i_t) \right] \quad \rightarrow \quad C_t^{-\sigma} = \beta \mathrm{E}_t \left[C_{t+1}^{-\sigma} (1+i_t) \frac{P_t}{P_{t+1}} \right]$$
$$C_t^{-\sigma} / P_t = \psi L_t^{\varphi} / W_t \quad \rightarrow \quad L_t = \left[(W_t / P_t) C_t^{-\sigma} / \psi \right]^{1/\varphi}$$

Deriving the simplified New Keynesian IS curve

1. Assume no investment and government spending, so that $C_t = Y_t$

$$Y_t^{-\sigma} = \frac{1}{1+\rho} \mathrm{E}_t \left[Y_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \right]$$

2. Apply logarithms and the first order approximation ("forget" about covariance)

$$-\sigma \ln Y_t \approx -\ln(1+\rho) - \sigma \mathrm{E}_t \ln Y_{t+1} + \ln(1+i_t) - \ln(1+\mathrm{E}_t \pi_{t+1})$$

$$\ln Y_t \approx \mathrm{E}_t \ln Y_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - \rho)$$

3. Denote counterfactual, "natural" variables in the flexible-prices world with *

$$\ln Y_t^* = \mathrm{E}_t \ln Y_{t+1}^* - \frac{1}{\sigma}(r_t^* - \rho)$$

4. Subtract the "natural" from the "actual" (where $x_t \equiv \ln Y_t - \ln Y_t^*$ is the **output gap**)

$$\ln Y_t - \ln Y_t^* = \mathrm{E}_t \left[\ln Y_{t+1} - \ln Y_{t+1}^* \right] - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - r_t^*)$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - r_t^*)$$

New Keynesian IS curve

Natural real interest rate (when $n = g = 0$)

$$r_t^* = \rho + \sigma \mathrm{E}_t \left[\Delta \ln Y_{t+1}^* \right]$$

In general case where $C_t \neq Y_t$, one can capture the influence of other expenditures as a demand shock u

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^* \right) + u_t$$

Ceteris paribus higher nominal interest rate leads to more negative output gap

A positive output gap ($x > 0$) means the level of output is higher than in the counterfactual flexible-price world ($Y > Y^*$)

Output gap in NKPC: algebra

Assume $C_t = Y_t$ and $Y_t = Z_t L_t$ to relate x_t with $\hat{mc}_t$

$$Y_t = Z_t L_t \quad \rightarrow \quad MC_t^R = \frac{W_t/P_t}{Z_t} \quad \rightarrow \quad W_t/P_t = Z_t MC_t^R$$

Labor market equilibirum

$$\psi L_t^{\varphi} C_t^{\sigma} = \psi \left(\frac{Y_t}{Z_t}\right)^{\varphi} Y_t^{\sigma} = W_t/P_t = Z_t MC_t^R \quad \rightarrow \quad \ln\psi + (\varphi+\sigma)\ln Y_t - \varphi \ln Z_t = \ln Z_t + \ln MC_t^R$$

$$\ln Y_t = \frac{1+\varphi}{\varphi+\sigma}\ln Z_t + \frac{1}{\varphi+\sigma}\ln MC_t^R - \frac{\ln\psi}{\varphi+\sigma}$$

In the flexible-prices world

$$\ln Y_t^* = \frac{1+\varphi}{\varphi+\sigma} Z_t + \frac{1}{\varphi+\sigma}\ln MC_{ss}^R - \frac{\ln\psi}{\varphi+\sigma}$$

Output gap

$$x_t = \ln Y_t - \ln Y_t^* = \frac{1}{\varphi+\sigma}\left(\ln MC_t^R - \ln MC_{ss}^R\right) = \frac{1}{\varphi+\sigma}\hat{mc}_t$$

Output gap in NKPC

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \hat{mc}_t$$

Output gap

$$x_t = \frac{1}{\varphi + \sigma} \hat{mc}_t$$

Final form of the NKPC

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where $\kappa \equiv \chi (\varphi + \sigma) > 0$, and e_t is a cost-push shock:
influence of non-wage costs of production when $Y_t \neq Z_t L_t$

Key equations of the New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve (where for simplicity $r_t^* = \rho$)

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - \rho \right) + u_t$$

A forward-looking system of three variables:
output gap x, inflation π and nominal interest rate i

Need an additional equation to close the system
$\hookrightarrow$ need to specify monetary policy rule

Monetary policy in the New Keynesian model

Optimal policy: long run

Two distortions in the basic model

1. Monopolistic competition: $P > MC$
2. Price dispersion: $P_1/P_2 \neq MC_1/MC_2$

The first distortion cannot be eliminated by monetary policy

But the second can, by keeping inflation rate at 0%

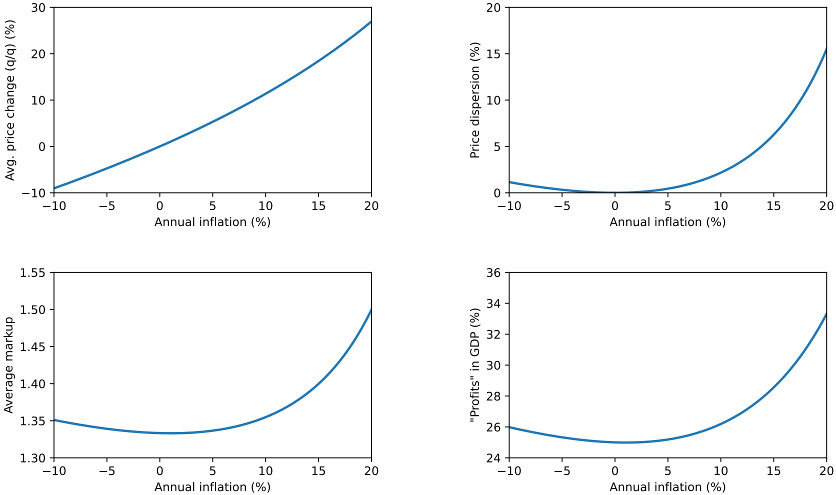
Welfare losses of annual inflation in 2-3% range
are very small, and other considerations matter

A positive inflation target (in advanced economies usually 2%)

- Decreases price dispersion under declining MC due to technological progress
- Increases labor market fluidity (nominal wage stickiness)
- Increases monetary policy space to reduce interest rate in recessions
- Lowers probability of hitting the effective lower bound on interest rates

Costs of inflation under Calvo scheme ($\mu = 1/3$ and $\theta = 0.75$)

Inflation other than 0% $\rightarrow$ price dispersion and lower effective output



Optimal policy: short run

If there are no cost-push shocks and sticky prices are the only distortion, then optimal monetary policy in the short run is to stabilize inflation perfectly

This would also perfectly stabilize output gap: divine coincidence, see Blanchard and Gali (2007)

If cost-push shocks occur and there are other distortions, like sticky wages, then optimal policy becomes more complicated, e.g. has to also stabilize wage inflation

Divine coincidence no longer holds
(impossible to stabilize all variables at the same time)

Optimal policy: loss function

Due to many distortions optimal policy involves trade-offs

Rotemberg and Woodford (1998): when real imperfections are present, the second order approximation to social welfare is (where $\phi > 0$ is a function of model parameters)

$$W = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\phi x_t^2 + \pi_t^2 \right) \right]$$

Consistent with behavior of central banks, who aim to stabilize inflation and output gaps

Optimal policy under discretion

Under optimal discretionary policy (ODP) the central bank is not able to influence expectations about future policy

Optimization boils down to solving a series of static problems

$$\begin{aligned} \min \quad & \phi x_t^2 + \pi_t^2 \\ \text{subject to} \quad & \pi_t = \beta \pi_t^e + \kappa x_t + e_t \end{aligned}$$

Note that expectations π_t^e are taken as given

Solution:

$$\pi_t = -\frac{\phi}{\kappa} x_t$$

After an inflationary cost-push shock the central bank allows the output gap to become negative and inflation to excees the target (how exactly? more on that later)

If the central bank perfectly implements ODP, identifying the NKPC slope using macro-level data becomes impossible, see McLeay and Tenreyro (2019)

Inflation bias

What if the policymakers wanted to maintain a positive output gap $\omega > 0$?

$$W^{\omega} = \phi \left(x_t - \omega\right)^2 + \pi_t^2$$

Consider for ease of exposition a slight alteration of the NKPC

$$\pi_t = \pi_t^e + \kappa x_t + e_t$$

where inflation is not a function of next-period inflation expectations,
but current-period inflation expectations (i.e. expectations formed
at the end of period $t-1$, before t-period shocks and policy actions are revealed)

Rewrite the NKPC (where $a \equiv 1/\kappa$ and $\epsilon_t \equiv -e_t/\kappa$)

$$x_t = \frac{\pi_t - \pi_t^e - \epsilon_t}{\kappa} \equiv a\left(\pi_t - \pi_t^e\right) + \epsilon_t$$

Central bank's choice

Optimization problem taking expectations as given

$$\begin{aligned} \min \quad & W^{\omega} = \phi \left(x_t - \omega\right)^2 + \pi_t^2 \\ \text{subject to} \quad & x_t = a\left(\pi_t - \pi_t^e\right) + \epsilon_t \end{aligned}$$

Plug constraint into objective

$$\min \quad W^{\omega} = \phi \left(a\left(\pi_t - \pi_t^e\right) + \epsilon_t - \omega\right)^2 + \pi_t^2$$

First Order Condition

$$\frac{\partial L}{\partial \pi_t} = 2\phi\left(a\left(\pi_t - \pi_t^e\right) + \epsilon_t - \omega\right) \cdot a + 2\pi_t = 0$$

Rearranging the FOC gives the desired inflation rate

$$\pi_t = \frac{a^2 \phi \pi_t^e + a\phi\left(\omega - \epsilon_t\right)}{1 + a^2 \phi}$$

Equilibrium rate of inflation

Under rational expectations

$$\pi_t^e \equiv \mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-1}\left[\frac{a^2\phi \mathrm{E}_{t-1}\pi_t + a\phi\left(\omega - \epsilon_t\right)}{1 + a^2\phi}\right] \rightarrow (1 + a^2\phi)\mathrm{E}_{t-1}\pi_t = a^2\phi \mathrm{E}_{t-1}\pi_t + a\phi\omega$$

Agents expect inflation exceeding declared target

$$\pi_t^e = \mathrm{E}_{t-1}\pi_t = a\phi\omega > 0$$

Resulting in actual inflation

$$\pi_t = a\phi\omega - \frac{a\phi}{1 + a^2\phi}\epsilon_t = a\phi\omega + \frac{a\phi}{1 + a^2\phi}\frac{e_t}{\kappa}$$

Because private agents understand the incentives facing the policymakers, average inflation is fully anticipated

Equilibrium produces average rate of inflation above target (inflation bias)

This has no systematic effect on output gap: $x_t = \frac{1}{1+a^2\phi}\epsilon_t = -\frac{1}{1+a^2\phi}\frac{e_t}{\kappa}$

Counteracting inflation bias

Appointment of a "hawkish" central bank governor

"Hawks" place an additional weight ($\delta > 0$) on inflation stabilization compared with other members of the society

$$W^{\omega\delta} = \phi \left(x_t - \omega\right)^2 + \left(1 + \delta\right) \pi_t^2$$

The rate of inflation under discretion will equal (lower mean and variance)

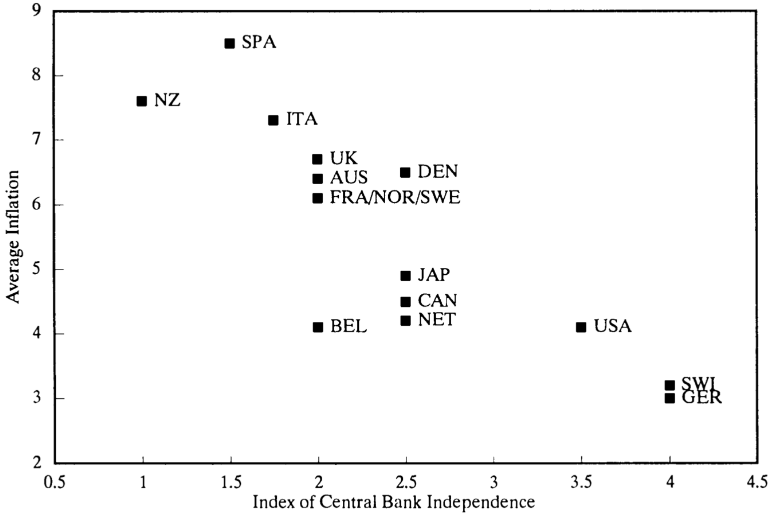
$$\pi_t = \frac{a\phi}{1 + \boldsymbol{\delta}} \omega + \frac{a\phi}{1 + \boldsymbol{\delta} + a^2\phi} \frac{e_t}{\kappa}$$

Targeting 0 output gap $(\omega = 0)$

The rate of inflation under discretion will equal the ODP outcome

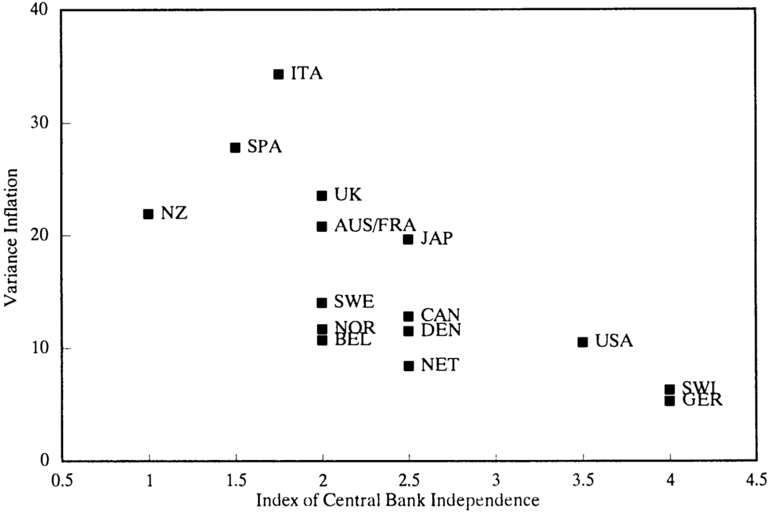
$$\pi_t = \frac{a\phi}{1 + a^2\phi} \frac{e_t}{\kappa} = -\frac{\phi}{\kappa} x_t$$

Central bank independence and average inflation (1955-1988)



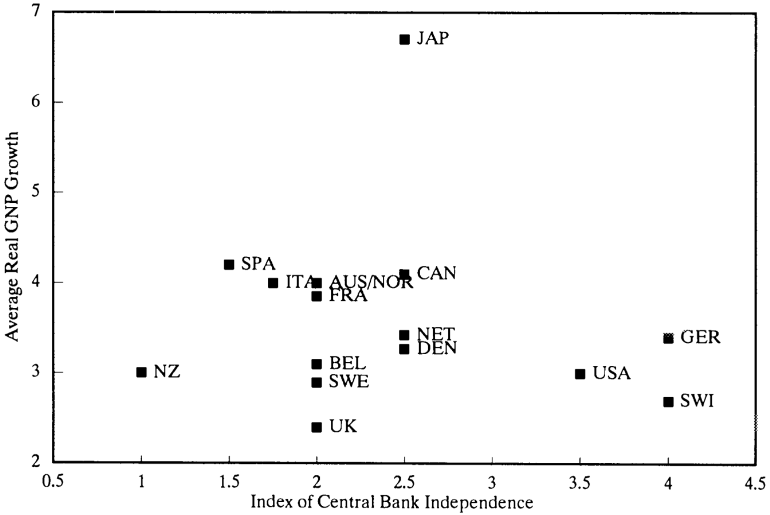
Alesina and Summers (1993)

Central bank independence and inflation volatility (1955-1988)



Alesina and Summers (1993)

Central bank independence and average real GNP growth (1955-1988)



Alesina and Summers (1993)

Optimal policy under commitment

Under **credible** commitment the central bank is able to influence expectations about future policy

The problem is now dynamic

$$\begin{aligned} \min \quad & \frac{1}{2}\sum_{t=0}^{\infty}\beta^t \,\mathrm{E}_0\left[\phi x_t^2 + \pi_t^2\right] \\ \text{subject to} \quad & \pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t \end{aligned}$$

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty}\beta^t \,\mathrm{E}_0\left[\frac{1}{2}\left(\phi x_t^2 + \pi_t^2\right) + \mu_t\left(\beta \pi_{t+1} + \kappa x_t + e_t - \pi_t\right)\right]$$

First order conditions

$$\begin{aligned} x_t: \quad & \beta^t \mathrm{E}_0\left[\phi x_t + \mu_t \kappa\right] = 0 & \rightarrow \quad & \mu_t = -\frac{\phi}{\kappa} x_t \\ \pi_t: \quad & \beta^{t-1}\mathrm{E}_0\left[\mu_{t-1}\beta\right] + \beta^t \mathrm{E}_0\left[\pi_t - \mu_t\right] = 0 & \rightarrow \quad & \pi_t = \mu_t - \mu_{t-1} \end{aligned}$$

Optimal policy under commitment

For the current period the past is not a constraint ($\mu_{-1} = 0$)

$$\pi_0 = \mu_0 = -\frac{\phi}{\kappa} x_0$$

Same as under discretion

For the future periods ($t \geq 1$) we get

$$\pi_t = \mu_t - \mu_{t-1} = -\frac{\phi}{\kappa} (x_t - x_{t-1})$$

Different than for today: will take the past into account

Optimal commitment policy (OCP) means pursuing a discretionary policy today, but promising a non-discretionary policy from tomorrow on!

But when we'll arrive in the next period, we will be tempted to act as in the current period: time inconsistency

Optimal policy under commitment

OCP is time inconsistent – solutions?

1. Appoint very credible central bankers
2. To build credibility, adopt a timeless perspective: pretend that OCP has been applied long ago and apply the formula for $t \geq 1$ from the beginning

Which is better: OCP or ODP?

- Neither invokes inflation bias
- ODP generates **stabilization bias**, making economy more volatile

The superiority of commitment calls for a credible, long-term arrangement for the central bank that will sometimes act **against** short-term welfare

Stabilization bias: discretion vs commitment

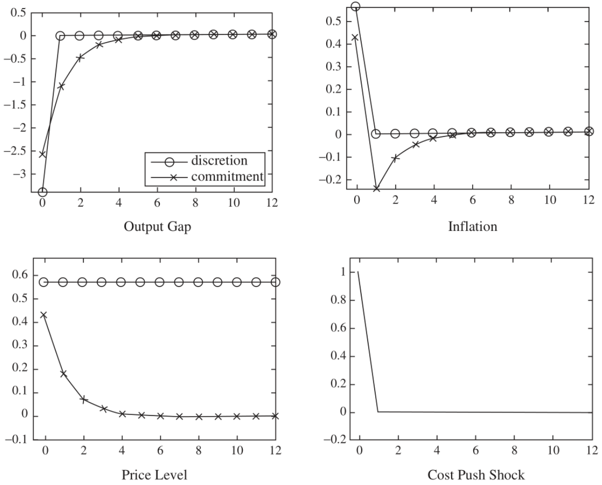


Figure 5.1 Optimal Responses to a Transitory Cost Push Shock

Gali (2008)

Stabilization bias: discretion vs commitment

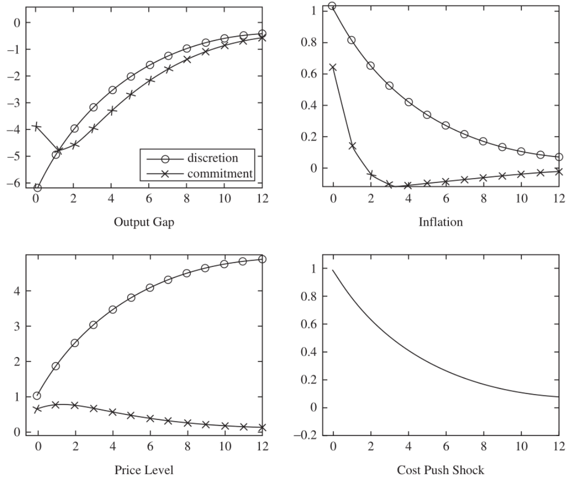


Figure 5.2 Optimal Responses to a Persistent Cost Push Shock

Gali (2008)

Instrument rules

We have been assuming that the central bank can "choose" x_t and π_t

In reality, the central bank can influence these variables indirectly, by e.g. changing the nominal interest rate

Recall the NKIS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + u_t$$

where the central bank can affect output gap by varying i

Note that monetary policy by changing i affects output gap "first" and inflation rate "second"

Determinacy concerns

Consider a persistent cost-push shock $e_t = \rho_e e_{t-1} + \epsilon_{e,t}$

Under ODP inflation and output gap are set to (α is a function of model parameters)

$$\pi_t = \phi \alpha e_t \quad \text{and} \quad x_t = -\kappa \alpha e_t$$

The CB could vary the level of i to try implementing the ODP

$$-\kappa\alpha \, e_t = -\kappa\alpha\rho_e e_t - \frac{1}{\sigma}\left(i_t - \phi\alpha\rho_e e_t - r_t^*\right)$$
$$i_t = r_t^* + \alpha\left(\kappa\sigma\left(1-\rho_e\right) + \phi\rho_e\right) e_t$$

But then our forward-looking system would have multiple solutions,
only one of which is consistent with ODP

Such instrument rule would be “too weak”

And would require observing e_t **perfectly** in real-time!

Taylor rules

Instead of constructing the instrument rule as function of shocks, construct the rule as function of endogenous variables

$$i_t = i^* + \gamma_\pi \, \pi_t$$

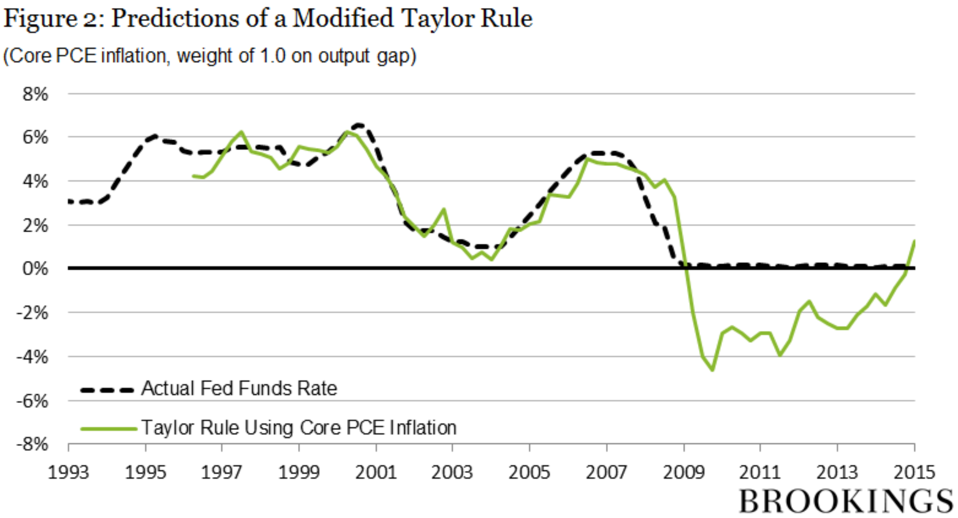
It can be shown that if only $\gamma_\pi > 1$, the system has a unique solution, and the central bank can “select” (among others) the ODP equilibrium

A more general Taylor rule allows for a non-zero inflation target π^*, reactions to output gap and smoothing of policy rate changes

$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i) \left(i^* + \gamma_\pi \, (\pi_t - \pi^*) + \gamma_x \, x_t \right)$$

A central bank should raise the interest rate when (forecasted) inflation is above target and / or (forecasted) output gap is positive

A Taylor rule can capture Fed's actual policy prior to 2009



Bernanke (2015)

Taylor principle and stability of inflation

Taylor principle: when inflation increases by 1 p.p., the central bank should raise the interest rate by $\gamma_\pi > 1$ p.p. (in a dynamic sense)

Failure to do so results in inflation instability

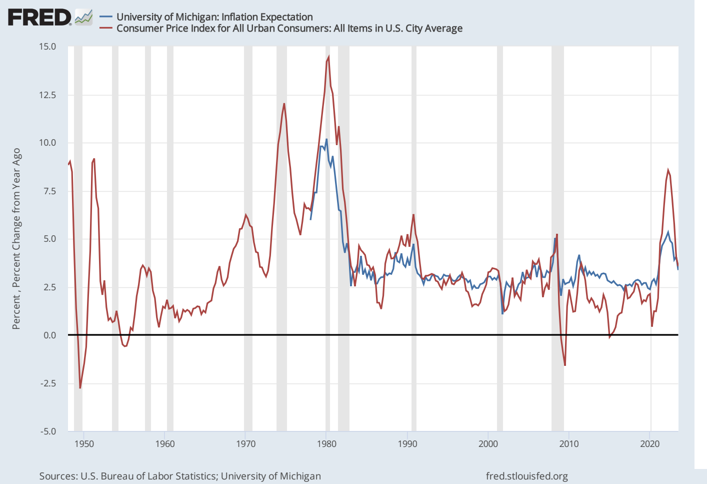
The estimate for the pre-Volcker rule is significantly less than unity. Monetary policy over this period was accommodating increases in expected inflation, in clear violation of the [Taylor principle – MB].

TABLE 1
ESTIMATES OF POLICY REACTION FUNCTION

	γ_π	γ_x	ρ
Pre-Volcker	0.83	0.27	0.68
	(0.07)	(0.08)	(0.05)
Volcker–Greenspan	2.15	0.93	0.79
	(0.40)	(0.42)	(0.04)

Clarida, Gali and Gertler (1999)

Inflation and household inflation expectations in the US



FRED

Basic three-equation New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^* \right) + u_t$$

Taylor rule

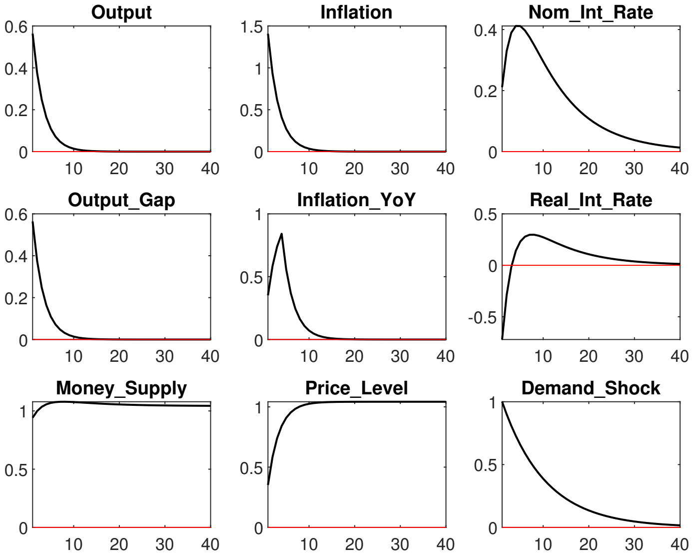
$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i) \left(i^* + \gamma_\pi \, (\pi_t - \pi^*) + \gamma_x \, x_t \right) + v_t$$

where v is an **exogenous** monetary policy shock

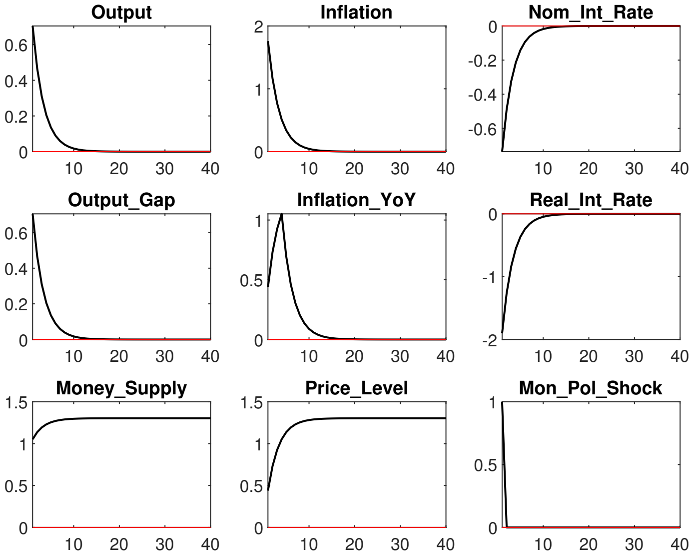
Plus (optionally) the natural real interest rate equation

$$r_t^* = \rho + \sigma \mathrm{E}_t \left[\Delta \ln Y_{t+1}^* \right]$$

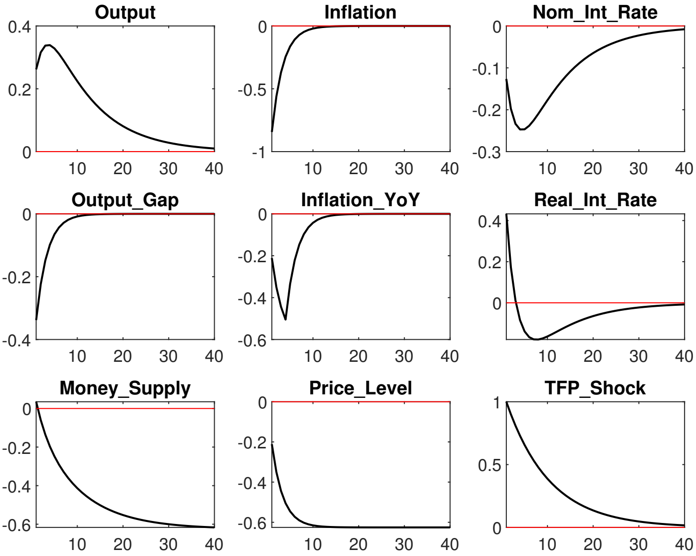
Demand shock ($u > 0$)



Monetary shock ($v < 0$)

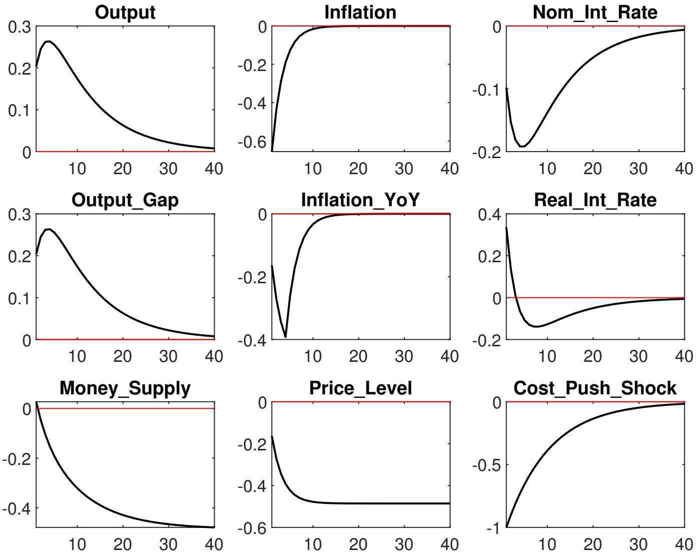


TFP shock ($z > 0$)

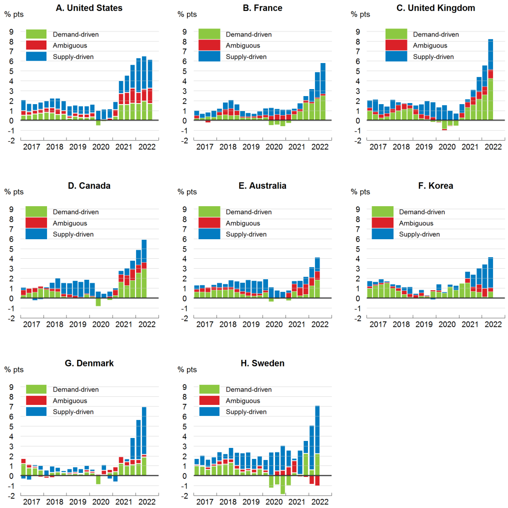


Output
0.4
0.2
0
10
20
30
40
Inflation
0
-0.5
-1
Nom_Int_Rate
-0.1
-0.2
-0.3
Output_Gap
-0.2
-0.4
Inflation_YoY
-0.6
Real_Int_Rate
Money_Supply
Price_Level
TFP_Shock
1
0.5

Cost-push shock ($e < 0$)



Model-based inflation shock decomposition

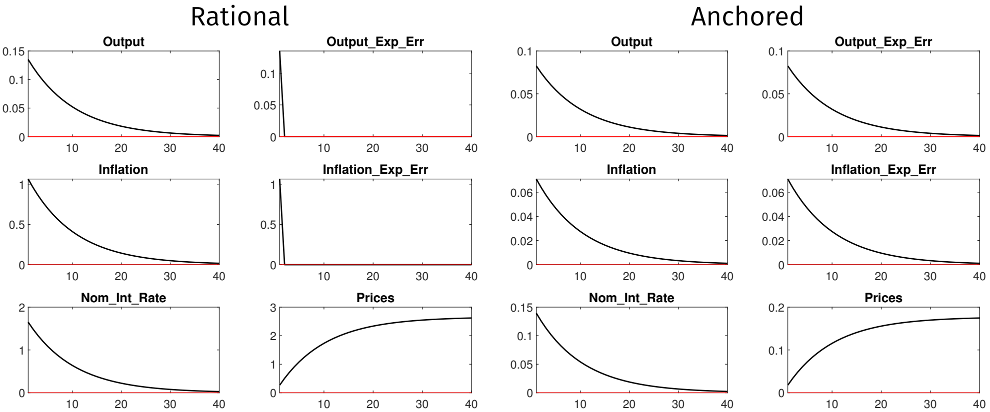


Role of expectations

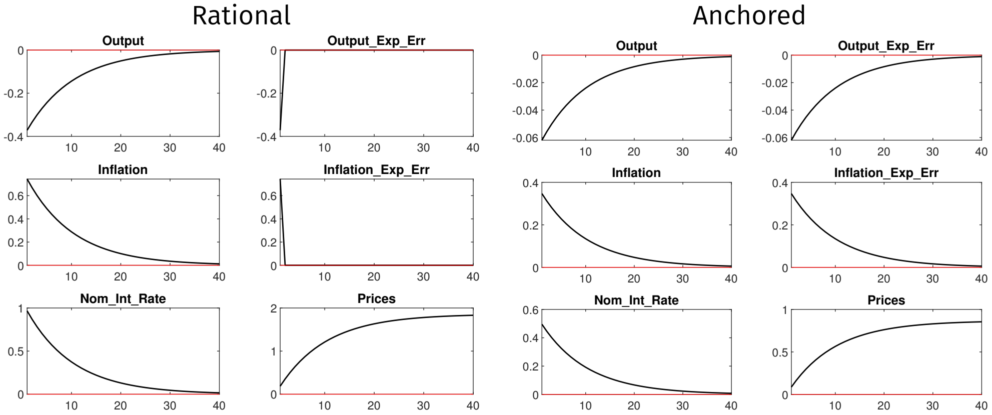
Modern monetary policy: management of expectations

Woodford (2005, p. 3): For not only do expectations about policy matter, but, at least under current conditions, very little *else* matters

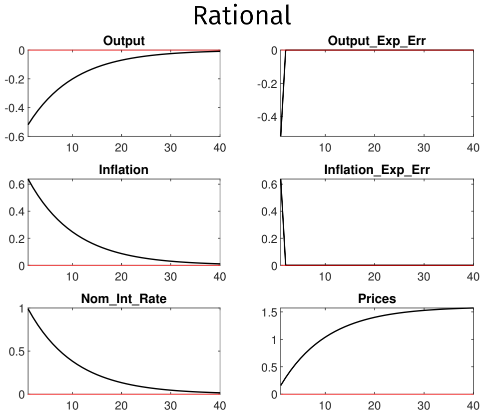
Response to demand shock under alternative expectations

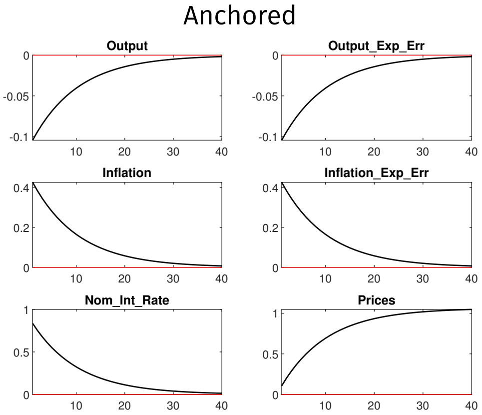


Response to cost-push shock under alternative expectations



Response to TFP shock under alternative expectations





Applied New Keynesian models

Shocks affect the economy in the directions indicated by empirical evidence

Basic model is too stylized to take it directly to data

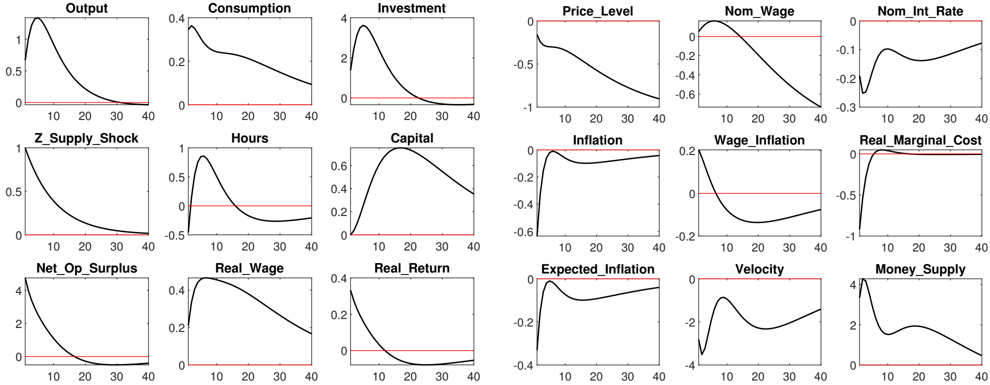
Some standard extensions introduced to applied NK models:

- Nominal wage stickiness
- Indexation of prices and wages or strategic complementarities delay the response of inflation to shocks
- Habits in the utility function delay the response of consumption to shocks
- Investment adjustment costs delay the response of investment to shocks

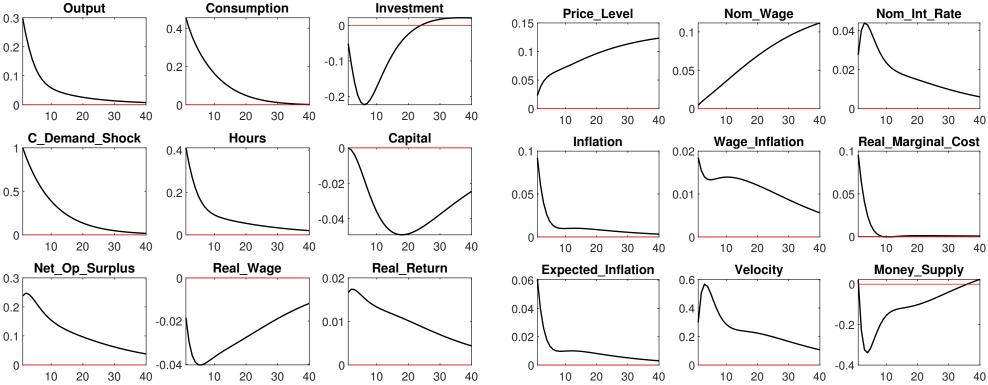
More complicated extensions:

- Financial frictions: Bernanke, Gertler and Gilchrist 1999, Kiyotaki and Moore 1997, Iacoviello 2005
- Unemployment: Gertler, Sala and Trigari 2008, Gali 2010
- Household heterogeneity (HANK): Kaplan, Moll and Violante (2018)

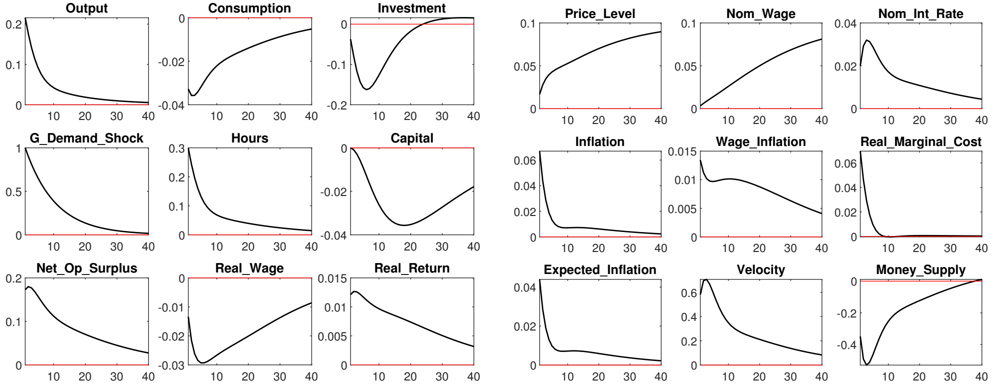
Extended NK model: TFP / cost-push shock



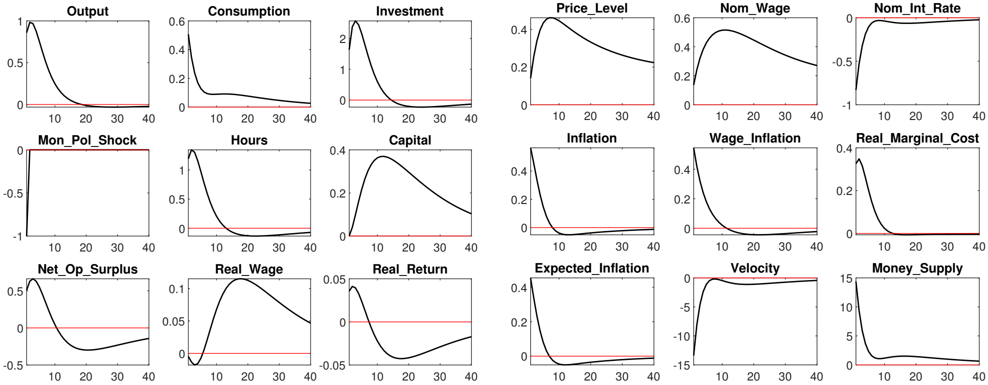
Extended NK model: consumption demand shock



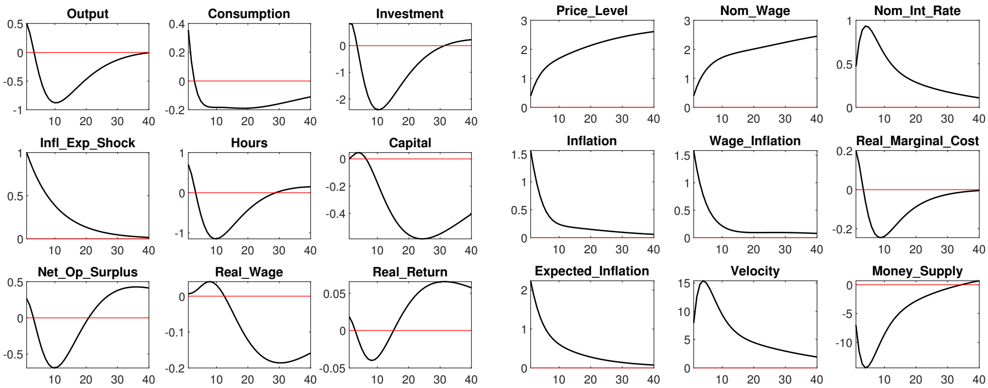
Extended NK model: government demand shock



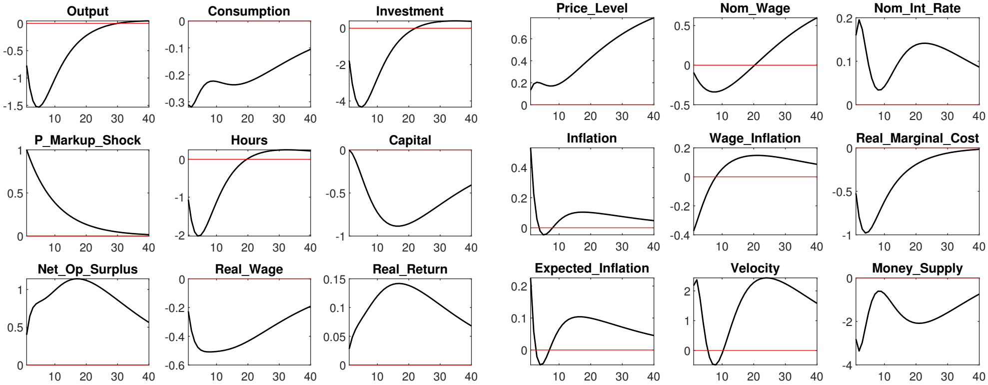
Extended NK model: exogenous interest rate cut



Extended NK model: exogenous rise in inflation expectations



Extended NK model: rise in firms' markup ("greedflation")



Extended NK model: rise in wage markup