University of Warsaw
Faculty of Economic Sciences

Unemployment

Advanced Macroeconomics QF: Lecture 10

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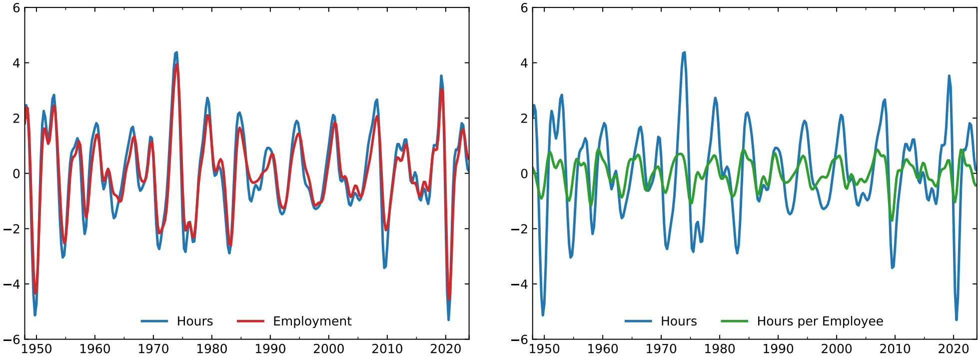
Labor input over the business cycle

Most of the variation in labor input is on the *extensive* margin (employment-unemployment) rather than on the *intensive* margin (hours worked by individual employees)



Labor input over the business cycle

Most of the variation in labor input is on the *extensive* margin (employment-unemployment) rather than on the *intensive* margin (hours worked by individual employees)

$$L = N \cdot h \quad \rightarrow \quad \ln L = \ln N + \ln h$$

$$\text{Var}\,(\ln L) = \text{Var}\,(\ln N) + \text{Var}\,(\ln h) + 2 \cdot \text{Cov}\,(\ln N,\, \ln h)$$

Variance-covariance matrix of Hodrick-Prescott deviations

	L	N	h
Total hours worked L	**2.92**		
Employment N		**2.16**	**0.26**
Hours per employee h		**0.26**	**0.25**

About 70% of variance of total hours worked is accounted for by variance of employment level and only 7% is accounted for by variance of hours worked by individual employees

Search and matching

At the same time there are job-seeking unemployed workers and worker-seeking firms

Labor markets are decentralized and active search is needed

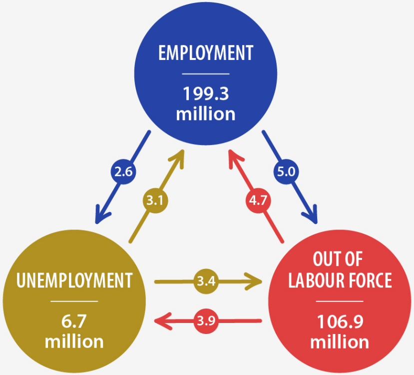
Search friction leads to unemployment even in the steady state

Peter Diamond, Dale Mortensen and Christopher Pissarides were awarded the Nobel Prize in 2010 for developing the search and matching model

Labor market status and flows in the EU

Transitions in labour market status in the EU, Q2 2024 - Q3 2024

(population aged 15-74, in millions)



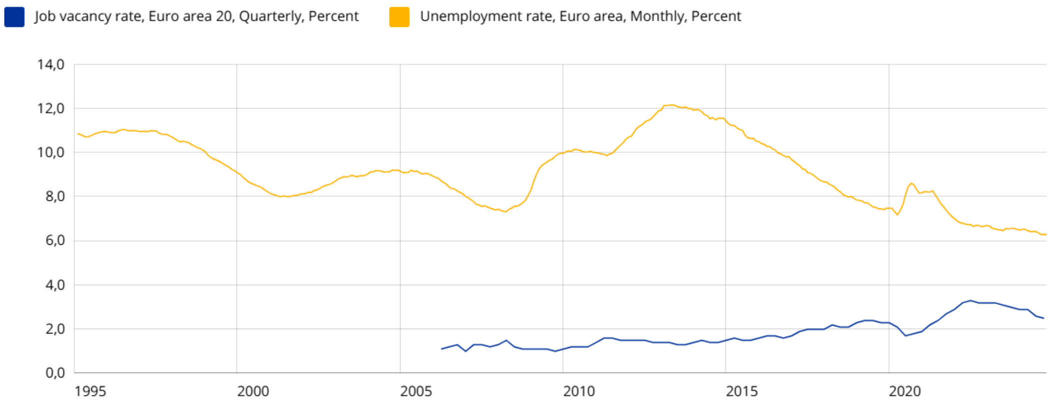
Labor market status change probabilities in the EU

Changes in the labour market status in the EU in Q3 2024

(% of initial status in Q2 2024, population aged 15-74)

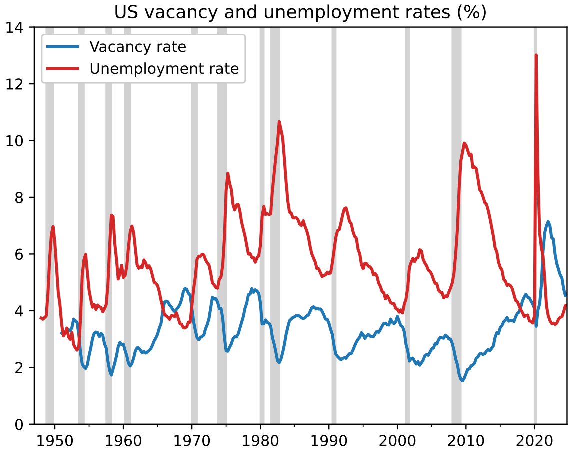
INITIAL STATUS IN Q2 2024	*NEW STATUS IN Q3 2024*		
	EMPLOYMENT	UNEMPLOYMENT	OUT OF LABOUR FORCE
EMPLOYMENT	96.3%	1.3%	2.4%
UNEMPLOYMENT	23.4%	50.6%	26.0%
OUT OF LABOUR FORCE	4.1%	3.3%	92.6%

Unemployment and vacancy rates in the EA

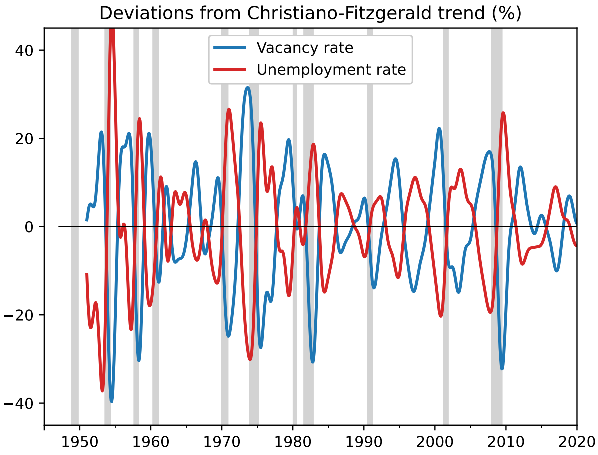


ECB Data Portal: vacancy rate and unemployment rate

Unemployment and vacancy rates in the USA



Labor market fluctuations in the USA (pre-covid)



Matching function

Firms create open job positions (openings, vacancies)

Unemployed workers search for jobs

Both jobs and workers are heterogeneous: not every possible match is attractive

Matching function captures this feature

New matches M are a function of the pool of vacancies V and pool of unemployed U

$$M = \chi V^{\eta} U^{1-\eta}$$

where $\chi > 0$ and $\eta \in (0, 1)$

Job finding and job filling rates

Unemployed workers care about the job finding rate f

$$f = \frac{M}{U} = \chi \left(\frac{V}{U} \right)^{\eta} = \chi \theta^{\eta}$$

where $\theta \equiv V/U$ is called the labor market tightness

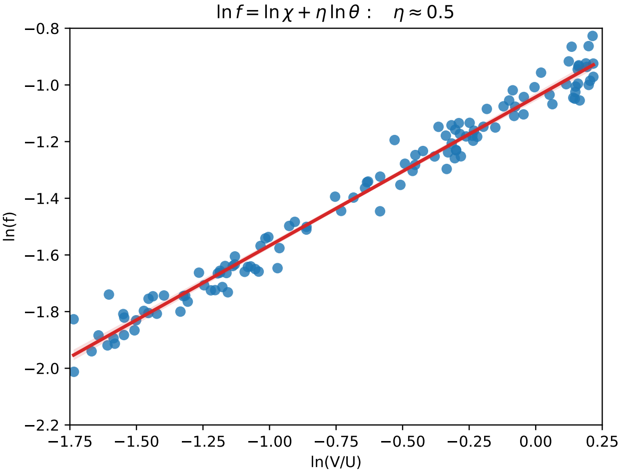
Firms with vacancies care about the job filling rate q

$$q = \frac{M}{V} = \chi \left(\frac{V}{U} \right)^{\eta - 1} = \chi \theta^{\eta - 1}$$

Dual externality from congestion:

- High number of unemployed decreases f and increases q
- High number of vacancies increases f and decreases q

Estimating the slope of the matching function (USA 2010-2019, monthly data)



Employment dynamics and steady state unemployment rate

For simplicity we are going to ignore flows between labor market activity and inactivity

Existing matches are destroyed at exogenous rate s

Timing convention: assume that new matches M_t are already productive in period t

$$N_t = N_{t-1} - sN_{t-1} + M_t$$

By definition, labor force $\bar{N}$ is the sum of employed N and unemployed U

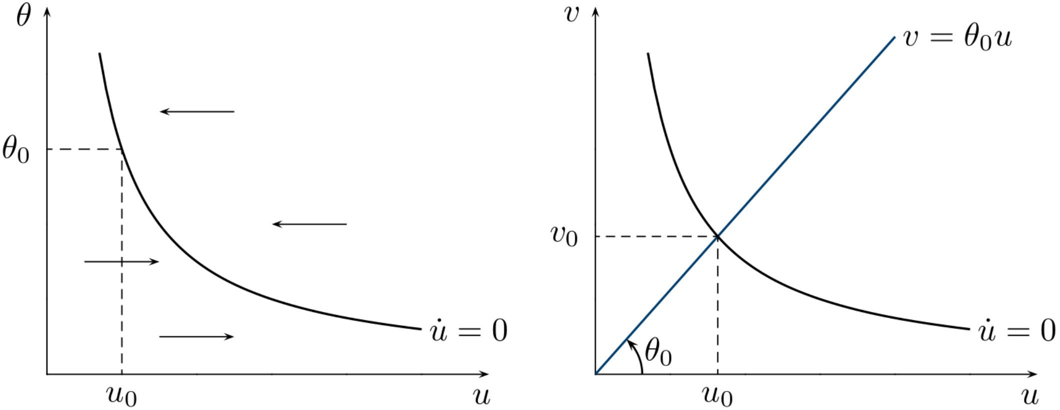
$$\bar{N} = N_t + U_t \quad \rightarrow \quad N_t = \bar{N} - U_t \quad \text{and} \quad U_t = \bar{N} - N_t$$

Steady state unemployment rate is a function of separation and job finding rates

$$\begin{aligned} U &= \bar{N} - \left[(1-s)\left(\bar{N} - U\right) + f\left(\theta\right)U\right] \quad | \quad : \bar{N} \\ u &= 1 - (1-s)(1-u) + f(\theta)\,u \\ u &= \frac{s}{s + f(\theta)} \end{aligned}$$

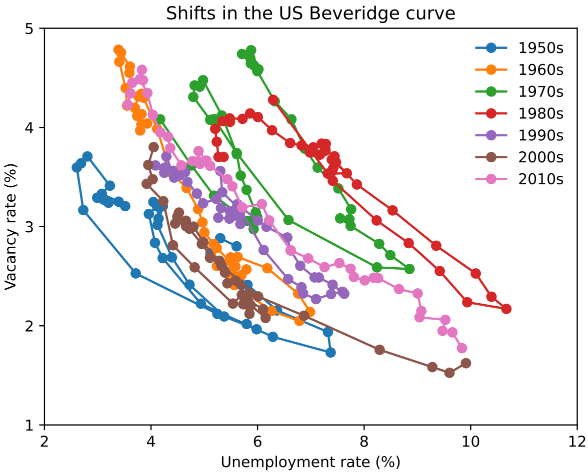
Beveridge curve: negative relationship between unemployment and vacancy rates

Beveridge curve: theory

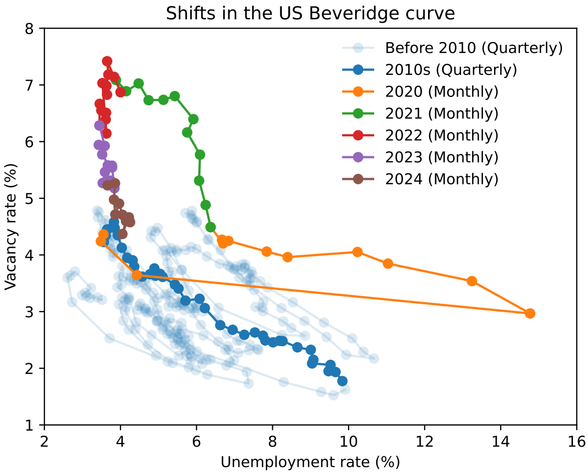


Graph by Leszek Wincenciak

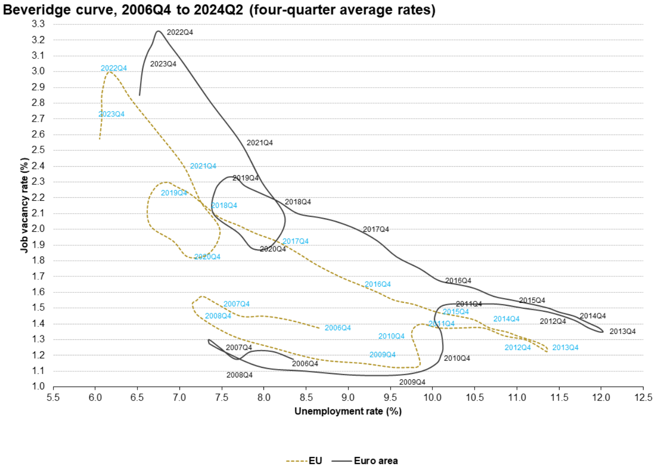
Beveridge curve: US data



Beveridge curve: US data

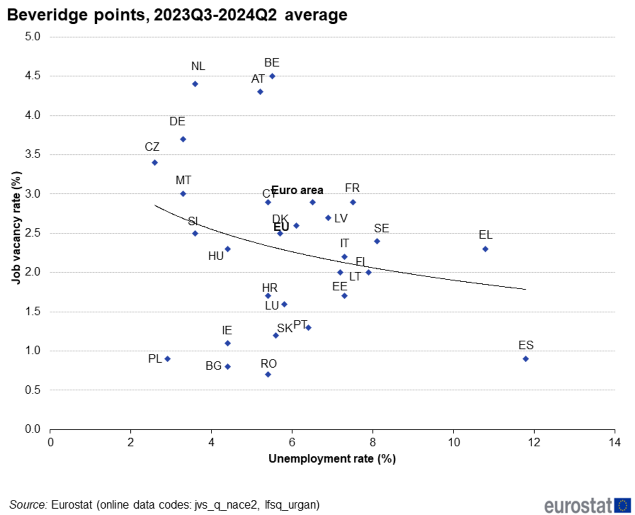


Beveridge curve: EU data



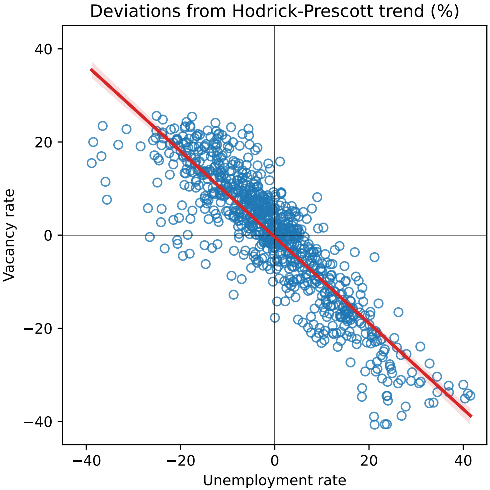
Source: Eurostat (online data codes: jvs_q_nace2, lfsq_urgan)

Beveridge curve: EU data

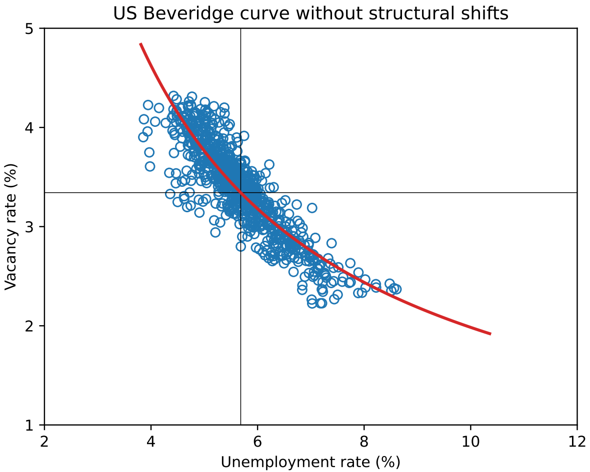


Beveridge curve: back to US data

Detrending with e.g. Hodrick-Prescott filter “takes out” structural shifts



Beveridge curve: "estimation" for US



Firms

Firms aim to maximize their expected value. Labor input is equal to employment N.

Maintaining a vacancy (filled at rate q) costs κ: advertising, interviewing, training, etc.

$$\begin{aligned}
\max_{\{K_t, N_t, V_t\}_{t=0}^{\infty}} \quad & \mathrm{E}_0\left[\sum_{t=0}^{\infty} R_{0,t}^{-1} D_t\right] \\
\text{subject to} \quad & D_t = Z_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - (r_t + \delta) K_t - \kappa V_t \\
& N_t = (1-s) N_{t-1} + q_t V_t \\
& R_{0,t} \equiv (1+r_1) \cdot (1+r_2) \cdot \ldots \cdot (1+r_t), \quad R_{0,0} \equiv 1
\end{aligned}$$

Lagrangian

$$\mathcal{L} = \mathrm{E}_0\left[\sum_{t=0}^{\infty} R_{0,t}^{-1} \left\{Z_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - (r_t+\delta) K_t - \kappa V_t + \mathcal{J}_t \left[(1-s) N_{t-1} + q_t V_t - N_t\right]\right\}\right]$$

Lagrange multiplier $\mathcal{J}_t$ captures the value of a marginal employed worker

Firms

Expanded Lagrangian from the perspective of time period t onward

$$\mathcal{L} = Z_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - (r_t + \delta) K_t - \kappa V_t + \mathcal{J}_t \left[(1-s) N_{t-1} + q_t V_t - N_t\right]$$
$$+ \mathrm{E}_t \left[\frac{1}{1+r_{t+1}} \left\{D_{t+1} + \mathcal{J}_{t+1} \left[(1-s) N_t + q_{t+1} V_{t+1} - N_{t+1}\right]\right\}\right] + \ldots$$

First Order Conditions

$$K_t: \quad \alpha Z_t K_t^{\alpha-1} N_t^{1-\alpha} - (r_t + \delta) = 0 \quad \rightarrow \quad r_t = \alpha Z_t K_t^{\alpha-1} N_t^{1-\alpha} - \delta$$

$$V_t: \quad -\kappa + \mathcal{J}_t q_t \quad \rightarrow \quad \mathcal{J}_t = \frac{\kappa}{q_t}$$

$$N_t: \quad \overbrace{(1-\alpha) Z_t K_t^{\alpha} N_t^{-\alpha}}^{MPN_t} - w_t - \mathcal{J}_t + \mathrm{E}_t \left[\frac{1}{1+r_{t+1}} \mathcal{J}_{t+1} (1-s)\right]$$

$$\hookrightarrow \quad \mathcal{J}_t = \underbrace{MPN_t - w_t}_{\text{within-period gain}} + \underbrace{\mathrm{E}_t \left[\frac{1-s}{1+r_{t+1}} \mathcal{J}_{t+1}\right]}_{\text{continuation value}}$$

Firms

Marginal benefit of an extra employee is equalized with average search cost

$$\mathcal{J}_t = \frac{\kappa}{q_t}$$

Steady state wage (Job / vacancy creation condition, JC)

$$\mathcal{J} = MPN - w + \frac{1-s}{1+r}\mathcal{J}$$
$$w = MPN + \left(\frac{1-s}{1+r} - 1\right)\mathcal{J}$$
$$w = MPN - \frac{s+r}{1+r}\frac{\kappa}{q\left(\theta\right)}$$

If $\kappa = 0$ (no search costs) then $\mathcal{J}_t = 0$ and we're back to Walrasian case with $w_t = MPN_t$

If $\kappa > 0$ then firms need to "break even" after search costs, resulting in $w_t < MPN_t$

Firms are willing to post more vacancies if the $MPN - w$ gap is large

Households

Workers belong to a "big family" of households that solves the following problem

$$
\begin{aligned}
\max_{\{K_t, N_t, A_{t+1}\}_{t=0}^{\infty}} \quad & U_0 = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - N_t \psi \frac{\bar{h}^{1+\varphi}}{1+\varphi} \right) \right] \\
\text{subject to} \quad & C_t + A_{t+1} = w_t N_t + \bar{b}(\bar{N} - N_t) + (1 + r_t) A_t - T_t \\
& N_t = (1-s) N_{t-1} + f_t(\bar{N} - N_{t-1})
\end{aligned}
$$

where $\bar{b}$ are unemployment benefits financed via lump-sum taxes T

Lagrangian (with $\bar{h}$ normalized to 1)

$$
\mathcal{L} = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{C_t^{1-\sigma}}{1-\sigma} - \psi N_t + \lambda_t \mathcal{E}_t \left[(1-s) N_{t-1} + f_t(\bar{N} - N_{t-1}) - N_t \right] \\ + \lambda_t \left[w_t N_t + b(\bar{N} - N_t) + (1 + r_t) A_t - T_t - C_t - A_{t+1} \right] \end{array} \right\} \right]
$$

Multiplier $\mathcal{E}_t$ captures the net value of a marginal employed household member

Households

Expanded Lagrangian from the perspective of time period t onward

$$\mathcal{L} = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi N_t + \lambda_t \mathcal{E}_t \left[(1-s)\, N_{t-1} + f_t(\bar{N} - N_{t-1}) - N_t\right]$$
$$+ \lambda_t \left[w_t N_t + b(\bar{N} - N_t) + (1+r_t)\, A_t - T_t - C_t - A_{t+1}\right]$$
$$+ \beta \mathrm{E}_t \left[C_{t+1}^{1-\sigma} / \,(1-\sigma) - \psi N_{t+1} + \lambda_{t+1} \mathcal{E}_{t+1} \left[(1-s)\, N_t + f_{t+1}(\bar{N} - N_t) - N_{t+1}\right]\right]$$
$$+ \beta \mathrm{E}_t \left[\lambda_{t+1} \left[w_{t+1} N_{t+1} + b(\bar{N} - N_{t+1}) + (1 + r_{t+1})\, A_{t+1} - T_{t+1} - C_{t+1} - A_{t+2}\right]\right] + \ldots$$

First Order Conditions

$$C_t : \quad C_t^{-\sigma} - \lambda_t = 0 \quad \rightarrow \quad \lambda_t = C_t^{-\sigma}$$

$$A_{t+1} : \quad -\lambda_t + \beta \mathrm{E}_t \left[\lambda_{t+1}\,(1 + r_{t+1})\right] = 0 \quad \rightarrow \quad \lambda_t = \beta \mathrm{E}_t \left[\lambda_{t+1}\,(1 + r_{t+1})\right]$$

$$N_t : \quad -\psi + \lambda_t \left[w_t - \bar{b}\right] - \lambda_t \mathcal{E}_t + \beta \mathrm{E}_t \left[\lambda_{t+1} \mathcal{E}_{t+1} \left[(1-s) - f_{t+1}\right]\right] = 0$$

$$\hookrightarrow \quad \mathcal{E}_t = \underbrace{w_t - \psi/\lambda_t - \bar{b}}_{\text{within-period gain}} + \underbrace{\mathrm{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t}\,(1 - s - f_{t+1})\, \mathcal{E}_{t+1}\right]}_{\text{continuation value}}$$

Households

From the asset FOC we get that

$$\mathrm{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1+r_{t+1}\right)\right]=1$$

Define the outside option of a worker as the sum of unemployed benefit and utility from extra leisure when unemployed

$$b_t \equiv \bar{b}+\psi/\lambda_t=\bar{b}+\psi C_t^{\sigma}$$

The net value of being employed is given by the difference between wage and outside option, plus the continuation value

$$\mathcal{E}_t=w_t-b_t+\mathrm{E}_t\left[\frac{1-s-f_{t+1}}{1+r_{t+1}}\mathcal{E}_{t+1}\right]$$

Wage setting

Since $MPN > b$, both firms and workers benefit from the matches

The negotiated wage can be anywhere between the outside option b and the marginal product of an employee MPN (plus an extra term capturing saving on future search)

Nash bargaining allows to model any sensible outcome of negotiations

Let $\gamma \in [0, 1]$ denote the relative bargaining power of firms

The negotiated wage w is the solution of the problem

$$\max_{w_t} \quad \mathcal{J}_t\left(w_t\right)^{\gamma} \cdot \mathcal{E}_t\left(w_t\right)^{1-\gamma}$$

Solving the problem (see slides at the end) results in

$$w_t = \gamma b_t + (1-\gamma)\left\{MPN_t + \mathrm{E}_t\left[\frac{\kappa\theta_{t+1}}{1+r_{t+1}}\right]\right\}$$

Intuitively: $w \to b$ if $\gamma \to 1$ and $w \to MPN + \kappa\theta/\left(1+r\right)$ if $\gamma \to 0$

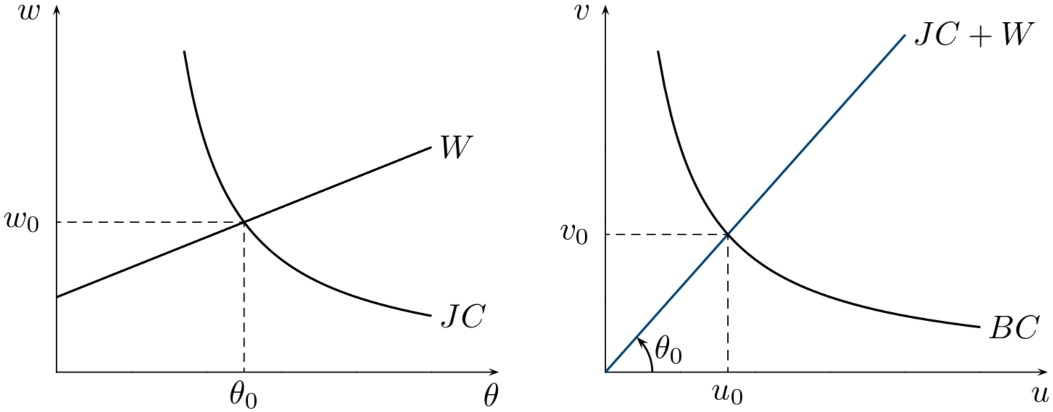
Steady state: key equations

In the steady state the model is fully summarized by the following three key equations

$$
\begin{aligned}
\text{Beveridge curve (BC)} &: \quad u = \frac{s}{s + f(\theta)} \\
\text{Job / vacancy creation (JC)} &: \quad w = MPN - \frac{r+s}{1+r}\frac{\kappa}{q(\theta)} \\
\text{Wage setting (W)} &: \quad w = \gamma b + (1-\gamma)\left(MPN + \frac{\kappa\theta}{1+r}\right)
\end{aligned}
$$

that have a convenient graphical interpretation

Steady state: graphical solution

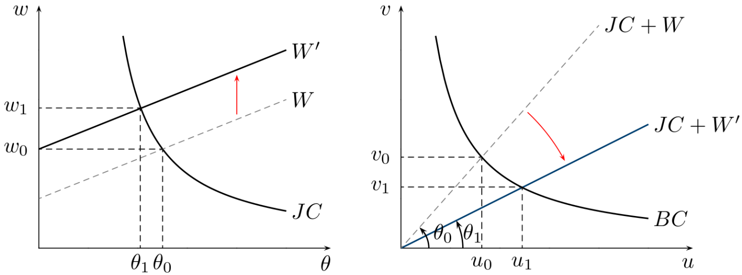


Graph by Leszek Wincenciak

Comparative statics

Increase in unemployment benefits ($b\uparrow$) or in workers' bargaining power ($\gamma\downarrow$)

- Increase in real wage w
- Decrease in labor market tightness θ
- Decrease in vacancy rate v
- Increase in unemployment rate u

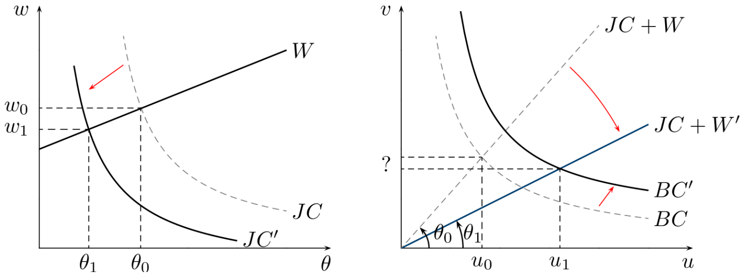


Graph by Leszek Wincenciak

Comparative statics

Increase in separation rate ($s \uparrow$) or a decrease in matching efficiency ($\chi \downarrow$)

- Decrease in real wage w
- Decrease in labor market tightness θ
- Ambiguous effect on vacancy rate v
- Increase in unemployment rate u

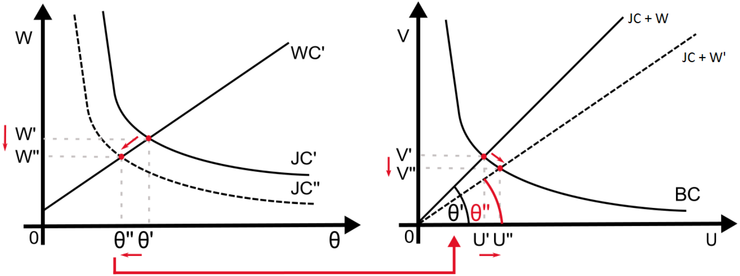


Graph by Leszek Wincenciak

Comparative statics

Increase in impatience ($\rho \uparrow$ / $\beta \downarrow$) or an increase in market interest rate ($r \uparrow$)

- Decrease in real wage w
- Decrease in labor market tightness θ
- Decrease in vacancy rate v
- Increase in unemployment rate u

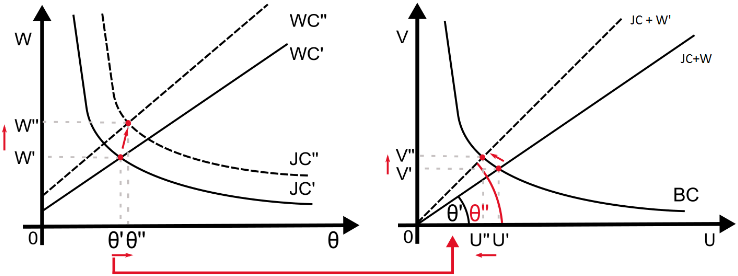


Graph by Matthias Hertweck

Comparative statics

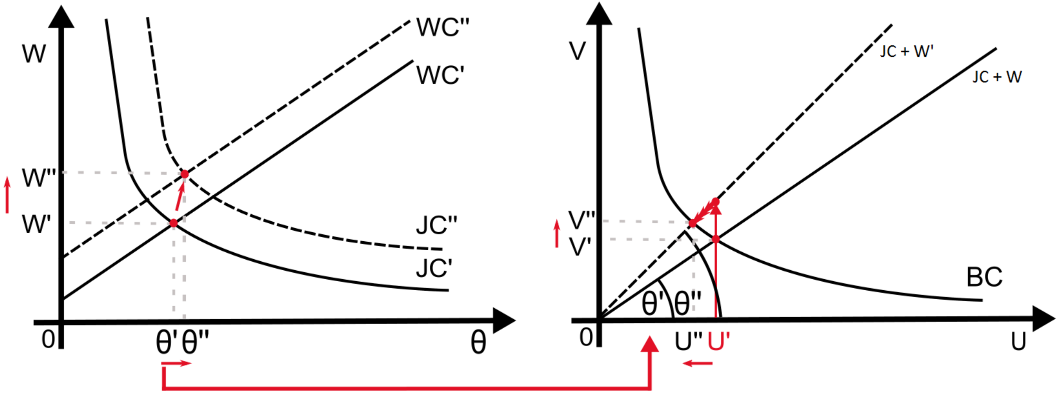
Increase in marginal product of an employee ($MPN \uparrow$)

- Increase in real wage w
- Increase in labor market tightness θ
- Increase in vacancy rate v
- Decrease in unemployment rate u



Graph by Matthias Hertweck

Transitional dynamics: permanent positive productivity shock



Graph by Matthias Hertweck

Parameters (monthly frequency)

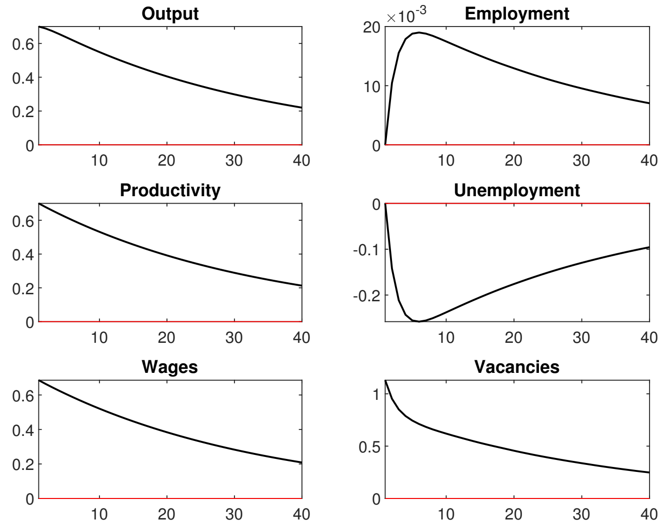
Values come from Shimer (2005)

	Description	Value
χ	matching efficiency	0.45
η	matching elasticity of v	0.28
s	separation rate	0.033
β	discount factor	0.99
MPN	steady state marginal product	1
κ	vacancy cost	0.21
b	unemployment benefit	0.4
γ	firm bargaining power	0.28

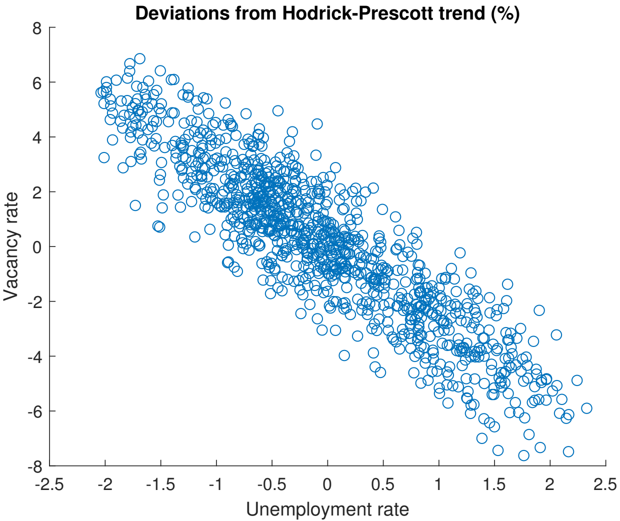
Implied steady state values (monthly frequency)

	Description	Value
u	unemployment rate	0.0687
v	vacancy rate	0.0674
θ	tightness	0.98
f	job finding rate	0.448
q	job filling rate	0.456
w	wage	0.98

Impulse response functions



Model generated Beveridge curve



Summary

We have a realistic model of the labor market

Able to match both steady state (averages) and some cyclical properties

Replicates the negative slope of the Beveridge curve

Shortcomings

- Not enough variation in employment
- Beveridge curve too steep
- Too much variation in wages

Alternative parametrizations

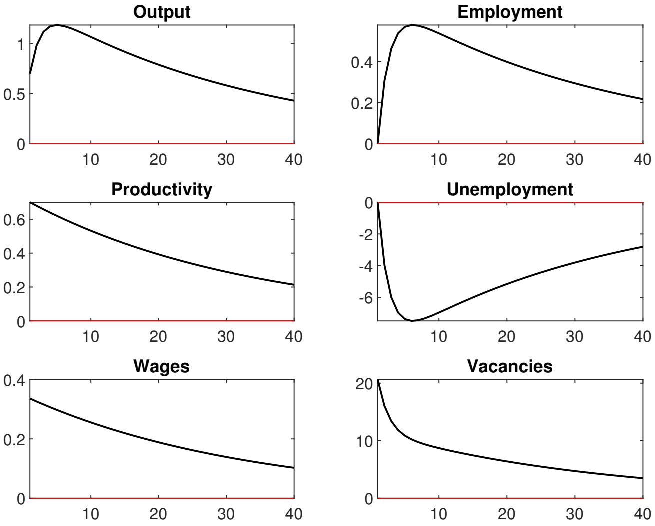
Values come from Hagedorn & Manovskii (2008)

	Description	Value
η	matching elasticity of v	0.45
b	outside option	0.965
γ	firm bargaining power	0.928

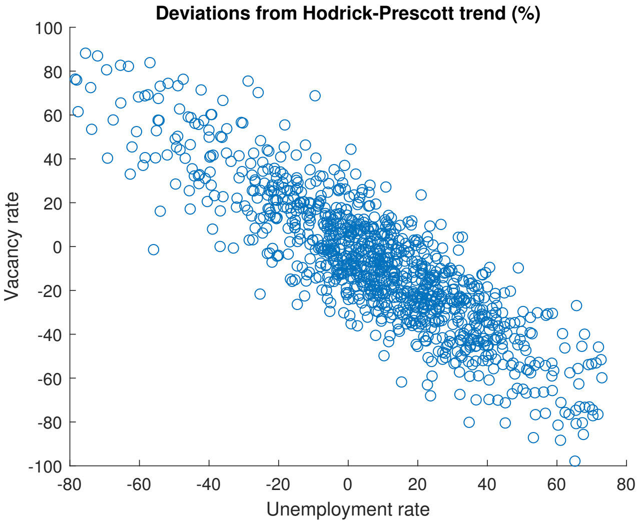
Additionally, Mortensen & Nagypal (2007) set $\eta = 0.54$

- Firms have very strong bargaining position
- But outside option includes leisure utility: match surplus is low and very volatile

Impulse response functions (alternative parametrizations)



Model generated Beveridge curve (alternative parametrizations)



Summary

Alternative parametrizations yield better results

Both unemployment and employment become more volatile

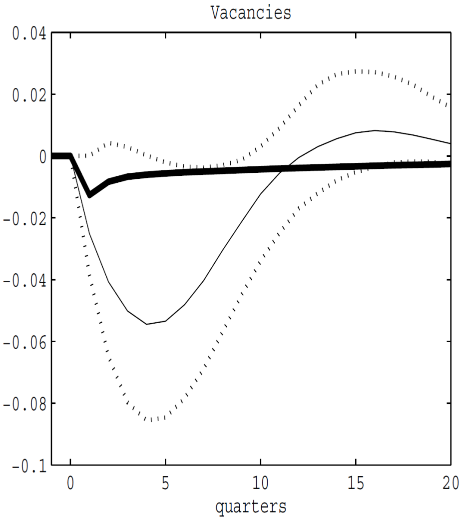
Volatility of wages is diminished

Key problem for the basic search and matching model identified:
period-by-period Nash bargaining

Further extensions make alternative assumptions about the wage setting process

Dynamics of vacancies

Fujita (2004): model IRF for vacancies is counterfactual



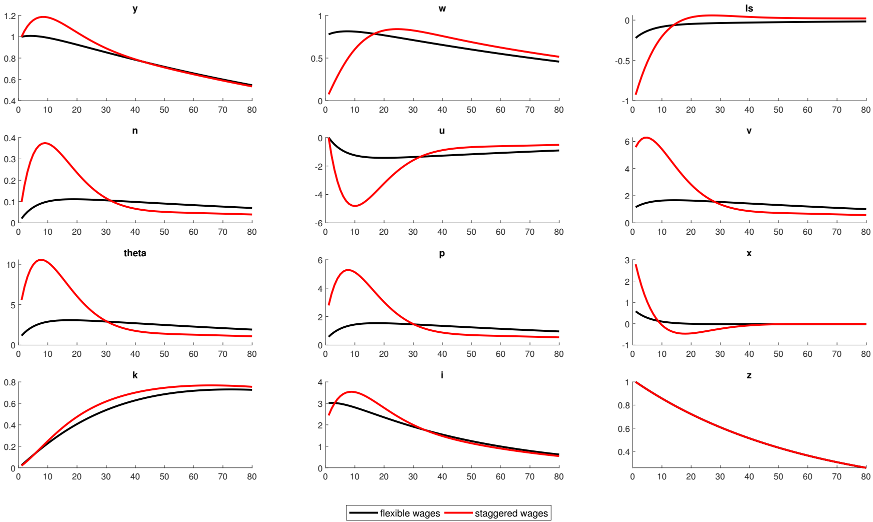
Alternative hiring cost function

We have assumed linear vacancy costs

Gertler & Trigari (2009) assume convex costs in terms of hiring rate $x \equiv M/N$

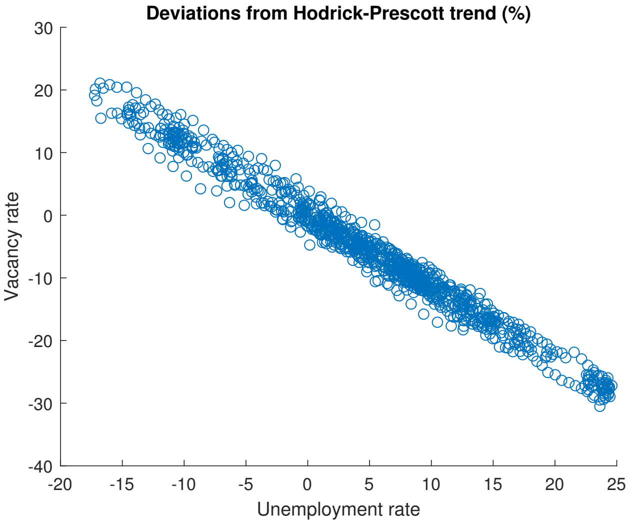
They also consider multi-period wage contracts:
within each period only a fraction of wage contracts are renegotiated

Gertler & Trigari: Impulse response functions

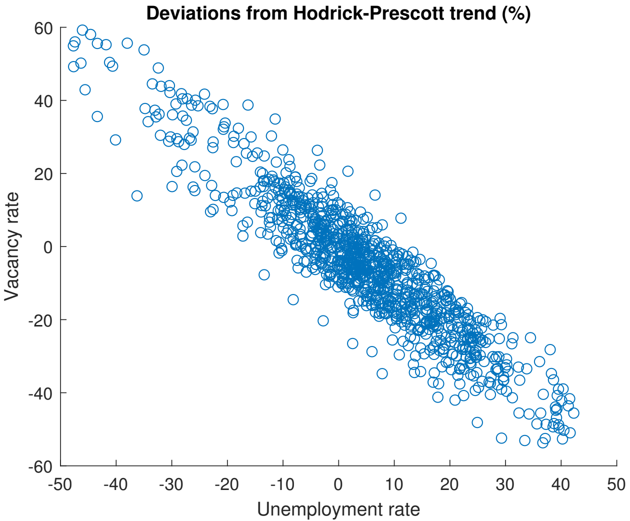


Monthly period frequency

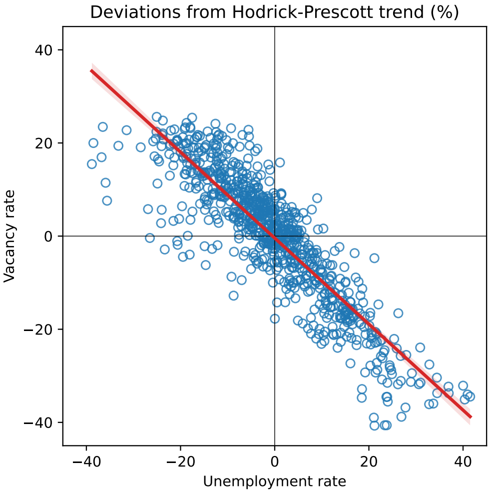
Gertler & Trigari: Beveridge curve (flexible wages)



Gertler & Trigari: Beveridge curve (staggered wages)



Beveridge curve: data



Gertler & Trigari: business cycle statistics

	y	w	ls	n	u	v	θ	a	i	c
	A. U.S. Economy, 1964:1–2005:1									
Relative standard deviation	1.00	.52	.51	.60	5.15	6.30	11.28	.61	2.71	.41
Autocorrelation	.87	.91	.73	.94	.91	.91	.91	.79	.85	.87
Correlation with y	1.00	.56	−.20	.78	−.86	.91	.90	.71	.94	.81
	B. Model Economy, $\lambda = 0$ (Flexible Wages)									
Relative standard deviation	1.00	.87	.09	.10	1.24	1.58	2.72	.93	3.11	.37
Autocorrelation	.81	.81	.58	.92	.92	.86	.90	.78	.80	.85
Correlation with y	1.00	1.00	−.54	.59	−.59	.98	.92	1.00	.99	.93
	C. Model Economy, $\lambda = 8/9$ (3 Quarters)									
Relative standard deviation	1.00	.56	.57	.35	4.44	5.81	9.84	.71	3.18	.35
Autocorrelation	.84	.95	.65	.90	.90	.82	.88	.76	.86	.86
Correlation with y	1.00	.66	−.56	.77	−.77	.91	.94	.97	.99	.90
	D. Model Economy, $\lambda = 11/12$ (4 Quarters)									
Relative standard deviation	1.00	.48	.58	.44	5.68	7.28	12.52	.64	3.18	.34
Autocorrelation	.85	.96	.68	.91	.91	.86	.90	.74	.88	.86
Correlation with y	1.00	.55	−.59	.78	−.78	.93	.95	.95	.99	.90

Summary

After adding multi-period contracts, Gertler & Trigari obtain a very good empirical match of the RBC model with search & matching features

This is one of the best matches for single-shock models

Key to the success was:

- Convex vacancy posting
- Staggered (multi-period) wage contracts

Possible further extensions

Endogenous (non-constant) separation rate

Hours per worker adjustments

On-the-job search

Alternative bargaining and wage expectations schemes

Derivation of the wage setting equation

The negotiated wage is the solution of the problem

$$\max_{w_t} \quad \mathcal{J}_t\left(w_t\right)^{\gamma} \cdot \mathcal{E}_t\left(w_t\right)^{1-\gamma}$$

Derivatives of $\mathcal{J}_t$ and $\mathcal{E}_t$ with respect to wage w_t

$$\mathcal{J}_t = MPN_t - w_t + \mathrm{E}_t\left[\frac{1-s}{1+r_{t+1}}\mathcal{J}_{t+1}\right] \quad \rightarrow \quad \frac{\partial \mathcal{J}_t}{\partial w_t} = -1$$

$$\mathcal{E}_t = w_t - b_t + \mathrm{E}_t\left[\frac{1-s-f_{t+1}}{1+r_{t+1}}\mathcal{E}_{t+1}\right] \quad \rightarrow \quad \frac{\partial \mathcal{E}_t}{\partial w_t} = 1$$

First order condition

$$\gamma \mathcal{J}_t^{\gamma-1} \cdot \frac{\partial \mathcal{J}_t}{\partial w_t} \cdot \mathcal{E}_t^{1-\gamma} + \mathcal{J}_t^{\gamma} \cdot (1-\gamma)\,\mathcal{E}_t^{-\gamma} \cdot \frac{\partial \mathcal{E}_t}{\partial w_t} = 0$$

$$\gamma \mathcal{E}_t = (1-\gamma)\,\mathcal{J}_t$$

Derivation of the wage setting equation

Plug in expressions for $\mathcal{E}_t$ and $\mathcal{J}_t$

$$\gamma\left\{w_t - b_t + \mathrm{E}_t\left[\frac{1-s-f_{t+1}}{1+r_{t+1}}\mathcal{E}_{t+1}\right]\right\} = (1-\gamma)\left\{MPN_t - w_t + \mathrm{E}_t\left[\frac{1-s}{1+r_{t+1}}\mathcal{J}_{t+1}\right]\right\}$$

$$w_t - \gamma b_t + \mathrm{E}_t\left[\frac{1-s-f_{t+1}}{1+r_{t+1}}\cdot\gamma\mathcal{E}_{t+1}\right] = (1-\gamma)\,MPN_t + (1-\gamma)\,\mathrm{E}_t\left[\frac{1-s}{1+r_{t+1}}\mathcal{J}_{t+1}\right]$$

$$w_t - \gamma b_t + \mathrm{E}_t\left[\frac{1-s-f_{t+1}}{1+r_{t+1}}(1-\gamma)\,\mathcal{J}_{t+1}\right] = (1-\gamma)\,MPN_t + (1-\gamma)\,\mathrm{E}_t\left[\frac{1-s}{1+r_{t+1}}\mathcal{J}_{t+1}\right]$$

$$w_t = \gamma b_t + (1-\gamma)\,MPN_t + (1-\gamma)\,\mathrm{E}_t\left[\frac{f_{t+1}}{1+r_{t+1}}\mathcal{J}_{t+1}\right] \quad \Big| \quad \mathcal{J}_t = \frac{\kappa}{q_t}$$

$$w_t = \gamma b_t + (1-\gamma)\left\{MPN_t + \mathrm{E}_t\left[\frac{f_{t+1}}{1+r_{t+1}}\frac{\kappa}{q_{t+1}}\right]\right\}$$

$$w_t = \gamma b_t + (1-\gamma)\left\{MPN_t + \mathrm{E}_t\left[\frac{\kappa\theta_{t+1}}{1+r_{t+1}}\right]\right\}$$