University of Warsaw
Faculty of Economic Sciences

Consumption & asset pricing

Advanced Macroeconomics QF: Lecture 1

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University of Warsaw

Course organization

Website & contact information

- Course website: coin.wne.uw.edu.pl/mbielecki
  $\hookrightarrow$ Advanced Macroeconomics QF Lectures
- Lecture slides and/or notes available prior to the relevant lecture
- E-mail: m.p.bielecki@uw.edu.pl
- Office hours by appointment

Assessment

You will be graded on the basis of

- Final exam (70 points): closed book, problems similar to homeworks
- Homeworks (30 points): 5 problem sets, worth 6 points each
  - at least two weeks to submit solutions
  - can be submitted in groups of 2

Points from the final exam and homeworks add up

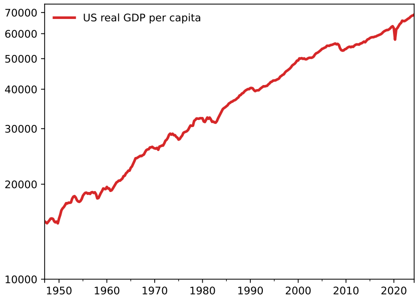
You need at least 50 points to pass the course

Score	[0, 50)	[50, 60)	[60, 70)	[70, 80)	[80, 90)	[90, 100]
Grade	2	3	3.5	4	4.5	5

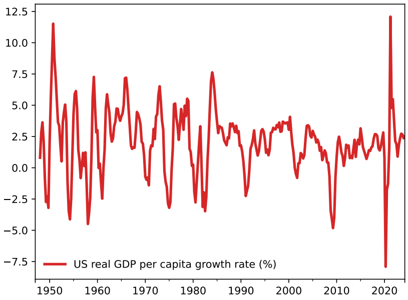
Topics of interest

We want to understand the mechanisms behind

Long-run growth



Business cycles



using the tools of modern macroeconomics

Course structure

- Microeconomic Foundations
  - Consumption & asset pricing
  - Investment
- Economic Growth
  - Growth facts & Solow-Swan model
  - Overlapping generations model
  - Ramsey-Cass-Koopmans model
  - Endogenous growth models
- Business Cycles
  - Business cycle facts & Real Business Cycles model
  - Models of unemployment
  - New Keynesian model
  - Monetary policy design
  - Financial frictions

Questions?

Intertemporal consumption choice

Utility Maximization Problem

The household maximizes utility from consumption in two periods

$$\max_{c_1, c_2, a} \quad U = \ln c_1 + \beta \ln c_2$$

$$\text{subject to} \quad c_1 + a = y_1$$

$$c_2 = y_2 + (1 + r)\, a$$

Logarithmic utility for easy derivations, discount factor $\beta \in [0, 1]$

Exogenous variables: incomes y_1, y_2 and the real interest rate r

Choice variables: consumption c_1, c_2 and assets at the end of period 1 a

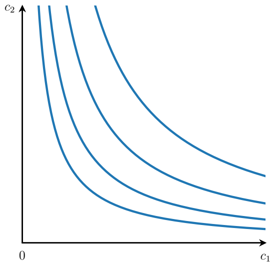
Lifetime budget constraint:

$$a = \frac{c_2 - y_2}{1 + r} \quad \rightarrow \quad c_1 + \frac{c_2 - y_2}{1 + r} = y_1 \quad \rightarrow \quad c_1 + \frac{c_2}{1 + r} = y_1 + \frac{y_2}{1 + r}$$

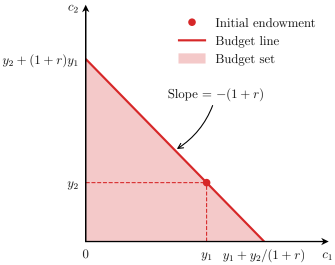
Utility Maximization Problem: graphical interpretation

We are looking for a specific **indifference curve** that is just tangent to the **budget line**. The point of tangency is the **optimal consumption** choice:

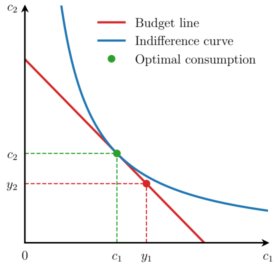
Indifference curve map



Lifetime budget constraint



Graphical solution



Method of Lagrange multipliers

Set up the Lagrangian

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left[y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right]$$

Derive the first order conditions (FOCs)

$$c_1: \quad \frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} + \lambda [-1] = 0 \quad \rightarrow \quad \lambda = \frac{1}{c_1}$$

$$c_2: \quad \frac{\partial \mathcal{L}}{\partial c_2} = \beta \cdot \frac{1}{c_2} + \lambda \left[-\frac{1}{1+r} \right] = 0 \quad \rightarrow \quad \lambda = \beta (1+r) \frac{1}{c_2}$$

Obtain the optimality condition (**Euler equation**)

$$\frac{1}{c_1} = \beta (1+r) \frac{1}{c_2} \quad \rightarrow \quad \boldsymbol{c_2 = \beta (1+r) c_1}$$

Utility Maximization Problem: solution

Plug the Euler equation into the lifetime budget constraint

$$c_2 = \beta (1+r)\, c_1$$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$c_1 + \beta c_1 = y_1 + \frac{y_2}{1+r}$$

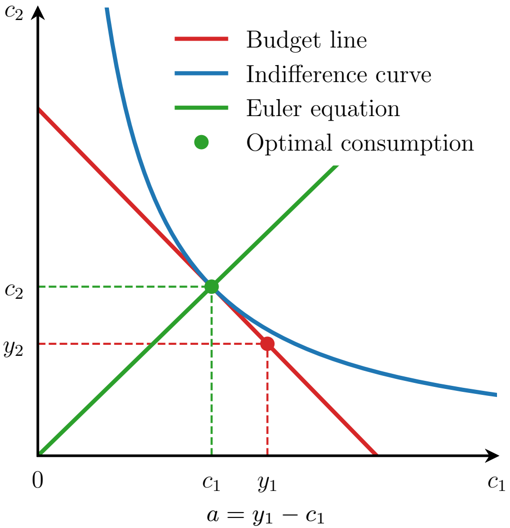
Optimal levels of consumption and assets

$$c_1 = \frac{1}{1+\beta}\left[y_1 + \frac{y_2}{1+r}\right]$$

$$c_2 = \frac{\beta}{1+\beta}\left[(1+r)\, y_1 + y_2\right]$$

$$a = y_1 - c_1 = \frac{1}{1+\beta}\left[\beta y_1 - \frac{y_2}{1+r}\right]$$

Utility Maximization Problem solution: graphical interpretation



Comparative Statics

Consumer is more patient (higher β)

$$\frac{\partial c_1}{\partial \beta} < 0, \quad \frac{\partial c_2}{\partial \beta} > 0, \quad \frac{\partial a}{\partial \beta} > 0$$

Higher income in the first period

$$\frac{\partial c_1}{\partial y_1} > 0, \quad \frac{\partial c_2}{\partial y_1} > 0, \quad \frac{\partial a}{\partial y_1} > 0$$

Higher (expected) income in the second period

$$\frac{\partial c_1}{\partial y_2} > 0, \quad \frac{\partial c_2}{\partial y_2} > 0, \quad \frac{\partial a}{\partial y_2} < 0$$

Comparative Statics: changes in real interest rate r

Substitution effect: as consumption in the future gets „cheaper", induces the agent to consume more in the second period and less in the first period

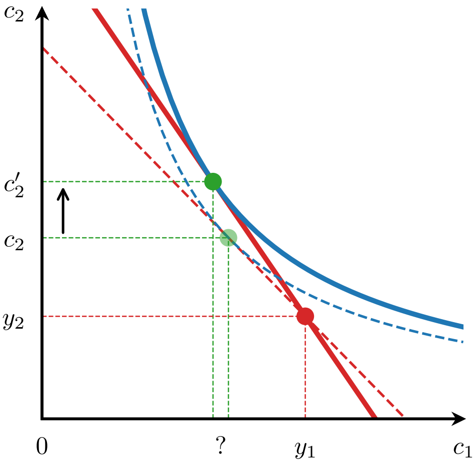
Income effect depends on the desired assets prior to interest rate change:

- Saver ($a > 0$): expansion of the budget set induces increases in consumption in both periods
- Borrower ($a < 0$): contraction of the budget set induces decreases in consumption in both periods

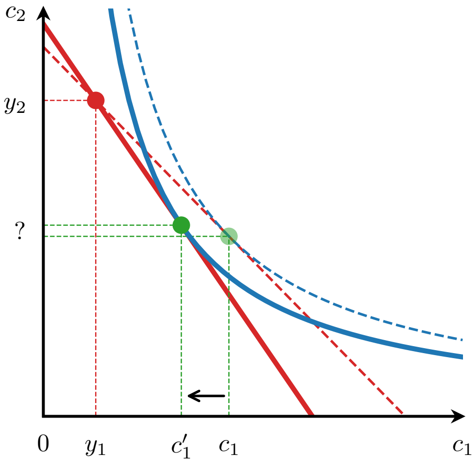
Effects of an increase in r	Saver			Borrower		
	c_1	c_2	a	c_1	c_2	a
Substitution	$-$	$+$	$+$	$-$	$+$	$+$
Income	$+$	$+$	$-$	$-$	$-$	$+$
Net	?	$+$	?	$-$	?	$+$

Comparative Statics: changes in real interest rate r

Saver



Borrower



Effects of changes in interest rate in the data

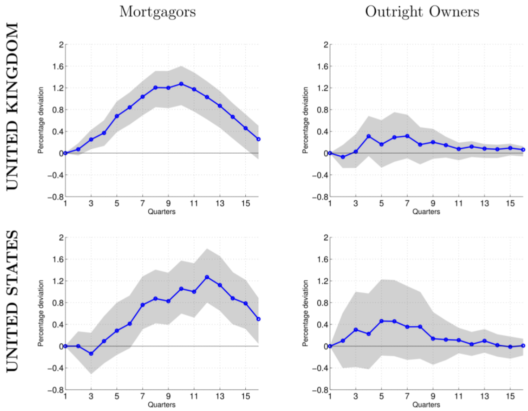


Figure 5: Dynamic effects of a 25 basis point unanticipated interest rate cut on the expenditure of durable goods by housing tenure group. Grey areas are bootstrapped 90% confidence bands. Top row: UK (FES/LCFS data). Bottom row: US (CEX data).

Cloyne, Ferreira, Surico (2016) *Monetary policy when households have debt*

Additional constraints

Borrowing constraint

Now the agent cannot have negative assets

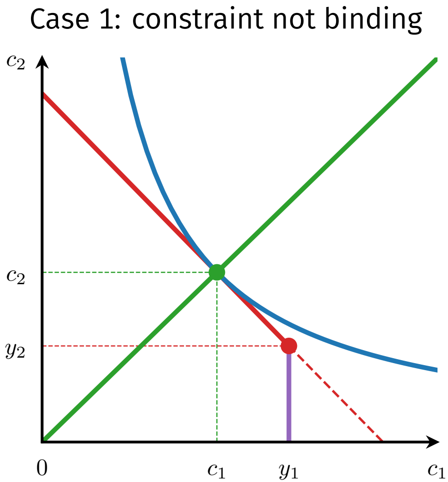
$$\begin{aligned} \max_{c_1, c_2, a} \quad & U = \ln c_1 + \beta \ln c_2 \\ \text{subject to} \quad & c_1 + a = y_1 \\ & c_2 = y_2 + (1+r)\, a \\ & \boldsymbol{a \geq 0} \end{aligned}$$

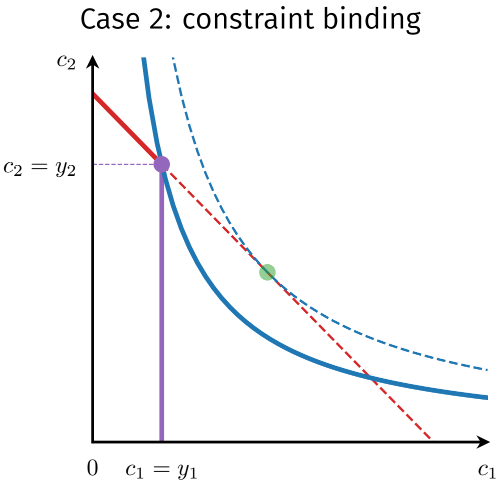
Either the agent would choose $a > 0$ and the constraint is not binding

Or they would like to choose $a < 0$ and the constraint is binding:

$$a = 0, \quad c_1 = y_1, \quad c_2 = y_2$$

Borrowing constraint: graphical interpretation





In Case 2 the agent changes current consumption following any change in income

Two interest rates

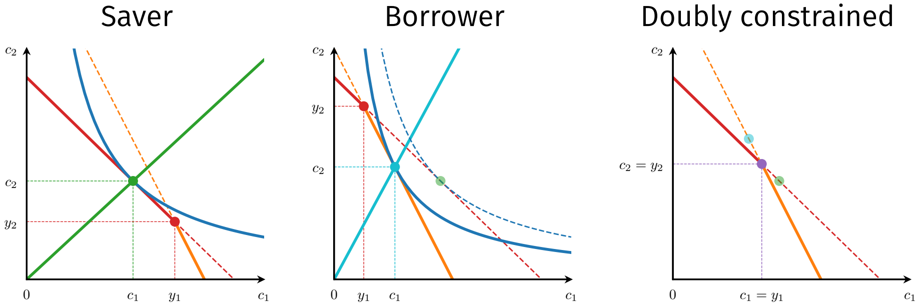
A similar, more realistic set-up is when the agent can freely borrow amount b, but at a higher interest rate $r^b > r$

$$
\begin{aligned}
\max_{c_1, c_2, a, b} \quad & U = \ln c_1 + \beta \ln c_2 \\
\text{subject to} \quad & c_1 + a = y_1 + b \\
& c_2 + (1 + r^b)b = y_2 + (1 + r)\, a \\
& \boldsymbol{a \geq 0} \\
& \boldsymbol{b \geq 0}
\end{aligned}
$$

We now have three (sensible) cases:

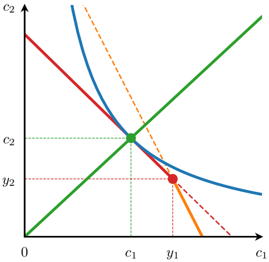
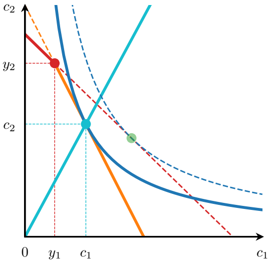
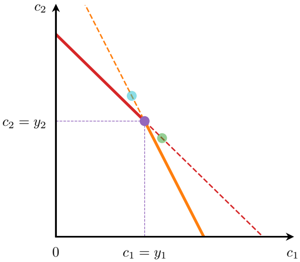
1. Saver: ($a > 0,\ b = 0$)
2. Borrower: ($a = 0$, $b > 0$)
3. Doubly constrained: ($a = 0$, $b = 0$)

Two interest rates: graphical interpretation



In the third case the agent behaves (locally) as if borrowing constrained

Sensitivity of c_1 to monetary policy (MP) & fiscal policy (FP) changes

Saver	Borrower	Doubly constrained
		
MP: low FP: zero	**MP: high** FP: low	MP: zero **FP: high**

Uncertainty & asset pricing

Uncertainty in income

Consider a two-period expected utility maximization problem

$$\begin{aligned} \max_{c_1, c_2, a} \quad & U = \ln c_1 + \beta \mathrm{E}\left[\ln c_2\right] \\ \text{subject to} \quad & c_1 + a = y_1 \\ & c_2 = y_2 + (1+r)\, a \end{aligned}$$

First period income is certain and equals y

Second period income will be equal to either $y+e$ or $y-e$:

$$y_2 = \begin{cases} y + e & \text{with probability } 1/2 \\ y - e & \text{with probability } 1/2 \end{cases}$$

Uncertainty in income

Assume $\beta = 1$ and $r = 0$ for simplicity

Use budget constraints to express consumption levels

$$c_1 = y - a$$

$$c_2 = \begin{cases} y + e + a & \text{with probability } 1/2 \\ y - e + a & \text{with probability } 1/2 \end{cases}$$

Rewrite the problem as choosing the optimal a alone:

$$\max_a \quad U = \ln(y - a) + \frac{1}{2}\ln(y + e + a) + \frac{1}{2}\ln(y - e + a)$$

First order condition:

$$-\frac{1}{y - a} + \frac{1}{2}\frac{1}{y + e + a} + \frac{1}{2}\frac{1}{y - e + a} = 0$$

Precautionary saving

$$a = \frac{1}{2}\left(\sqrt{y^2 + 2e^2} - y\right)$$

When second period income is certain ($e = 0$) then (given $\beta = 1$ and $r = 0$) the household holds no assets in optimum and enjoys smooth consumption over time, since $c_1 = c_2 = y$

When there is uncertainty about second period income ($e > 0$), the household accumulates **precautionary savings** to self-insure against the scenario of low income in the second period

The more uncertain second period income is, the higher is the stock of accumulated assets:

$$\frac{\partial a}{\partial e} = \frac{1}{2} \cdot \frac{1}{2\sqrt{y^2 + 2e^2}} \cdot 2 \cdot 2e = \frac{e}{\sqrt{y^2 + 2e^2}} > 0$$

Uncertainty in future income and ex-post rate of return

$$
\begin{aligned}
\max_{c_1, c_2, a} \quad & U = \ln c_1 + \beta \mathrm{E}\left[\ln c_2\right] \\
\text{subject to} \quad & c_1 + a = y_1 \\
& c_2 = y_2 + (1 + r_2)\, a
\end{aligned}
$$

Set up the Lagrangian

$$
\mathcal{L} = \ln \boldsymbol{c_1} + \beta \mathrm{E}\left[\ln \boldsymbol{c_2}\right] + \lambda_1 \left[y_1 - \boldsymbol{c_1} - \boldsymbol{a}\right] + \mathrm{E}\left[\lambda_2 \left[y_2 + (1 + r_2)\, \boldsymbol{a} - \boldsymbol{c_2}\right]\right]
$$

First order conditions (FOCs)

$$
\begin{aligned}
c_1: \quad & \frac{\partial \mathcal{L}}{\partial c_2} = \frac{1}{c_1} - \lambda_1 = 0 & \rightarrow \quad & \lambda_1 = \frac{1}{c_1} \\
c_2: \quad & \frac{\partial \mathcal{L}}{\partial c_2} = \mathrm{E}\left[\beta \frac{1}{c_2}\right] - \mathrm{E}\left[\lambda_2\right] = 0 & \rightarrow \quad & \lambda_2 = \beta \frac{1}{c_2} \\
a: \quad & \frac{\partial \mathcal{L}}{\partial a} = -\lambda_1 + \mathrm{E}\left[\lambda_2 \left(1 + r_2\right)\right] = 0 & \rightarrow \quad & \lambda_1 = \mathrm{E}\left[\lambda_2 \left(1 + r_2\right)\right]
\end{aligned}
$$

Uncertainty in future income and ex-post rate of return

Resulting optimality condition

$$\frac{1}{c_1} = \mathrm{E}\left[\beta \frac{1}{c_2}\left(1 + r_2\right)\right]$$

We need to be extra careful not to break any expectation operators!

Rewrite the Euler equation in the following way

$$1 = \mathrm{E}\left[\beta \frac{c_1}{c_2}\left(1 + r_2\right)\right] \equiv \mathrm{E}\left[\beta \frac{u'\left(c_2\right)}{u'\left(c_1\right)} \cdot \left(1 + r_2\right)\right]$$

This is an asset pricing equation. Here the price of a unit of savings is one unit of first period consumption. The payoff from having an asset in the second period will be $(1 + r_2)$. The term $\beta \cdot c_1/c_2$ (or $\beta \cdot u'\left(c_2\right)/u'\left(c_1\right)$ in the general case) is called the stochastic discount factor and measures the relative marginal utility of consumption across periods.

Asset pricing: general case

Investors can buy or sell as much of the payoff x_2 as they wish, at a price p_1

$$\begin{aligned} \max_{c_1,\, c_2,\, a} \quad & U = u\left(c_1\right) + \mathrm{E}\left[\beta u\left(c_2\right)\right] \\ \text{subject to} \quad & c_1 + p_1 \cdot a = y_1 \\ & c_2 = y_2 + x_2 \cdot a \end{aligned}$$

Set up the Lagrangian

$$\mathcal{L} = u\left(c_1\right) + \mathrm{E}\left[\beta u\left(c_2\right)\right] + \lambda_1\left[y_1 - \boldsymbol{c_1} - p_1 \cdot \boldsymbol{a}\right] + \mathrm{E}\left[\lambda\left[y_2 + x_2 \cdot \boldsymbol{a} - \boldsymbol{c_2}\right]\right]$$

Resulting optimality condition

$$p_1 \cdot u'\left(c_1\right) = \mathrm{E}\left[\beta u'\left(c_2\right) \cdot x_2\right] \quad \rightarrow \quad p_1 = \mathrm{E}\left[\beta \frac{u'\left(c_2\right)}{u'\left(c_1\right)} \cdot x_2\right] \equiv \mathrm{E}\left[m_2 \cdot x_2\right]$$

Pricing a bond: a simplified example

Utility function is logarithmic, $\beta = 0.95$ and $c_1 = 1$

Second period consumption can take two values: high $c_2^h = 1.1$ and low $c_2^l = 0.9$, with $q = 0.5$ being the probability of the low state

Use $p_1 = \mathrm{E}\left[m_2 \cdot x_2\right]$ to price bonds and stocks in this economy

Stochastic discount factor

$$\mathrm{E}\left[m_2\right] = \mathrm{E}\left[\beta \frac{u'\left(c_2\right)}{u'\left(c_1\right)}\right] = \beta \mathrm{E}\left[\frac{c_1}{c_2}\right] = \beta\left[q \cdot \frac{c_1}{c_2^l} + (1-q) \cdot \frac{c_1}{c_2^h}\right] \approx 0.9596$$

Price and return of a bond that pays off $x_2^b = 1$ with certainty

$$p_1^b = \mathrm{E}[m_2 \cdot x_2^b] = \mathrm{E}\left[m_2 \cdot 1\right] \approx 0.9596$$

$$1 + r_2^b = \frac{x_2^b}{p_1^b} = \frac{1}{0.9596} \approx 1.0421 \quad \rightarrow \quad r_2^b \approx 4.2\%$$

Pricing a stock: a simplified example

A stock pays dividend $d_2^h = 1.2$ in high state and $d_2^l = 0.8$ in low state, with a resale value of $p_2^s = 0$ for simplicity (so that $\mathrm{E}[x_2^s] = 1$)

$$p_1^s = \mathrm{E}[m_2 \cdot x_2^s] = \mathrm{E}[m_2 \cdot (d_2 + p_2^s)] = \mathrm{E}[m_2 \cdot d_2]$$

Important to remember that (unless SDF m_2 and d_2 are independent)

$$\mathrm{E}[m_2 \cdot d_2] \neq \mathrm{E}[m_2] \cdot \mathrm{E}_1[d_2]$$

The stock price and expected return are calculated as follows

$$p_1^s = \beta \left[q \frac{c_1}{c_2^l} d_2^l + (1-q) \frac{c_1}{c_2^h} d_2^h \right] \approx 0.9404$$

$$\mathrm{E}[1 + r_2^s] = \frac{\mathrm{E}[x_2^s]}{p_1^s} = \frac{1}{0.9404} \approx 1.0634 \quad \rightarrow \quad \mathrm{E}[r_2^s] \approx 6.3\%$$

Equity risk premium

The stock is cheaper than a bond, although their expected payoffs are identical

This is because stock dividends and the SDF exhibit negative covariance (while stock dividends and future consumption exhibit positive covariance)

Investors receive higher payoff in the state where consumption is high anyway, and a lower payoff when consumption is already low

The expected return on the stock needs then to be higher to motivate investors to hold the risky asset

$$\mathrm{E}[r_2^s - r_2^b] \approx 2.1\%$$

Current research suggests that the majority of equity risk premium arises due to the possibility of drawdowns in the 10-30% range, typical for recessions where income (consumption) risk increases significantly

Ricardian Equivalence (and how to break it)

Government budget constraints

$$g_1 = \tau_1 + b_1$$
$$g_2 + (1+r)\, b_1 = \tau_2$$

where g_1 and g_2 are public expenditure (per person) in periods 1 i 2, τ_1 and τ_2 are lump-sum taxes, and b_1 is issuance of government bonds (per person) financing deficit in period 1 and bought back in period 2

It's a simplified version of the full dynamic problem:

$$\sum_{t=1}^{\infty} \frac{g_t - \tau_t}{(1+r)^t} = b_0 + \lim_{t\to\infty} \frac{b_t}{(1+r)^t}$$

assuming the government does not go bankrupt: $\lim_{t\to\infty} \left[b_t / (1+r)^t \right] = 0$

Households' problem

Households solve their problem

$$\max_{c_1,\, c_2,\, a_1} \quad U = \ln c_1 + \beta \ln c_2$$

$$\text{subject to} \quad c_1 + a_1 = y_1 - \tau_1$$

$$c_2 = y_2 - \tau_2 + (1 + r)\, a_1$$

where assets a_1 comprise of bonds b_1 and other assets $\tilde{a}_1$

Lifetime budget constraint

$$c_1 + \frac{c_2}{1 + r} = y_1 - \tau_1 + \frac{y_2 - \tau_2}{1 + r}$$

Households' problem: solution

Set up the Lagrangian

$$\mathcal{L} = \ln \boldsymbol{c_1} + \beta \ln \boldsymbol{c_2} + \lambda \left[y_1 - \tau_1 + \frac{y_2 - \tau_2}{1+r} - \boldsymbol{c_1} - \frac{\boldsymbol{c_2}}{1+r} \right]$$

Derive the first order conditions (FOCs)

$$\begin{aligned} c_1 &: \quad \frac{1}{c_1} - \lambda = 0 \qquad \rightarrow \qquad \lambda = \frac{1}{c_1} \\ c_2 &: \quad \beta \frac{1}{c_2} - \frac{\lambda}{1+r} = 0 \qquad \rightarrow \qquad \lambda = \beta \left(1+r\right) \frac{1}{c_2} \end{aligned}$$

Optimality condition (Euler equation)

$$c_2 = \beta \left(1+r\right) c_1$$

Households' problem: solution

Budget constraints once again

$$c_1 + b_1 + \tilde{a}_1 = y_1 - \tau_1 \quad \text{and} \quad b_1 = g_1 - \tau_1 \quad \rightarrow \quad \tilde{a}_1 = y_1 - g_1 - c_1$$

$$c_2 = y_2 - \tau_2 + (1+r)(b_1 + \tilde{a}_1) \quad \text{and} \quad b_1 = \frac{\tau_2 - g_2}{1+r} \quad \rightarrow \quad c_2 = y_2 - g_2 + (1+r)\,\tilde{a}_1$$

Lifetime budget constraint

$$c_2 = y_2 - g_2 + (1+r)(y_1 - g_1 - c_1) \quad \rightarrow \quad c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r}$$

After plugging in the Euler equation

$$c_1 = \frac{1}{1+\beta}\left[y_1 - g_1 + \frac{y_2 - g_2}{1+r}\right] \quad \text{and} \quad c_2 = \frac{\beta}{1+\beta}\left[(1+r)(y_1 - g_1) + (y_2 - g_2)\right]$$

$$a_1 = y_1 - \tau_1 - c_1 \quad \text{and} \quad \tilde{a}_1 = y_1 - g_1 - c_1 \quad \text{and} \quad b_1 = g_1 - \tau_1$$

Changes in sequence of taxes do not influence consumption choices!

Additionally, assets change 1:1 with changes in supply of government bonds

Assumptions behind the Ricardian Equivalence result

All assets have the same rate of return (in expectation)

Taxes are non-distortionary

Changes in taxes are symmetric across households (no redistribution)

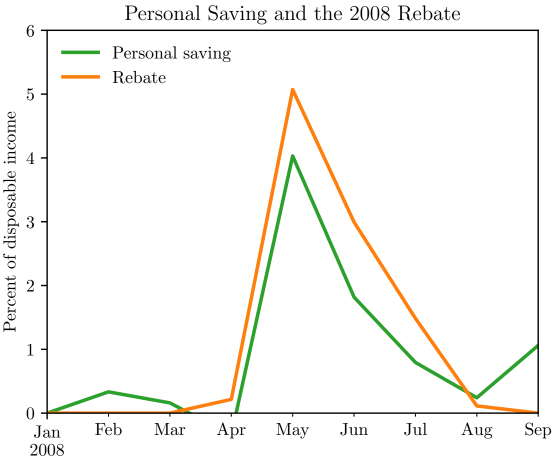
New public debt is repaid within current households' lifetime

Households are aware of the government budget constraints

Households are not borrowing constrained

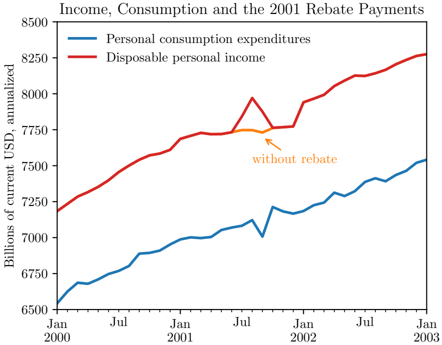
Households have time-consistent preferences

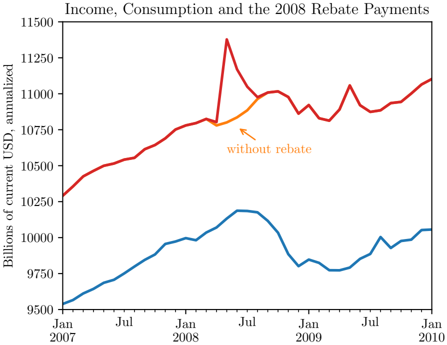
2008 tax rebates and savings



Taylor (2009), US Bureau of Economic Analysis.

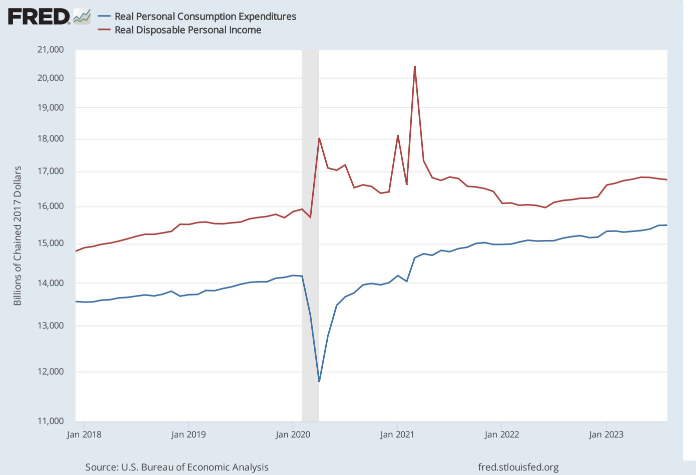
2001 and 2008 tax rebates and consumption





Taylor (2009), US Bureau of Economic Analysis.

Real consumption and disposable income during the pandemic



FRED

2001 and 2008 tax rebates and consumption

PCE Regressions with Rebate Payments

Lagged PCE	0.794	0.832
	(0.057)	(0.056)
Rebate payments	**0.048**	**0.081**
	(0.055)	**(0.054)**
Disposable personal income (w/o rebate)	0.206	0.188
	(0.056)	(0.055)
Oil price ($/bbl lagged 3 months)		-1.007
		(0.325)
R^2	0.999	0.999

Taylor (2009)

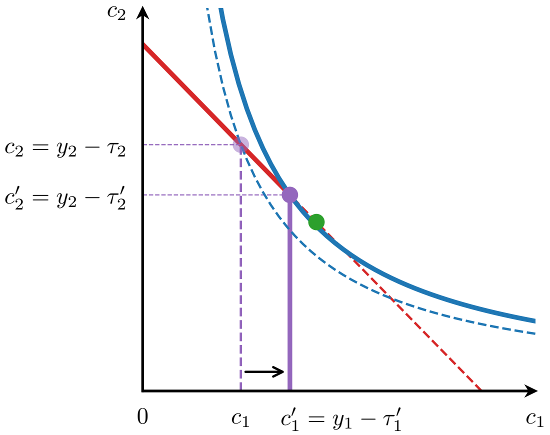
Heterogeneous reaction to tax rebates

Responses to 2001 and 2008 Rebate Surveys

	2001		2008	
	Number	Percent	Number	Percent
Mostly spend	256	**21.8**	447	**19.9**
Mostly save	376	32.0	715	31.8
Mostly pay off debt	544	46.2	1083	48.2
Will not get rebate	223		212	
Don't know / refused	45		61	
Total	1444	100	2518	100

Shapiro and Slemrod (2003), Shapiro and Slemrod (2009)

Borrowing constrained consumers



Until disposable income moves beyond the green point, consumption increases 1:1 due to tax rebates / extra transfers

Households with low liquid assets

Households with current consumption almost equal to current income and with almost no liquid assets are „hand-to-mouth”

Lusardi et al. (2011), Broda and Parker (2012): 30-40% US households have liquid assets below two months’ income. But these are not necessarily „poor” people!

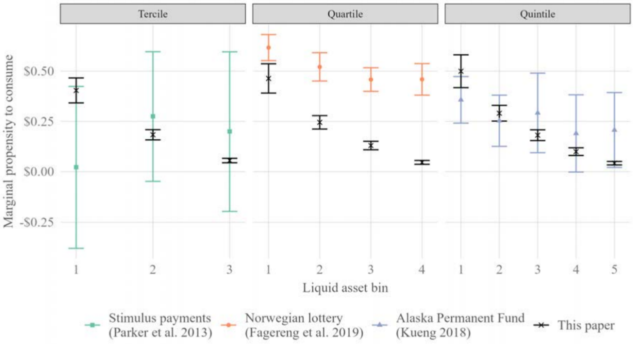
Kaplan and Violante (2014): in US microdata around 10% of households are „poor hand-to-mouth”, but around 33% are „wealthy hand-to-mouth”: with positive net worth allocated into illiquid assets (houses, pension funds, etc.)

They construct a model with two types of assets (low-return liquid and high-return illiquid), with transaction costs between them

In their model around 25% households spend immediately a small unforeseen extra income transfer, but if the transfer is large enough, they convert it into illiquid assets, behaving as „standard” consumers

Marginal Propensity to Consume from current income vs liquid assets

Figure 4: Marginal Propensity to Consume by Asset Buffer



Note: This figure compares the estimates of heterogeneity by assets in the passthrough of income shocks to consumption. Parker et al. (2013), Fagereng, Holm and Natvik (2021), and Kueng (2018) use terciles, quartiles, and quintiles respectively. To enable comparability with these prior papers, we calculate the marginal propensity to consume (instead of the elasticity of consumption to income) using their respective bin cutoffs. Our paper, Parker et al. (2013), and Kueng (2018) measure the MPC on nondurables. Fagereng, Holm and Natvik (2021) measures the MPC on total consumption. See Section 3.6 for details.

Ganong et al. (2023)

Finite planning horizon

Older households might expect that the higher future taxes will affect the economy after they die

Spending the 2008 Rebate, by Age

Age group	Percent mostly spending
29 or less	11.7
30–39	14.2
40–49	16.9
50–64	19.9
65 or over	28.4

Shapiro and Slemrod (2009)