Marcin Bielecki, Advanced Macroeconomics QF, Fall 2024
Homework 4 – deadline: 2nd February, 23:59

Problem 1 (2 points)

Consider the following model. Households maximize expected utility subject to the standard budget constraint:

$$\max_{\{C_t,\,L_t,\,K_{t+1}\}_{t=0}^{\infty}} \quad U = \mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\ln C_t - \psi\frac{L_t^{1+\varphi}}{1+\varphi}\right)\right]$$
$$\text{subject to} \quad C_t + K_{t+1} = w_t L_t + (1+r_t)\,K_t$$

Firms maximize profits subject to the Cobb-Douglas production function:

$$\max \quad D_t = Y_t - w_t L_t - (r_t+\delta)\,K_t$$
$$\text{subject to} \quad Y_t = Z_y K_t^{\alpha} L_t^{1-\alpha}$$

where $\delta = 1$ (capital deprecates fully). The technology constant z evolves according to the process:

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \epsilon_{Z,t}$$

(a) Derive the first order conditions of the households and their optimality conditions.

(b) Derive the first order conditions of the firm and expressions for prices in equilibrium.

(c) Find the steady state of the system, assuming that $L^* = 1$. Find the corresponding value of ψ.

(d) Assuming that household behavior can be expressed as $C_t = (1-s)\,Y_t$ where s is a constant, find the value of s as a function of model parameters. **Hint: start with the Euler equation.**

(e) Show that $L_t = L^*$. Find the expression for K_{t+1} as a function of variables at time t.

Problem 2 (2.5 points)

Consider the effects of permanently increasing government expenditure in the RBC model. Households maximize expected utility subject to the standard budget constraint:

$$\max_{\{C_t,\,L_t,\,K_{t+1}\}_{t=0}^{\infty}} \quad U = \mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\ln C_t + \chi\ln G_t - \psi\frac{L_t^{1+\varphi}}{1+\varphi}\right)\right]$$
$$\text{subject to} \quad C_t + K_{t+1} = w_t L_t + (1+r_t)\,K_t - T_t$$

The problem of the firms is the same as in the previous problem. The government maintains a balanced budget at all times, and the government spending is a constant fraction of GDP:

$$T_t = G_t = \omega \cdot Y_t$$

(a) Derive the first order conditions of the households and their optimality conditions.

(b) Find the steady state of the system. **Caution: this time we cannot assume that $L^* = 1$. You'll need to use at first variables divided by labor input: K/L, Y/L, C/L, etc.**

(c) Suppose now that the government permanently increases its expenditure to $\omega' > \omega$. What is the effect on this higher government spending on GDP in the steady state?

(d) What is the steady state effect of the change from (c) on capital, consumption and labor input? If $\chi = 0$, what would be the impact of higher government expenditure on welfare (utility)?

(e) The RBC model postulates that households dislike working. Why is then unemployment a problem? Can you argue for a setup where higher labor input is welfare improving?

Problem 3 (1.5 points)

In class we considered a model where permanent changes to marginal productivity of labor reduced the unemployment rate. This would imply that with trend productivity growth unemployment would trend downward over time. Let's modify our model so that it is consistent with stationary unemployment in face of trend productivity growth. Suppose that the flow cost of a vacancy κ and the outside option of a worker (unemployment benefit for simplicity) b are functions of the trend wage rate w (instead of being exogenous). In particular, assume that $\kappa_t = \kappa_0 w_t$ and $b_t = b_0 w_t$.

(a) Determine the formula for job creation and wage setting along the balanced growth path (steady state).

(b) How do labor market tightness and wages along the balanced growth path react to productivity changes?

(c) Does a continuous growth of productivity lead to a decrease in the long run unemployment rate?

Problem 4 (2 points)

Consider the Rotemberg scheme where costs of price changes resulted in the New Keynesian Phillips Curve due to losses of customer loyalty. The costs of price changes are given by:

$$\phi \left(p_t - p_t^e\right)^2$$

Assume that customers consider "inflationary" price increases as fair:

$$p_t^e = p_{t-1} + \pi_{t-1}$$

The loss function of the firm is given by:

$$L = \sum_{j=0}^{\infty} \beta^j \cdot \mathrm{E}_t \left[\left(p_{t+j} - p_{t+j}^*\right)^2 + \phi \left(p_{t+j} - p_{t+j}^e\right)^2 \right]$$

(a) Derive the first order condition with respect to price p_t.

(b) Find the formula for the inflation rate in this economy (New Keynesian Phillips Curve).

(c) What are the consequences of the backward-looking component of inflation in times of high inflation?

Problem 5 (2 points)

Suppose that you have the following simplified New Keynesian model. The two main non-policy equations of the model can be written:

$$x_t = \mathrm{E}_t x_{t+1} - (i_t - \mathrm{E}_t \pi_{t+1})$$
$$\pi_t = \mathrm{E}_t \pi_{t+1} + x_t$$

where x is output gap, i is the nominal interest rate and π is inflation rate. The central bank obeys a strict inflation targeting rule. In particular, let π_t^* be an exogenous inflation target. The central bank will adjust i_t so that $\pi_t = \pi_t^*$ is consistent with these equations holding. Assume that the inflation target follows an exogenous AR(1) process:

$$\pi_t^* = \rho_\pi \pi_{t-1}^* + \epsilon_t, \quad \rho_\pi \in [0, 1]$$

(a) Derive an analytic expression for i_t as a function of π_t^*.

(b) Suppose that ρ_π is 0, approximating day-to-day monetary policymaking. In which direction must the central bank adjust i_t in order to achieve a decrease in π_t?

(c) If ρ_π is sufficiently close to 1, approximating the long-run situation. How does i_t behave after a long-run decline in π_t?

Problem 6 (2 points)

Consider a simplified New Keynesian model:

$$x_t = \mathrm{E}_t x_{t+1} - (i_t - \mathrm{E}_t \pi_{t+1} - r^*) + u_t$$
$$\pi_t = \mathrm{E}_t \pi_{t+1} + x_t$$

where x is the output gap, i is the nominal interest rate, π is the inflation rate, $r^* > 0$ is the natural real rate of interest, and u is a demand shock.
In time period t the economy is affected by a strong negative demand shock: $u_t = -r^* - v$, which lasts for one period only. The central bank, subject to the zero lower bound constraint, sets $i_t = 0$. After the shock recedes, in time period $t+1$ it will be possible to set $x_{t+1} = \pi_{t+1} = 0$. Additionally, the central bank credibly commits to maintain $x_{t+j} = \pi_{t+j} = 0$ for all $j \geq 2$.

(a) What level of nominal interest rate in $t+1$ will be set by the central bank aiming to minimize $\left(\pi_{t+1}^2 + x_{t+1}^2\right)$?

(b) What will be the levels of output gap and inflation in period t if the agents expect the central bank to act according to (a)?

(c) Assume the central bank credibly commits to set $i_{t+1} = r^* - e$. What will be the levels of output gap and inflation in periods $t+1$ and t?

(d) What is the optimal level of e for a central bank aiming to minimize $\frac{1}{2}\left[\left(\pi_t^2 + x_t^2\right) + \left(\pi_{t+1}^2 + x_{t+1}^2\right)\right]$? Why does the optimal value of e differ from zero?