University of Warsaw
Faculty of Economic Sciences

New Keynesian Model

Advanced Macroeconomics IE: Lecture 19

Marcin Bielecki

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University of Warsaw

Nominal rigidities and sticky prices

Monopolistic competition

Under perfect competition firms are selling homogeneous goods and are price-takers

In reality many firms sell differentiated products and are able to set their own prices

Convenient framework: **monopolistic competition**

- Households enjoy consuming many different goods (“love for variety”)
- Firms’ market power depends on elasticity of substitution
- Perfect competition: infinitely high elasticity of substitution
- One sector: all firms “compete” with each other
- Can easily extend to a multisector setup: higher elasticity of substitution within industries, smaller across industries

Monopolistic competition

Households have "love for variety" utility function

Demand function for the j-th good

$$y_j = (P/P_j)^{(1+\mu)/\mu}\, y \quad \leftrightarrow \quad P_j = P y^{\mu/(1+\mu)} y_j^{-\mu/(1+\mu)}$$

where P is the aggregate price index, P_j is the j-th good price, y is real GDP, and $\mu > 0$ is a parameter inversely related to the elasticity of substitution

If production function exhibits Constant Returns to Scale
then nominal marginal cost MC_j is independent of quantity produced

Profit maximization problem

$$\max_{P_j,\, y_j} \quad D_j = P_j y_j - MC_j y_j$$
$$\text{subject to} \quad P_j = P y^{\mu/(1+\mu)} y_j^{-\mu/(1+\mu)}$$

Monopolistic competition: solution

Profit maximization problem

$$\max_{P_j\,,\,y_j} \quad D_j = P_j y_j - MC_j y_j$$

$$\text{subject to} \quad P_j = P y^{\mu/(1+\mu)} y_j^{-\mu/(1+\mu)}$$

Plug in the inverse demand function into the profit function

$$\max_{y_j} \quad D_j = P y^{\mu/(1+\mu)} y_j^{1-\mu/(1+\mu)} - MC_j y_j$$

First order condition with respect to y_j ($MR = MC$)

$$\left(1 - \frac{\mu}{1+\mu}\right) P y^{\mu/(1+\mu)} y_j^{-\mu/(1+\mu)} - MC_j = 0 \quad \rightarrow \quad \frac{1}{1+\mu} P_j = MC_j$$

"Markup pricing" is the profit-maximizing strategy

$$P_j^* = (1+\mu)\, MC_j$$

If we allow to differentiate elasticity of substitution across sectors we get

$$P_{j,t}^* = (1+\mu_{j,t})\, MC_{j,t}$$

Empirical evidence on markups in US and EA

Perfect competition can be rejected for almost all sectors in all countries
Markups are generally higher in services than manufacturing

Table 1. Weighted average markup, 1981-2004

Country	Manufacturing & Construction		Market Services		All (Manufacturing, Construction & Market Services)	
Germany	1.16	(0.01)*	1.54	(0.03)*	1.33	(0.01)*
France	1.15	(0.01)*	1.26	(0.02)*	1.21	(0.01)*
Italy	1.23	(0.01)*	1.87	(0.02)*	1.61	(0.01)*
Spain	1.18	(0.00)*	1.37	(0.01)*	1.26	(0.01)*
Netherlands	1.13	(0.01)*	1.31	(0.02)*	1.22	(0.01)*
Belgium	1.14	(0.00)*	1.29	(0.01)*	1.22	(0.01)*
Austria	1.20	(0.02)*	1.45	(0.03)*	1.31	(0.02)*
Finland	1.22	(0.01)*	1.39	(0.02)*	1.28	(0.01)*
Euro Area	1.18	(0.01)*	1.56	(0.01)*	1.37	(0.01)*
USA	1.28	(0.02)*	1.36	(0.03)*	1.32	(0.02)*

Christopoulou and Vermeulen (2008)

Prices do not change every period

Survey about price setting practices carried out by the Banco de Portugal

Firms in the sample are generally quicker to react to cost shocks,
in particular when they are positive, than to demand shocks

TABLE 1

Distribution of the price responses to demand and cost shocks

	Cost shocks		*Demand shocks*	
Price adjustment lag	*Positive*	*Negative*	*Positive*	*Negative*
1 – less than one week	4.7	3.5	2.8	4.8
2 – from one week to one month	16.8	15.2	12.2	16.8
3 – from one month to three months	25.0	25.7	19.3	23.4
4 – from three to six months	17.6	14.9	13.4	13.6
5 – from six months to one year	26.3	21.2	17.7	14.0
6 – more than one year	9.6	19.5	34.6	27.4
Total	100.0	100.0	100.0	100.0

Dias et al. (2014)

Stylized facts on price stickiness

Benefits of price stickiness: no need to survey all prices everytime we go to store, easy to plan expenditures ahead

Average price duration

- US: average time between price changes is 2-4 quarters
  Blinder et al. (1998), Klenow and Kryvstov (2008), Nakamura and Steinsson (2008)
- EA: average time between price changes is 4-5 quarters
  Dhyne et al. (2005), Altissimo et al. (2006)
- PL: average time between price changes is 4 quarters
  Macias and Makarski (2013)

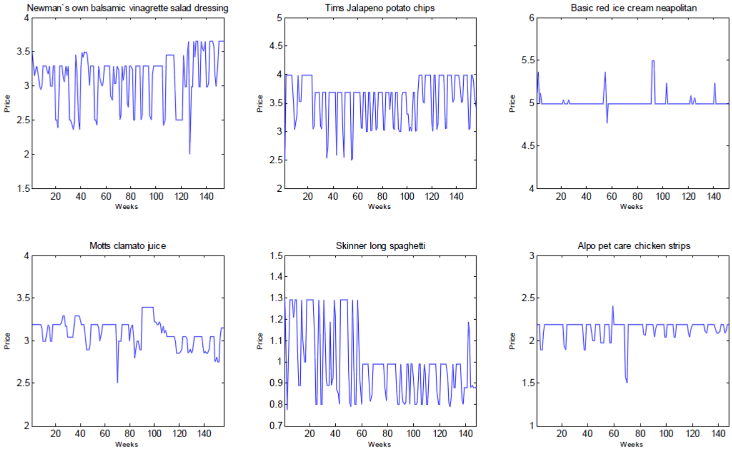
Cross-industry heterogeneity

- Prices of tradables less sticky than those of nontradables
- Retail prices usually more sticky than producer prices

Gagnon (2009): for inflation above 10-15% prices change more frequently with higher inflation

Example retail prices behavior

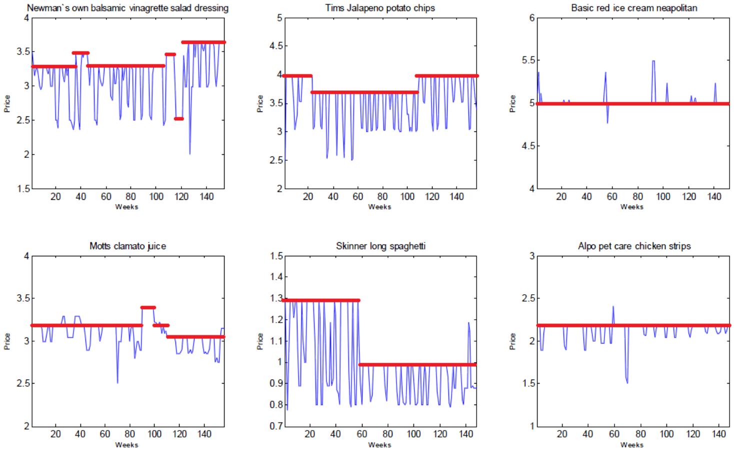
Raw retail scanner data



Koning (2015)

Example retail prices behavior

After “controlling” for short-lived sales prices: reference prices



Price stickiness depends on sector

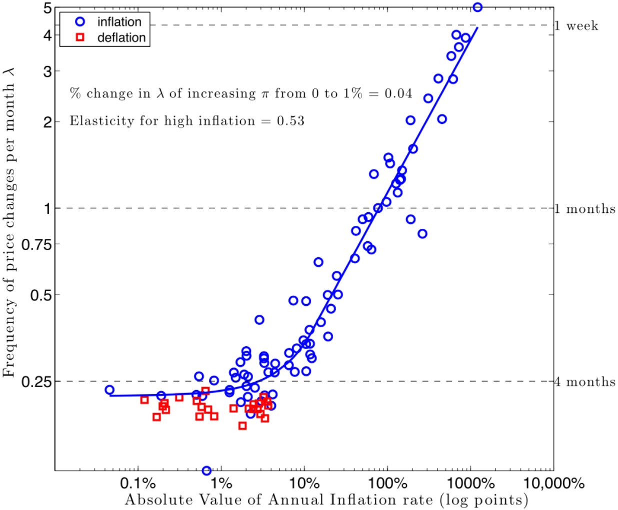
Table 4.1 Frequency of consumer price changes by product type, in %

Country	Unprocessed food	Processed food	Energy (oil products)	Non-energy industrial goods	Services	Total, country weights	Total, Euro area weights
Belgium	31.5	19.1	81.6	5.9	3.0	17.6	15.6
Germany	25.2	8.9	91.4	5.4	4.3	13.5	15.0
Spain	50.9	17.7	n.a.	6.1	4.6	13.3	11.5
France	24.7	20.3	76.9	18.0	7.4	20.9	20.4
Italy	19.3	9.4	61.6	5.8	4.6	10.0	12.0
Luxembourg	54.6	10.5	73.9	14.5	4.8	23.0	19.2
The Netherlands	30.8	17.3	72.6	14.2	7.9	16.2	19.0
Austria	37.5	15.5	72.3	8.4	7.1	15.4	17.1
Portugal	55.3	24.5	15.9	14.3	13.6	21.1	18.7
Finland	52.7	12.8	89.3	18.1	11.6	20.3	-
Euro Area	28.3	13.7	78.0	9.2	5.6	15.1	15.8

Source: Dhyne et al. (2005). Figures presented in this table are computed on the basis of the 50 product sample, with the only exception of Finland for which figures based on the entire CPI are presented. The total with country weights is calculated using country-specific weights for each item, the total with euro area weights using common euro area weights for each sub-index. No figures are provided for Finland because of a lack of comparability of the sample of products used in this country.

Altissimo, Ehrmann and Smets (2006)

Frequency of price changes when inflation is low vs when inflation is high



Alvarez i in. (2018)

New Keynesian Phillips Curve

Convex costs of price changes

Based on Rotemberg (1982)

Assumes that bigger price changes are more costly, e.g. due to losses in customer loyalty

The dynamic profit maximizing problem can be recast as a simpler problem of minimizing a loss function (where p is log of firm's price and $\phi > 0$)

$$L = \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \left[\left(p_{j,t+i} - p^*_{j,t+i} \right)^2 + \phi \left(p_{j,t+i} - p^e_{j,t+i} \right)^2 \right]$$

- $\mathrm{E}_t[(p_{j,t+i} - p^*_{j,t+i})^2]$ is the loss of profit by setting price other than $(1+\mu)\,MC$
- $\mathrm{E}_t[\phi(p_{j,t+i} - p^e_{j,t+i})^2]$ is the convex cost of price changes
- $p^e_{j,t}$ is the price expected by consumers, assume $p^e_{j,t} = p_{j,t-1}$

For simplicity assume firm symmetry: $p_{j,t} = p_t$ (imposed after solving the problem)

Rotemberg model: solution

Expand the loss function for convenience

$$L = \left(p_{j,t} - p^*_{j,t}\right)^2 + \phi\left(p_{j,t} - p_{j,t-1}\right)^2 + \beta \mathrm{E}_t\left[\left(p_{j,t+1} - p^*_{j,t+1}\right)^2 + \phi\left(p_{j,t+1} - p_{j,t}\right)^2\right] + \ldots$$

First order condition with respect to $p_{j,t}$

$$2\left(p_{j,t} - p^*_{j,t}\right) + 2\phi\left(p_{j,t} - p_{j,t-1}\right) + \beta \mathrm{E}_t\left[2\phi\left(p_{j,t+1} - p_{j,t}\right)(-1)\right] = 0$$

$$\left(p_{j,t} - p_{j,t-1}\right) = \beta \mathrm{E}_t\left[p_{j,t+1} - p_{j,t}\right] - \frac{1}{\phi}\left(p_{j,t} - p^*_{j,t}\right)$$

Since all firms set the same price, the inflation rate is $\pi_t \equiv p_t - p_{t-1} = p_{j,t} - p_{j,t-1}$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{\phi}\left(p^*_t - p_t\right)$$

Current inflation depends on inflation expecations!

Staggered price adjustment

Based on Calvo (1983)

In Rotemberg firms make many small price changes

In Calvo firms are not "allowed" to do so

- Firms can change their price only if they receive a "signal"
- Price remains unchanged with probability θ
- If a firm set the price in period t, then the price remains unchanged in period $t+i$ with probability θ^i
- Average price duration is $1/\left(1-\theta\right)$ periods
- Denote price set in period t with $\tilde{p}_{j,t}$ ("reset" price)

Firm's loss function

$$L = \sum_{i=0}^{\infty} \left(\beta\theta\right)^i \mathrm{E}_t \left[\left(\tilde{p}_{j,t} - p^*_{j,t+i} \right)^2 \right]$$

Calvo model: solution

First order condition

$$\sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t \left[2 \left(\tilde{p}_{j,t} - p^*_{j,t+i} \right) \right] = 0$$

$$\tilde{p}_{j,t} \sum_{i=0}^{\infty} (\beta\theta)^i = \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{j,t+i}$$

$$\tilde{p}_{j,t} \frac{1}{1-\beta\theta} = \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{j,t+i} = p^*_{j,t} + \beta\theta \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{j,t+i+1}$$

$$\tilde{p}_{j,t} = (1-\beta\theta) \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{j,t+i}$$

Reset price $\tilde{p}_{j,t}$ is the weighted average of today's and future prices that would be optimal in a frictionless setting

Dynamics of inflation in the Calvo scheme

Within each period a fraction θ of firms keeps prices unchanged, the remaining $1-\theta$ fraction resets prices symmetrically to $\tilde{p}_t$

$$p_t = \theta p_{t-1} + (1-\theta)\,\tilde{p}_t$$

After a series of algebraic manipulations we get that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(p_t^* - p_t\right)$$

Compare to Rotemberg's outcome

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{\phi}\left(p_t^* - p_t\right)$$

In both settings expectations on future inflation affect inflation today!

Comparing Rotemberg and Calvo

Identical (up to the first-order approximation) functional form, different economic conclusions: see Lombardo and Vestin (2007) and Ascari and Rossi (2012)

- Rotemberg: inflation is “costly” due to costs of changing prices, relative prices across firms are unaffected
- Calvo: inflation is “costly” since not every firm adjusts prices within each period, price dispersion arises
- Price dispersion introduces inefficiencies into the economy ($P_1/P_2 \neq MC_1/MC_2$)
- Welfare costs of inflation are higher in the Calvo scheme
- Calvo scheme fits data well under single-digit inflation (constant price change frequency), for higher inflation rates price adjustment models perform better

Dynamics of inflation in the Calvo scheme: algebra

$$p_t = (1-\theta)\,\tilde{p}_t + \theta p_{t-1}$$

$$\tilde{p}_t = \frac{p_t - \theta p_{t-1}}{1-\theta} = \frac{p_t - p_{t-1} + (1-\theta)\,p_{t-1}}{1-\theta} = \frac{\pi_t}{1-\theta} + p_{t-1}$$

$$\tilde{p}_t = (1-\beta\theta)\sum_{i=0}^{\infty}(\beta\theta)^i\,\mathrm{E}_t p^*_{t+i} \quad \rightarrow \quad \tilde{p}_t = (1-\beta\theta)\,p^*_t + \beta\theta\mathrm{E}_t\tilde{p}_{t+1}$$

$$\tilde{p}_t = (1-\beta\theta)\,p^*_t + \beta\theta\mathrm{E}_t\tilde{p}_{t+1} = (1-\beta\theta)\,p^*_t + \frac{\beta\theta}{1-\theta}\mathrm{E}_t\pi_{t+1} + \beta\theta p_t$$

$$(1-\theta)\,\tilde{p}_t = p_t - \theta p_{t-1} = \theta\,(p_t - p_{t-1}) + (1-\theta)\,p_t$$

$$\theta\,(p_t - p_{t-1}) = (1-\theta)\,(\tilde{p}_t - p_t)$$

$$\theta\pi_t = (1-\theta)\left[\frac{\beta\theta}{1-\theta}\mathrm{E}_t\pi_{t+1} + (1-\beta\theta)\,p^*_t + \beta\theta p_t - p_t\right]$$

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \frac{(1-\theta)\,(1-\beta\theta)}{\theta}\,(p^*_t - p_t)$$

New Keynesian Phillips Curve

Under both schemes we have that (where $\chi > 0$)

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \left(p_t^* - p_t\right)$$

Relate it to the expression for "optimal" price

$$P_t^* = (1+\mu)\, MC_t \quad \rightarrow \quad \ln P_t^* = \ln(1+\mu) + \ln MC_t \quad \rightarrow \quad p_t^* = \mu + mc_t^n$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \left(\mu + mc_t^n - p_t\right) \equiv \beta \mathrm{E}_t \pi_{t+1} + \chi \tilde{mc}_t^r$$

where $mc^n - p$ is the logarithm of **real marginal cost**, in the steady state equal to $-\mu$ and $\tilde{mc}^r$ is the percentage deviation of the real MC from the steady state

Gali and Gertler (1999) construct a proxy for the real marginal cost gap

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \quad \rightarrow \quad \tilde{mc}_t^r \equiv \frac{W_t/P_t}{\partial Y_t / \partial N_t} = \frac{W_t/P_t}{(1-\alpha)\, Y_t/N_t} = \frac{1}{1-\alpha} \frac{W_t N_t}{P_t Y_t}$$

And estimate the following NKPC

$$\pi_t = 0.942\, \mathrm{E}_t \pi_{t+1} + 0.023\, \tilde{mc}_t^r$$

NKPC slope estimates using industry-level data

Table 4: Structural estimates and slope of the sectoral Phillips curves: Sectoral estimates.

Sector:	θ_s	Ω_s	λ_s	$\bar{s}_s$
Transport equipment	0.582	0.077	0.281	0.033
	(0.009)	(0.015)	(0.019)	(0.012)
Electrical equipment	0.632	0.191	0.177	0.020
	(0.005)	(0.021)	(0.007)	(0.007)
Machinery equipment	0.668	0.087	0.154	0.019
	(0.021)	(0.041)	(0.029)	(0.012)
Wood, paper and printing	0.703	0.202	0.102	0.106
	(0.019)	(0.082)	(0.020)	(0.034)
Metals	0.728	0.127	0.091	0.099
	(0.016)	(0.056)	(0.013)	(0.016)
Rubber and plastic	0.714	0.367	0.075	0.106
	(0.018)	(0.047)	(0.012)	(0.011)
Textiles, apparel and leather	0.737	0.509	0.047	0.030
	(0.023)	(0.144)	(0.019)	(0.024)
Chemicals	0.779	0.330	0.043	0.275
	(0.011)	(0.031)	(0.005)	(0.066)
Food, beverages and tobacco	0.760	0.483	0.041	0.289
	(0.018)	(0.048)	(0.010)	(0.031)

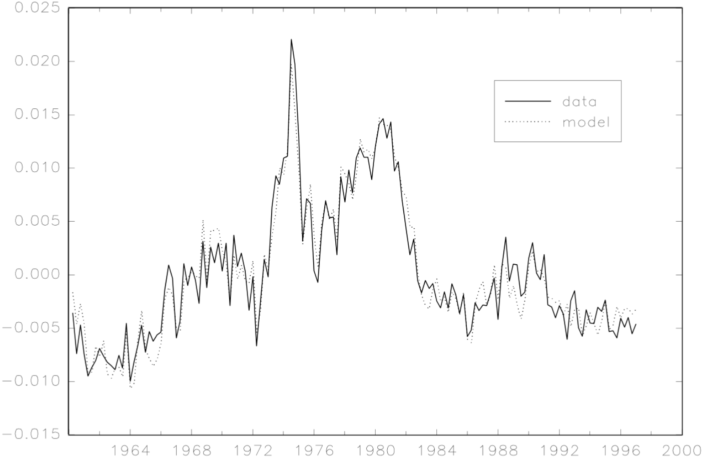
Notes. This table presents the estimates of the structural parameters (θ and Ω), the implied slope of the NKPC (λ), and the sector-specific Törnqvist weight ($\bar{s}$) for different manufacturing sectors. The estimates are obtained using Model C. For each sector, observations are weighted in the regression using Törnqvist weights. Standard errors (in parenthesis) are robust to heteroskedasticity and autocorrelation at the firm level.

Gagliardone et al. (2023), λ corresponds to χ

Inflation vs its one period ahead forecast from NKPC

FIGURE 6

Inflation, models vs. data ($\alpha_0 = 17.5$, $\alpha_1 = 1$)



Sbordone (2002)

New Keynesian IS curve

Households' problem

For simplicity consider a model with no physical capital,
where **nominal** bonds B yield the **nominal** interest rate i

$$\begin{aligned} \max \quad & U = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{h_t^{1+\varphi}}{1+\varphi} \right) \\ \text{subject to} \quad & P_t c_t + B_t = W_t h_t + (1+i_{t-1}) B_{t-1} + D_t \end{aligned}$$

Rewrite the budget constraint in real terms (divide by P_t)

$$c_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} h_t + (1+i_{t-1}) \frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{D_t}{P_t}$$

$$c_t + b_t = w_t h_t + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + d_t$$

Lagrangian

$$\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{h_t^{1+\varphi}}{1+\varphi} \right) + \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[w_t h_t + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + d_t - c_t - b_t \right]$$

Households' problem

Langrangian (already expanded "from the point of view" of period t)

$$\mathcal{L} = \beta^t \left(\frac{\boldsymbol{c_t}^{1-\sigma}}{1-\sigma} - \psi \frac{\boldsymbol{h_t}^{1+\varphi}}{1+\varphi} \right) + \beta^t \lambda_t \left[w_t \boldsymbol{h_t} + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + d_t - \boldsymbol{c_t} - \boldsymbol{b_t} \right]$$
$$+ \mathrm{E}_t \left[\beta^{t+1} \lambda_{t+1} \left[w_{t+1} h_{t+1} + \frac{1+i_t}{1+\pi_{t+1}} \boldsymbol{b_t} + d_{t+1} - c_{t+1} - b_{t+1} \right] \right] + \dots$$

First order conditions

$$\boldsymbol{c_t} : \quad \beta^t c_t^{-\sigma} - \beta^t \lambda_t = 0 \qquad \rightarrow \qquad \lambda_t = c_t^{-\sigma}$$
$$\boldsymbol{h_t} : \quad \beta^t \left(-\psi h_t^{\varphi} \right) + \beta^t \lambda_t w_t = 0 \qquad \rightarrow \qquad \lambda_t = \psi h_t^{\varphi} / w_t$$
$$\boldsymbol{b_t} : \quad \beta^t \lambda_t \left[-1 \right] + \mathrm{E}_t \left[\beta^{t+1} \lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}} \right] = 0 \qquad \rightarrow \qquad \lambda_t = \beta \mathrm{E}_t \left[\lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}} \right]$$

Households' problem

Euler equation

$$c_t^{-\sigma} = \beta \mathrm{E}_t \left[c_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \right]$$

Labor supply

$$c_t^{-\sigma} = \frac{\psi h_t^{\varphi}}{w_t} \quad \rightarrow \quad h_t^{\varphi} = \frac{w_t c_t^{-\sigma}}{\psi} \quad \rightarrow \quad h_t = \left(\frac{w_t c_t^{-\sigma}}{\psi} \right)^{1/\varphi}$$

$$w_t = \psi h_t^{\varphi} c_t^{\sigma}$$

Can also add a money demand equation to relate interest rate with **endogenous** money supply

$$M_t^d = P_t c_t \cdot i_t^{-\nu}$$

Deriving New Keynesian IS curve

Assume no investment and government spending, so that $c_t = y_t$

$$y_t^{-\sigma} = \frac{1}{1+\rho}\mathrm{E}_t\left[y_{t+1}^{-\sigma}\frac{1+i_t}{1+\pi_{t+1}}\right]$$

Apply logarithms and the first order approximation ("forget" about covariance)

$$-\sigma \ln y_t \approx -\ln(1+\rho) - \sigma\mathrm{E}_t \ln y_{t+1} + \ln(1+i_t) - \ln(1+\mathrm{E}_t\pi_{t+1})$$

$$\ln y_t \approx \mathrm{E}_t \ln y_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t\pi_{t+1} - \rho)$$

Denote counterfactual, "natural" variables in the flexible-prices world with *

$$\ln y_t^* = \mathrm{E}_t \ln y_{t+1}^* - \frac{1}{\sigma}(r_t^* - \rho)$$

Subtract the "natural" from the "actual" (where $x_t \equiv \ln y_t - \ln y_t^*$ is the **output gap**)

$$\ln y_t - \ln y_t^* = \mathrm{E}_t\left[\ln y_{t+1} - \ln y_{t+1}^*\right] - \frac{1}{\sigma}(i_t - \mathrm{E}_t\pi_{t+1} - r_t^*)$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t\pi_{t+1} - r_t^*)$$

New Keynesian IS curve

Natural real interest rate

$$r_t^* = \rho + \sigma \mathrm{E}_t \left[\Delta \ln y_{t+1}^*\right]$$

In general case where $c_t \neq y_t$, one can capture the influence of other expenditures as a demand shock $\tilde{u}$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + \mathrm{E}_t \left[\Delta \ln y_{t+1}^*\right] + \tilde{u}_t$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + u_t$$

Cet. par. higher nominal interest rate leads to more negative output gap

Caution: positive output gap ($x > 0$) means the level of output is higher than in the counterfactual flex-price world ($y > y^*$)

Output gap in NKPC: algebra

Assume $c_t = y_t$ and $y_t = z_t h_t$ to easily relate x_t with $\tilde{mc}_t^r$

$$y_t = z_t h_t \quad \rightarrow \quad MC_t = P_t w_t / z_t \quad \rightarrow \quad w_t = z_t \frac{MC_t}{P_t}$$

Labor supply

$$\psi h_t^{\varphi} c_t^{\sigma} = w_t = z_t \frac{MC_t}{P_t} \quad \rightarrow \quad \ln \psi + \varphi \ln h_t + \sigma \ln c_t = \ln z_t + mc_t^n - p_t$$

$$\ln \psi + \varphi \left(\ln y_t - \ln z_t \right) + \sigma \ln y_t = \ln z_t + mc_t^n - p_t$$

$$\ln y_t = \frac{1+\varphi}{\varphi+\sigma} \ln z_t + \frac{1}{\varphi+\sigma} \left(mc_t^n - p_t \right) - \frac{\ln \psi}{\varphi+\sigma}$$

In the flexible prices world

$$P_t = (1+\mu) MC_t \quad \rightarrow \quad p_t = \mu + mc_t^n \quad \rightarrow \quad \ln y_t^* = \frac{1+\varphi}{\varphi+\sigma} \ln z_t + \frac{1}{\varphi+\sigma} (-\mu) - \frac{\ln \psi}{\varphi+\sigma}$$

Output gap

$$x_t = \ln y_t - \ln y_t^* = \frac{1}{\varphi+\sigma} \left(mc_t^n - p_t - (-\mu) \right) = \frac{1}{\varphi+\sigma} \left(\mu + mc_t^n - p_t \right) = \frac{1}{\varphi+\sigma} \tilde{mc}_t^r$$

Output gap in NKPC

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \tilde{mc}_t^r$$

Output gap

$$x_t = \frac{1}{\varphi + \sigma} \tilde{mc}_t^r$$

Final form of NKPC

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where $\kappa \equiv \chi (\varphi + \sigma) > 0$, and e_t is a cost-push shock: influence of non-wage costs of production when $y_t \neq z_t h_t$

Key equations of the New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - \rho \right) + u_t$$

A forward-looking system of three variables:
output gap x, inflation π and nominal interest rate i

Need an additional equation to close the system
$\hookrightarrow$ need to specify monetary policy rule

Monetary policy in the New Keynesian model

Optimal policy: long run

Two distortions in the basic model

1. Monopolistic competition: $P > MC$
2. Price dispersion: $P_1/P_2 \neq MC_1/MC_2$

The first distortion cannot be eliminated by monetary policy

But the second can, by keeping inflation rate at 0%

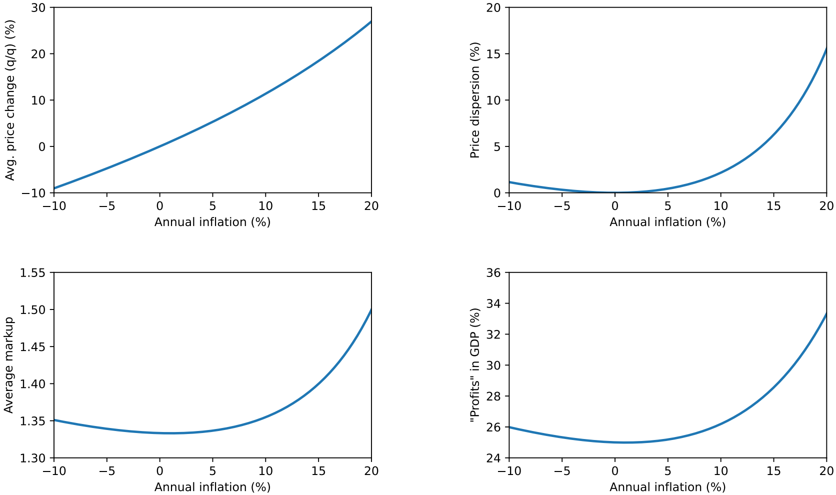
Welfare losses of annual inflation in 2-3% range
are very small, and other considerations matter

A positive inflation target (in advanced economies usually 2%)

- Decreases price dispersion under declining MC due to technological progress
- Increases labor market fluidity (nominal wage stickiness)
- Increases monetary policy space to reduce interest rate in recessions
- Lowers probability of hitting the effective lower bound on interest rates

Costs of inflation under Calvo scheme ($\mu = 1/3$ and $\theta = 0.75$)

Inflation other than 0% $\rightarrow$ price dispersion and lower effective output



Optimal policy: short run

If there are no cost-push shocks and sticky prices are the only distortion, then optimal monetary policy in the short run is to stabilize inflation perfectly

This would also perfectly stabilize output gap: divine coincidence, see Blanchard and Gali (2007)

If cost-push shocks occur and there are other distortions, e.g. sticky wages, then optimal policy becomes more complicated, e.g. has to also stabilize wage inflation

Divine coincidence no longer holds
(impossible to stabilize all variables at the same time)

Optimal policy: loss function

Due to many distortions optimal policy involves trade-offs

Rotemberg and Woodford (1998): when real imperfections are present, the second order approximation to social welfare loss is (where $\phi > 0$ is a function of model parameters)

$$L = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\phi x_t^2 + \pi_t^2 \right) \right]$$

Consistent with behavior of central banks, who aim to stabilize inflation and output gaps

Question arises whether policy should be conducted discretionary or under commitment

Optimal policy under discretion

Under optimal discretionary policy (ODP) the central bank is not able to influence expectations about future policy

Optimization boils down to solving a series of static problems

$$\begin{aligned} \min \quad & \phi x_t^2 + \pi_t^2 \\ \text{subject to} \quad & \pi_t = \beta \pi_t^e + \kappa x_t + e_t \end{aligned}$$

Note that expectations π_t^e are taken as given

Solution:

$$\pi_t = -\frac{\phi}{\kappa} x_t$$

Caution: This is a targeting rule, without specifying instruments

After an inflationary cost-push shock the central bank allows the output gap to become negative

Digression: Inflation bias

What if the policymakers wanted to maintain a positive output gap $o > 0$?

$$L = \phi \left(x_t - o \right)^2 + \pi_t^2$$

Consider also a slight alteration of the NKPC

$$\pi_t = \pi_t^e + \kappa x_t + e_t$$

where inflation is not a function of next-period inflation expectations, but current-period inflation expectations (i.e. expectations formed at the end of period $t-1$, before t-period shocks and policy actions are revealed)

Rewrite the NKPC (where $a \equiv 1/\kappa$ and $\epsilon_t \equiv -e_t/\kappa$)

$$x_t = \frac{\pi_t - \pi_t^e - \epsilon_t}{\kappa} \equiv a \left(\pi_t - \pi_t^e \right) + \epsilon_t$$

Central bank's choice

Optimization problem taking expectations as given

$$\begin{aligned} \min \quad & L = \phi \left(x_t - o\right)^2 + \pi_t^2 \\ \text{subject to} \quad & x_t = a\left(\pi_t - \pi_t^e\right) + \epsilon_t \end{aligned}$$

Plug constraint into objective

$$\min \quad L = \phi \left(a\left(\pi_t - \pi_t^e\right) + \epsilon_t - o\right)^2 + \pi_t^2$$

First order condition

$$\frac{\partial L}{\partial \pi_t} = 2\phi \left(a\left(\pi_t - \pi_t^e\right) + \epsilon_t - o\right) \cdot a + 2\pi_t = 0$$

Rearranging the FOC gives the desired inflation rate

$$\pi_t = \frac{a^2 \phi \pi_t^e + a\phi \left(o - \epsilon_t\right)}{1 + a^2 \phi}$$

Inflation in rational expectations equilibrium

Under rational expectations

$$\pi_t^e \equiv \mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-1}\left[\frac{a^2\phi\mathrm{E}_{t-1}\pi_t + a\phi\left(o-\epsilon_t\right)}{1+a^2\phi}\right] \rightarrow (1+a^2\phi)\mathrm{E}_{t-1}\pi_t = a^2\phi\mathrm{E}_{t-1}\pi_t + a\phi o$$

Agents expect inflation exceeding target

$$\pi_t^e = \mathrm{E}_{t-1}\pi_t = a\phi o > 0$$

Resulting in actual inflation

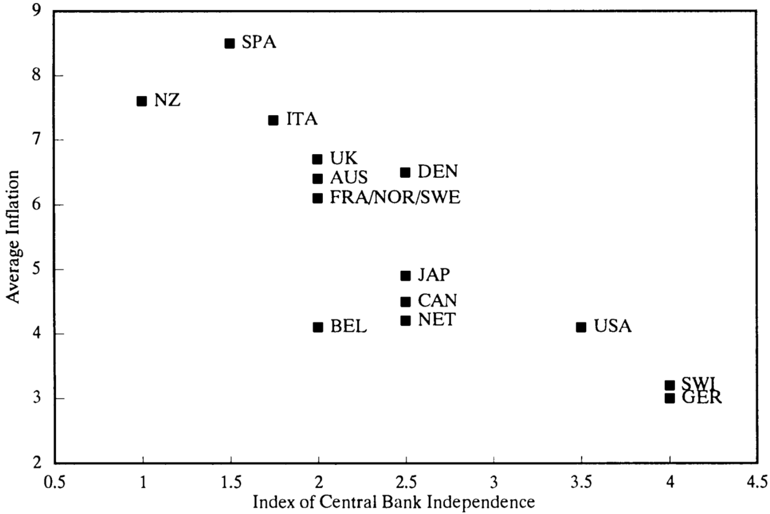
$$\pi_t = a\phi o - \frac{a\phi}{1+a^2\phi}\epsilon_t = a\phi o + \frac{a\phi}{1+a^2\phi}\frac{e_t}{\kappa}$$

Because private agents understand the incentives facing the central bank, average inflation is fully anticipated

Equilibrium produces average rate of inflation above target (inflation bias)

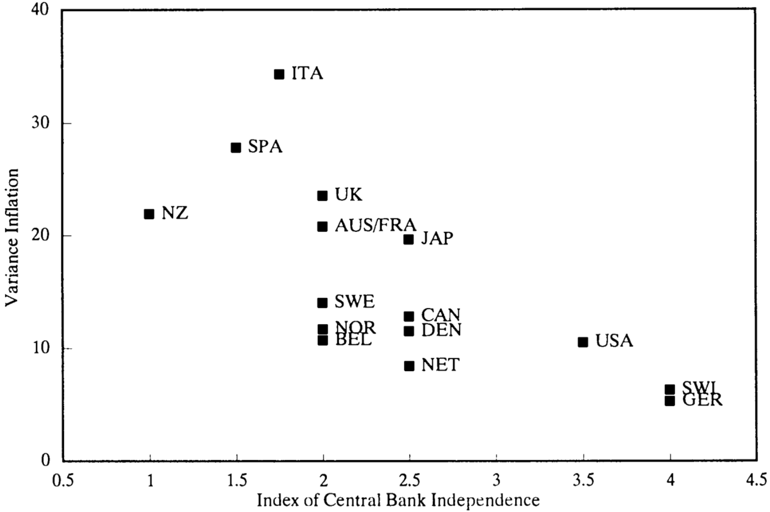
This has no systematic effect on output gap: $x_t = \frac{1}{1+a^2\phi}\epsilon_t = -\frac{1}{1+a^2\phi}\frac{e_t}{\kappa}$

Central bank independence and average inflation (1955-1988)



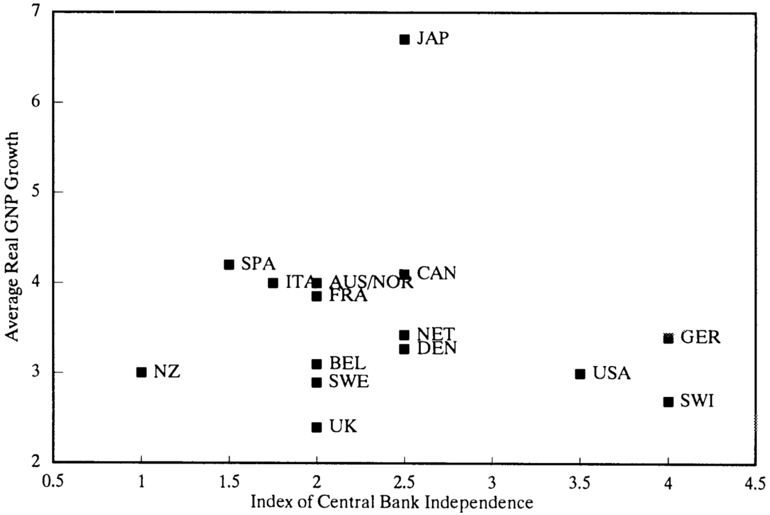
Alesina and Summers (1993)

Central bank independence and inflation volatility (1955-1988)



Alesina and Summers (1993)

Central bank independence and average real GNP growth (1955-1988)



Alesina and Summers (1993)

Counteracting inflation bias

Appointment of a "hawkish" central bank governor

"Hawks" place an additional weight ($\delta > 0$) on inflation stabilization compared with other members of the society

$$L = \phi \left(x_t - o\right)^2 + \left(1 + \delta\right) \pi_t^2$$

The rate of inflation under discretion will equal

$$\pi_t = \frac{a\phi}{1+\delta} o + \frac{a\phi}{1+\delta+a^2\phi} \frac{e_t}{\kappa}$$

Targeting 0 output gap $(o = 0)$

The rate of inflation under discretion will equal (ODP outcome)

$$\pi_t = \frac{a\phi}{1+a^2\phi} \frac{e_t}{\kappa} = -\frac{\phi}{\kappa} x_t$$

Optimal policy under commitment

Under **credible** commitment the central bank is able
to influence expectations about future policy

The problem is now dynamic

$$\min \quad \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\,\mathrm{E}_0\left[\phi x_t^2+\pi_t^2\right]$$

$$\text{subject to} \quad \pi_t=\beta\mathrm{E}_t\pi_{t+1}+\kappa x_t+e_t$$

Lagrangian

$$\mathcal{L}=\sum_{t=0}^{\infty}\beta^t\,\mathrm{E}_0\left[\frac{1}{2}\left(\phi x_t^2+\pi_t^2\right)+\mu_t\left(\beta\pi_{t+1}+\kappa x_t+e_t-\pi_t\right)\right]$$

First order conditions

$$x_t \quad : \quad \beta^t\,\mathrm{E}_0\left[\phi x_t+\mu_t\kappa\right]=0 \qquad \rightarrow \qquad \mu_t=-\frac{\phi}{\kappa}x_t$$

$$\pi_t \quad : \quad \beta^{t-1}\,\mathrm{E}_0\left[\mu_{t-1}\beta\right]+\beta^t\,\mathrm{E}_0\left[\pi_t-\mu_t\right]=0 \qquad \rightarrow \qquad \pi_t=\mu_t-\mu_{t-1}$$

Optimal policy under commitment

For the current period the past is not a constraint ($\mu_{-1} = 0$)

$$\pi_0 = \mu_0 = -\frac{\phi}{\kappa} x_0$$

Same as under discretion

For the future periods ($t \geq 1$) we get

$$\pi_t = \mu_t - \mu_{t-1} = -\frac{\phi}{\kappa} (x_t - x_{t-1})$$

Different than for today: will take the past into account

Optimal commitment policy (OCP) means pursuing a discretionary policy today, but promising a non-discretionary policy from tomorrow on!

But when we'll arrive in the next period, we will be tempted to act as in the current period: time inconsistency

Optimal policy under commitment

OCP is time inconsistent – solutions?

1. Appoint very credible central bankers
2. To build credibility, adopt a timeless perspective: pretend that OCP has been applied long ago and apply the formula for $t \geq 1$ from the beginning

Which is better: OCP or ODP?

- Neither invokes inflation bias
- ODP generates **stabilization bias**, making economy more volatile

The superiority of commitment calls for a credible, long-term arrangement for the central bank that will sometimes act **against** short-term welfare

Stabilization bias: discretion vs commitment

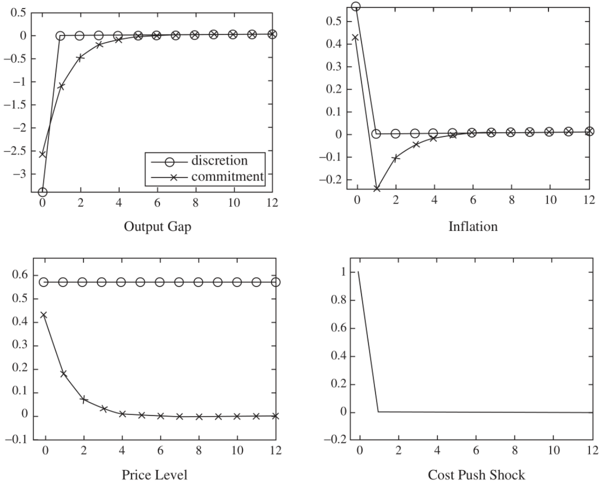


Figure 5.1 Optimal Responses to a Transitory Cost Push Shock

Gali (2008)

Stabilization bias: discretion vs commitment

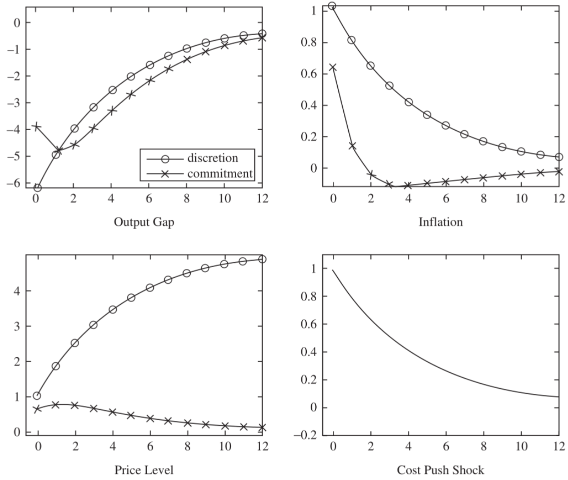


Figure 5.2 Optimal Responses to a Persistent Cost Push Shock

Gali (2008)

Digression: problems with identifying the NKPC

Recently, many papers have claimed that either

1. NKPC has become more "flat", or
2. NKPC relationship has disappeared at all

McLeay and Tenreyro (2019): Under ODP, observed inflation will be unrelated to the measure of slack in the economy!

Assume the cost-push shock follows an AR(1) process

$$e_t = \rho_e \, e_{t-1} + \epsilon_t$$

Combining above with NKPC and ODP rule, we get

$$\pi_t = \frac{\phi}{\kappa^2 + \phi\,(1 - \beta\rho_e)} e_t \equiv \phi\alpha \, e_t = \phi\alpha \left(\rho_e \, e_{t-1} + \epsilon_t\right)$$

Digression: problems with identifying the NKPC

Under ODP inflation is a function of cost-push shock

$$\pi_t = \phi\alpha\, e_t = \phi\alpha\left(\rho_e\, e_{t-1} + \epsilon_t\right)$$

But this means that current inflation will be very well forecastable using past inflation

$$\pi_t = \rho_e\, \pi_{t-1} + \phi\alpha\epsilon_t$$

Realized inflation and output gap will be **negatively** correlated

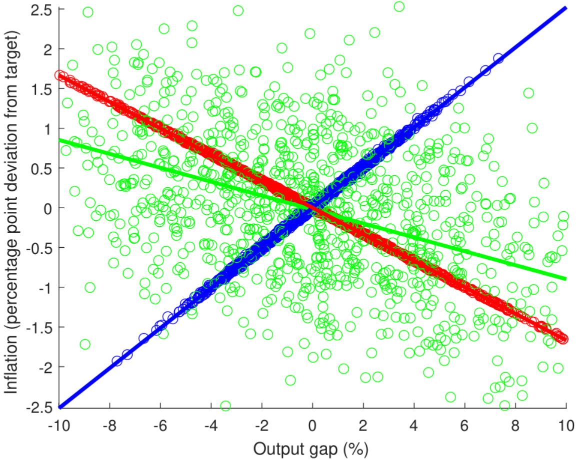
$$\pi_t = -\frac{\phi}{\kappa} x_t \quad \rightarrow \quad \pi_t = b x_t + \varepsilon_t$$

$$\hat{b} < 0 \quad \text{and} \quad \mathrm{Corr}\left(x_t,\, \varepsilon_t\right) \neq 0$$

Additionally, if there are shocks to monetary policy rule, any estimate of $\hat{b}$ is possible, including $\hat{b} > 0$ and $\hat{b} \approx 0$!

Estimate for $\hat{b}$ depends on relative magnitude of shocks

Figure 5: *Inflation/output gap correlation in model-simulated data: optimal discretion with shocks to the targeting rule*

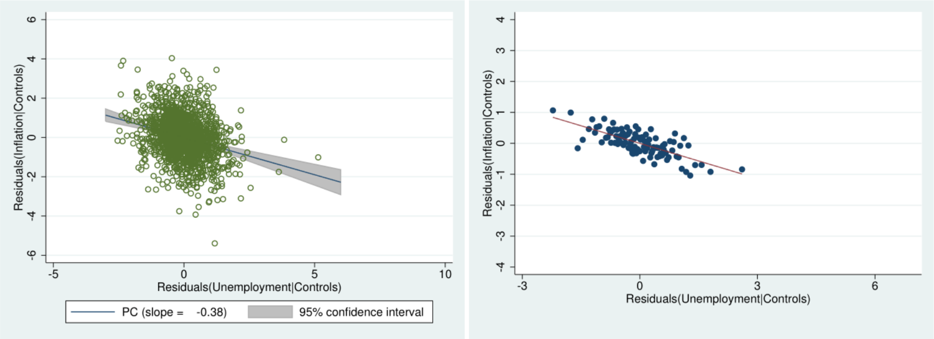


McLeay and Tenreyro (2019)

One solution: Identification using regional data

Figure 12: *Year and metro area fixed effects: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*

(a) *Raw residuals* **(b)** *Residuals grouped into bins*



Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (4) in table 3. See the notes to Figures 10a and 10b for details.

McLeay and Tenreyro (2019)

Instrument rules

We have been assuming that the CB can "choose" x_t and π_t

In reality, the CB can influence these variables indirectly,
by e. g. changing the nominal interest rate

Recall the NKIS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + u_t$$

where the CB can affect output gap by varying i

Note that monetary policy by changing i affects output gap "first"
and inflation rate "second"

Determinacy concerns

Under ODP inflation and output gap react to cost-push shock

$$\pi_t = \phi\alpha\, e_t \quad \text{and} \quad x_t = -\kappa\alpha\, e_t$$

The CB could vary the level of i to try implementing the ODP

$$-\kappa\alpha\, e_t = -\kappa\alpha\rho_e\, e_t - \frac{1}{\sigma}\left(i_t - \phi\alpha\rho_e\, e_t - r_t^*\right)$$
$$i_t = r_t^* + \alpha\left(\kappa\sigma\left(1-\rho_e\right) + \phi\rho_e\right) e_t$$

But then our forward-looking system would have multiple solutions, only one of which is consistent with ODP

Such instrument rule would be “too weak”

And would require observing e_t **perfectly** in real-time!

Taylor rules

Instead of constructing the instrument rule as function of shocks, construct the rule as function of endogenous variables

$$i_t = i^* + \gamma_\pi \, \pi_t$$

where $\gamma_\pi \equiv (1 - \rho_e)\, \kappa\sigma/\alpha + \rho_e$

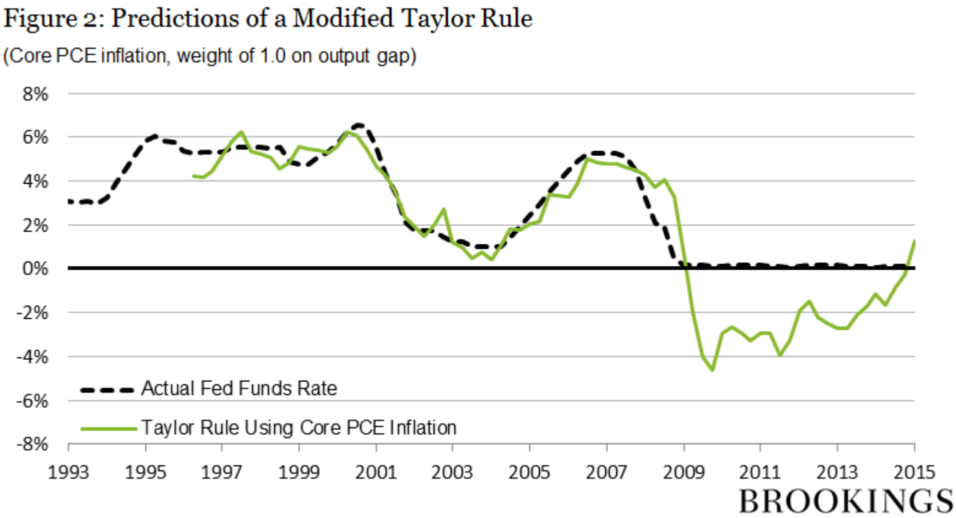
It can be shown that if only $\gamma_\pi > 1$, the system has a unique solution, and the central bank can "select" the ODP equilibrium

A more general Taylor rule allows for a non-zero inflation target π^*, reactions to output gap and smoothing of policy rate changes

$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i)\left(i^* + \gamma_\pi \, (\pi_t - \pi^*) + \gamma_x \, x_t\right)$$

A central bank should raise the interest rate when inflation is above target and / or output gap is positive

A Taylor rule can capture Fed's actual policy prior to 2009



Bernanke (2015)

Taylor principle and stability of inflation

Taylor principle: when inflation increases by 1 p.p., the central bank should raise the interest rate by $\gamma_\pi > 1$ p.p. (in a dynamic sense)

Failure to do so results in inflation instability

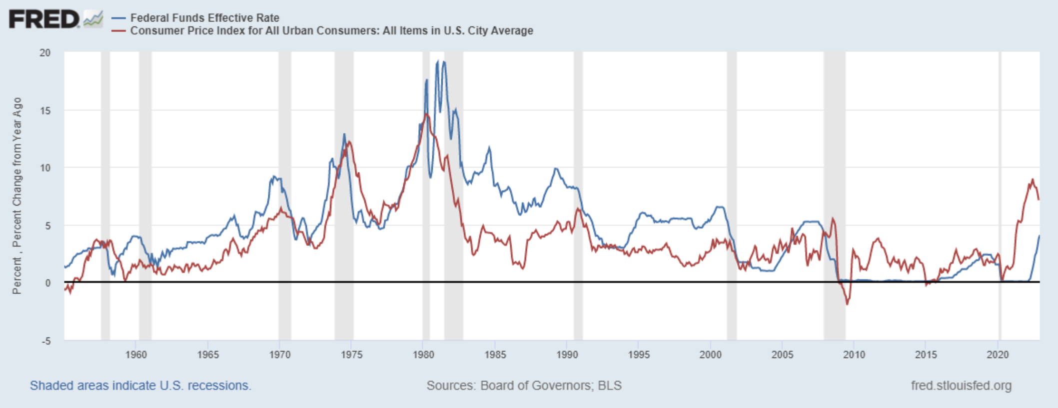
The estimate for the pre-Volcker rule is significantly less than unity. Monetary policy over this period was accommodating increases in expected inflation, in clear violation of the [Taylor principle – MB].

TABLE 1
ESTIMATES OF POLICY REACTION FUNCTION

	γ_π	γ_x	ρ
Pre-Volcker	0.83	0.27	0.68
	(0.07)	(0.08)	(0.05)
Volcker–Greenspan	2.15	0.93	0.79
	(0.40)	(0.42)	(0.04)

Clarida, Gali and Gertler (1999)

Interest rate and inflation in the US



St Louis Fed

Basic three-equation New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^* \right) + \tilde{u}_t$$

Taylor rule

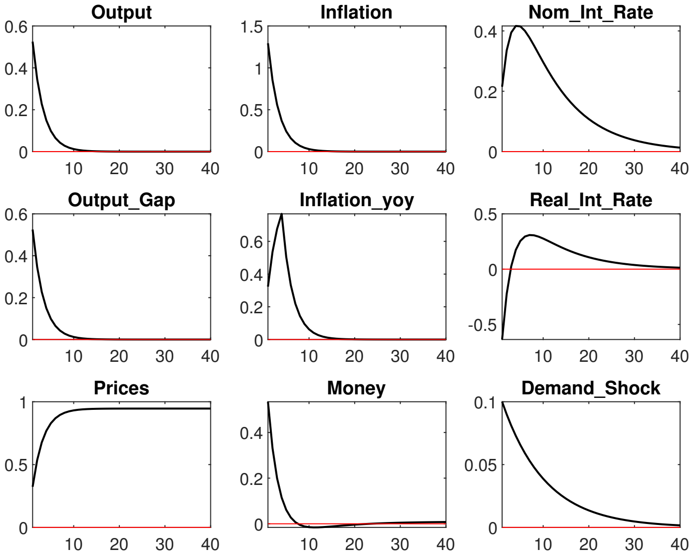
$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i) \left(i^* + \gamma_\pi \, (\pi_t - \pi^*) + \gamma_x \, x_t \right) + v_t$$

where v is an **exogenous** monetary policy shock

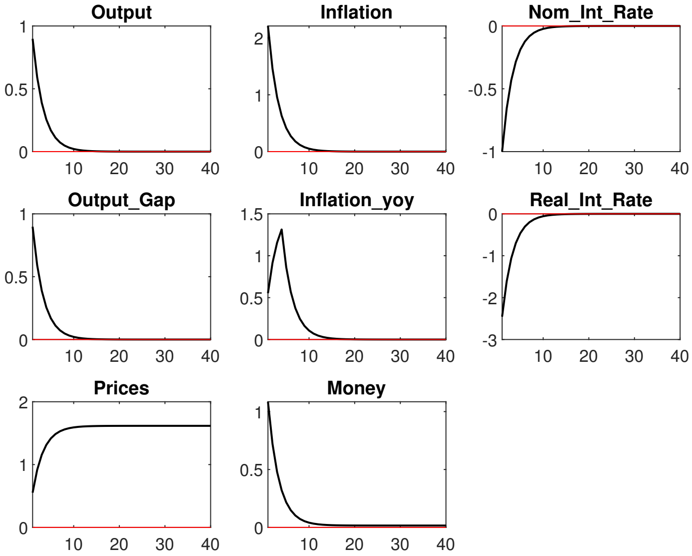
Plus (optionally) the natural real interest rate equation

$$r_t^* = \rho + \sigma \mathrm{E}_t \left[\Delta \ln y_{t+1}^* \right]$$

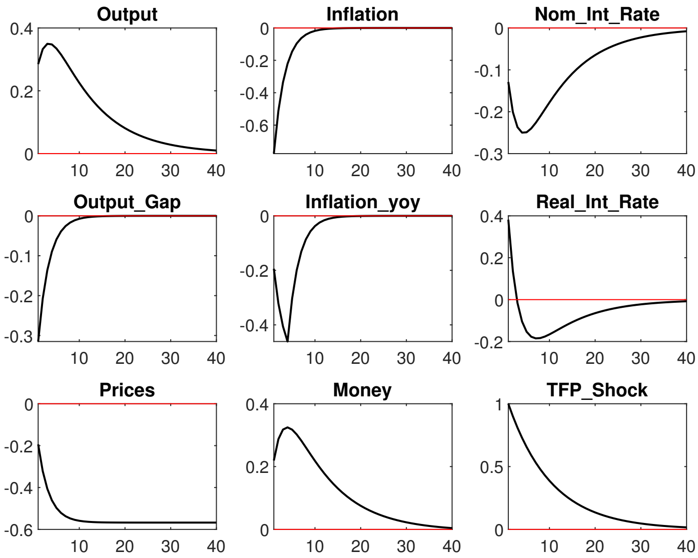
Positive demand shock ($\tilde{u} > 0$)



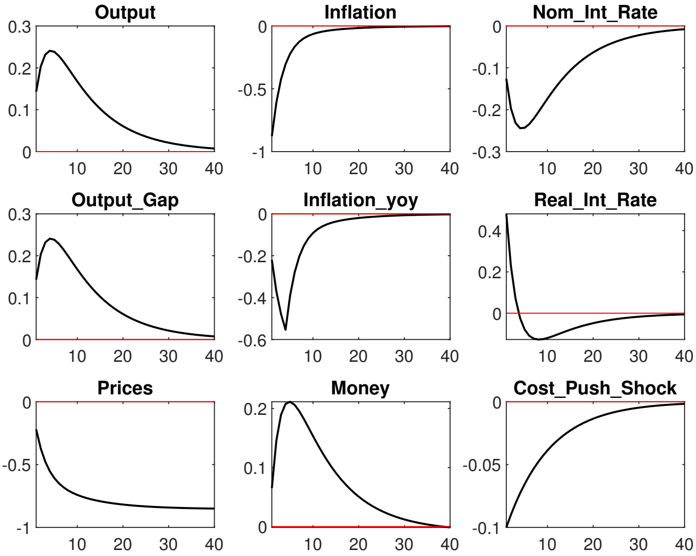
Positive monetary shock ($v < 0$)



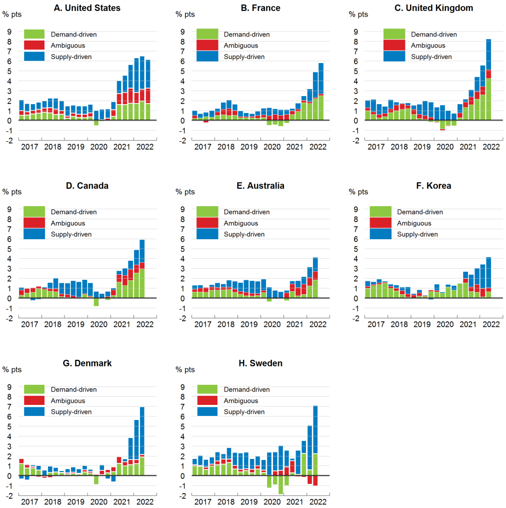
Positive TFP shock ($z > 0$)



Positive cost-push shock ($e < 0$)



Model-based inflation shock decomposition

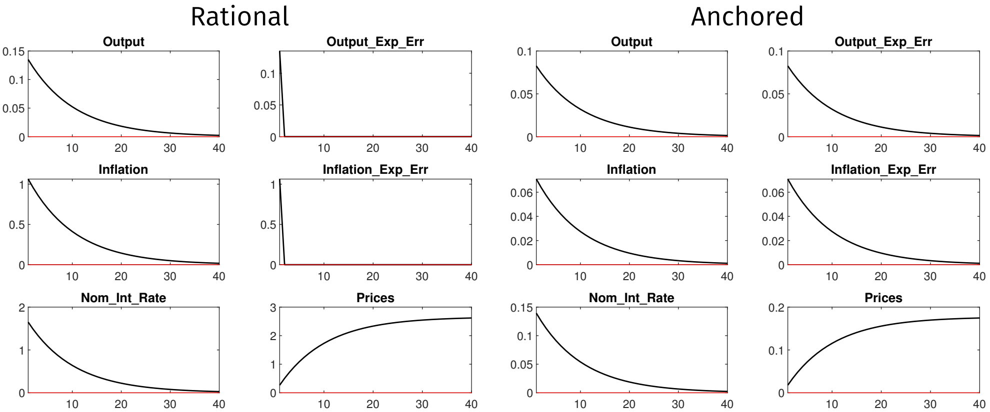


Role of expectations

Modern monetary policy: management of expectations

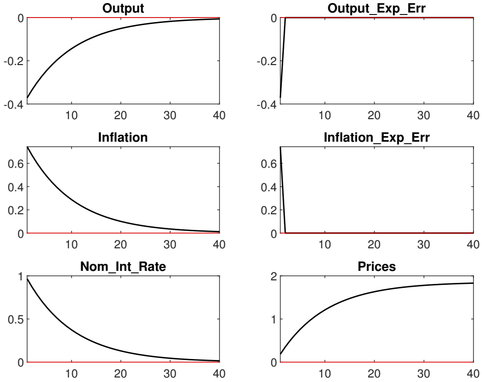
Woodford (2005, p. 3): For not only do expectations about policy matter, but, at least under current conditions, very little *else* matters

Response to demand shock under alternative expectations

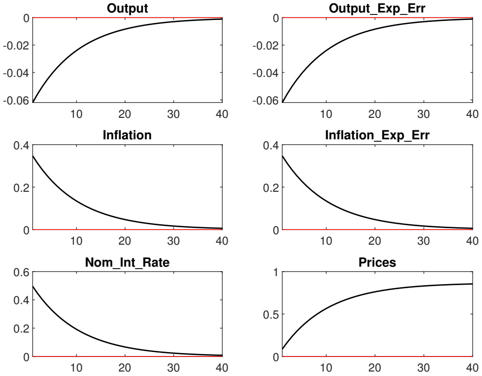


Response to cost-push shock under alternative expectations

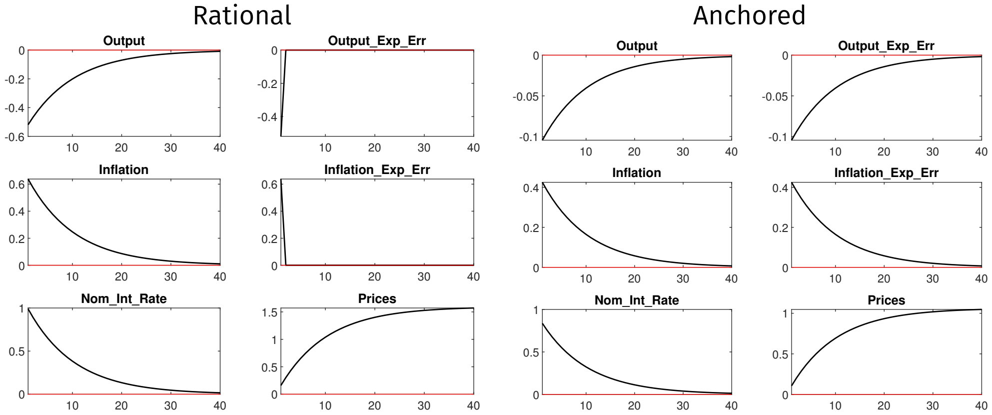
Rational



Anchored



Response to TFP shock under alternative expectations



Applied New Keynesian models

Shocks affect the economy in the directions indicated by empirical evidence

Basic model is too stylized to take it directly to data

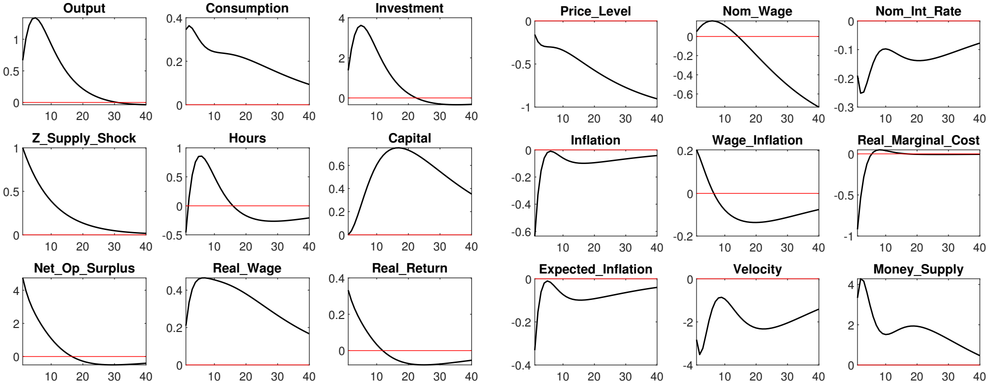
Some standard extensions introduced to applied NK models:

- Nominal wage stickiness
- Indexation of prices and wages or strategic complementarities delay the response of inflation to shocks
- Habits in the utility function delay the response of consumption to shocks
- Investment adjustment costs delay the response of investment to shocks

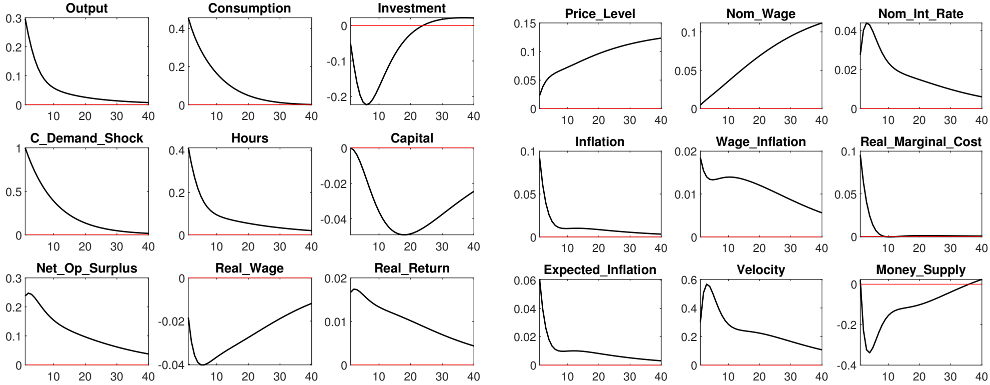
More complicated extensions:

- Financial frictions: Bernanke, Gertler and Gilchrist 1999, Kiyotaki and Moore 1997, Iacoviello 2005
- Unemployment: Gertler, Sala and Trigari 2008, Gali 2010
- Household heterogeneity (HANK): Kaplan, Moll and Violante (2018)

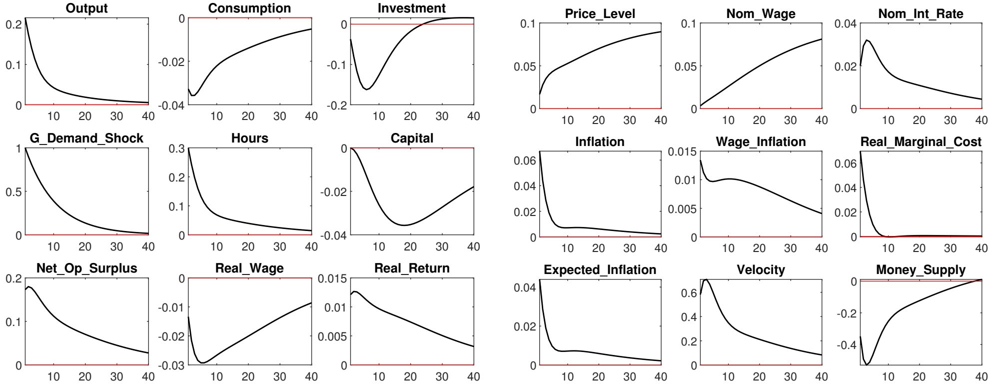
Extended NK model: TFP / cost-push shock



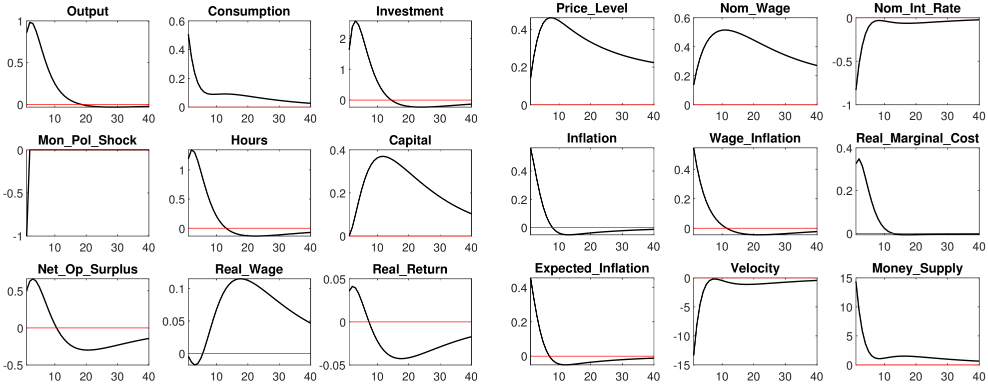
Extended NK model: consumption demand shock



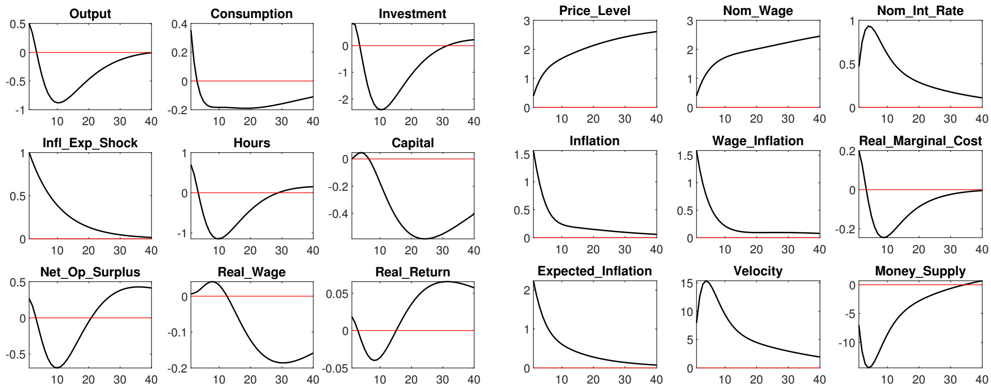
Extended NK model: government demand shock



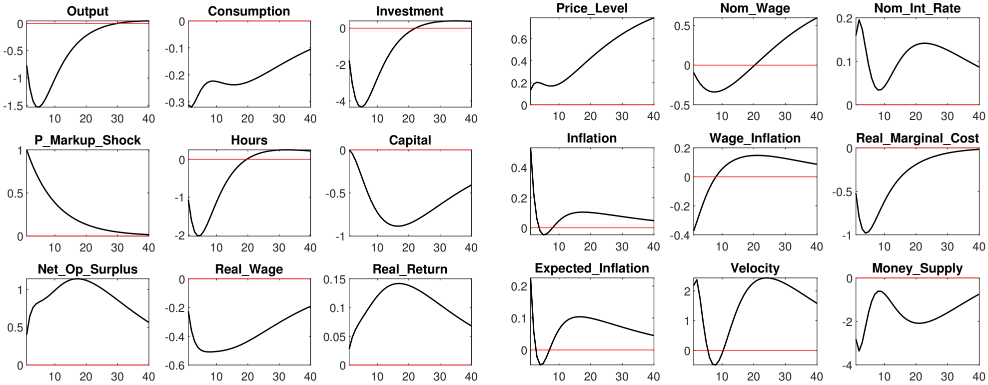
Extended NK model: exogenous interest rate cut



Extended NK model: exogenous rise in inflation expectations



Extended NK model: rise in firms' markup ("greedflation")



Extended NK model: rise in wage markup