Growth facts. Solow-Swan model

Advanced Macroeconomics IE: Lecture 8

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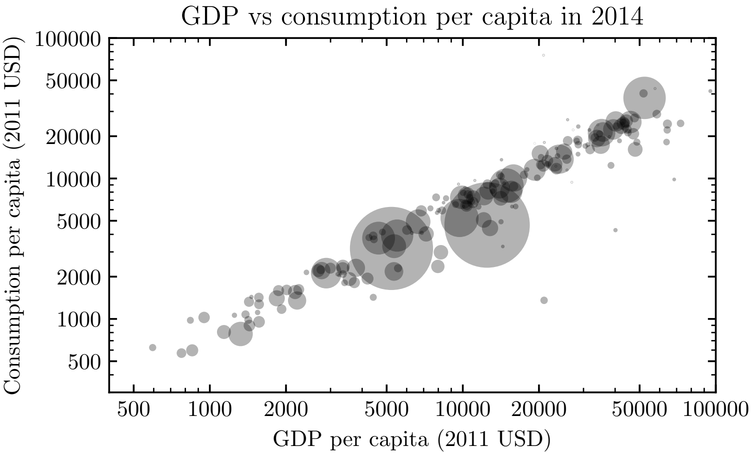
In case you need a review of basic concepts

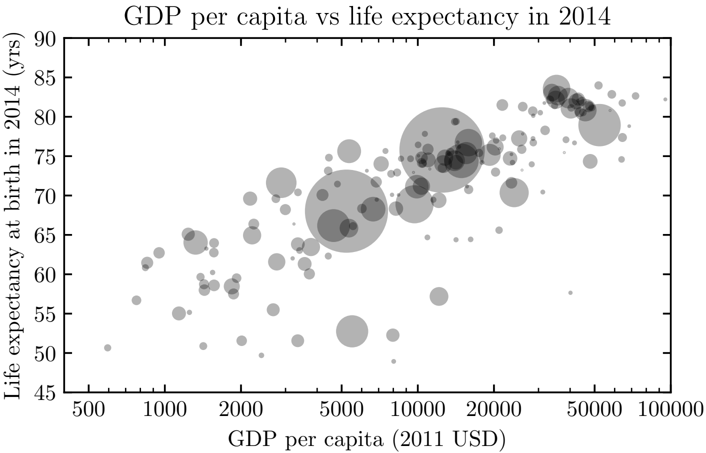
https://www.mruniversity.com/courses/principles-economics-macroeconomics

Chapters 1-3

GDP per capita and welfare: consumption

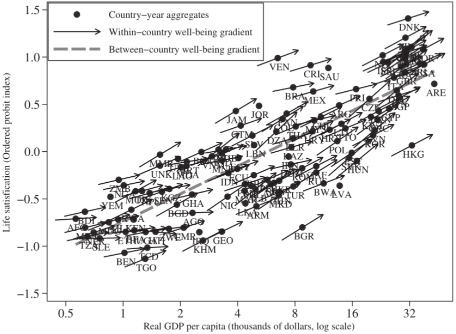


GDP per capita and welfare: life duration



GDP per capita and welfare: life satisfaction

Figure 11. Within-Country and Between-Country Estimates of the Life Satisfaction–Income Gradient: Gallup World Poll[a]

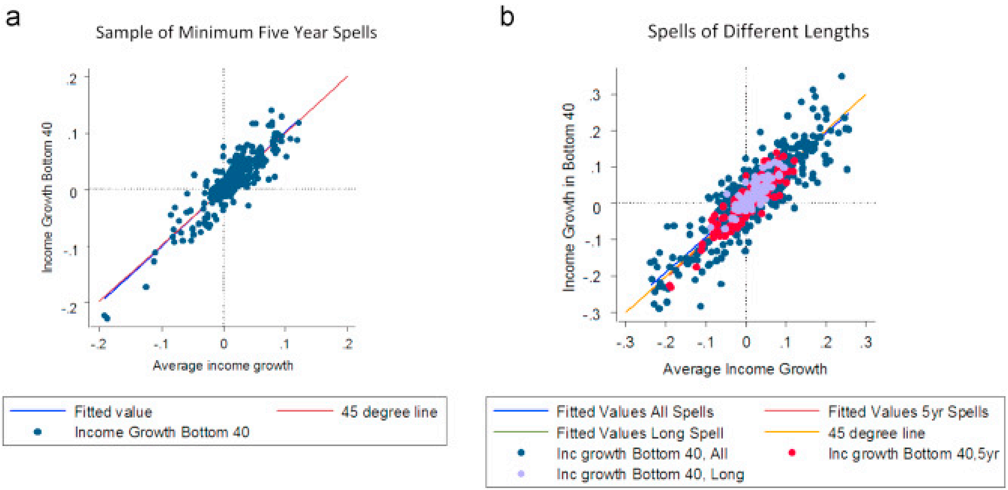


Source: Gallup World Poll, 2006; authors' regressions. Sources for GDP per capita are described in the text.

a. Each solid circle plots life satisfaction against GDP per capita for one of 131 developed and developing countries. The slope of the arrow represents the satisfaction-income gradient estimated for that country from a country-specific ordered probit of satisfaction on the log of annual real household income, controlling for gender, a quartic in age, and their interaction. Usable household income data were unavailable for eighteen countries. The dashed line represents the between-country satisfaction-income gradient estimated from an OLS regression of the satisfaction index on the logarithm of real GDP per capita. GDP per capita is at purchasing power parity in constant 2000 international dollars.

Stevenson and Wolfers (2013) Economic Growth and Subjective Well-Being: Reassessing the Easterlin Paradox

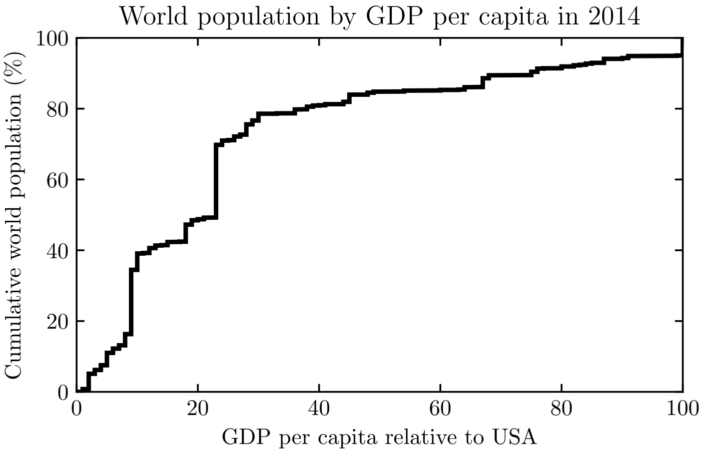
Growth in total income vs income of bottom 40%



Dollar, Kleineberg and Kraay (2016) *Growth still is good for the poor*

Growth fact 1

There is enormous variation in GDP per capita across economies



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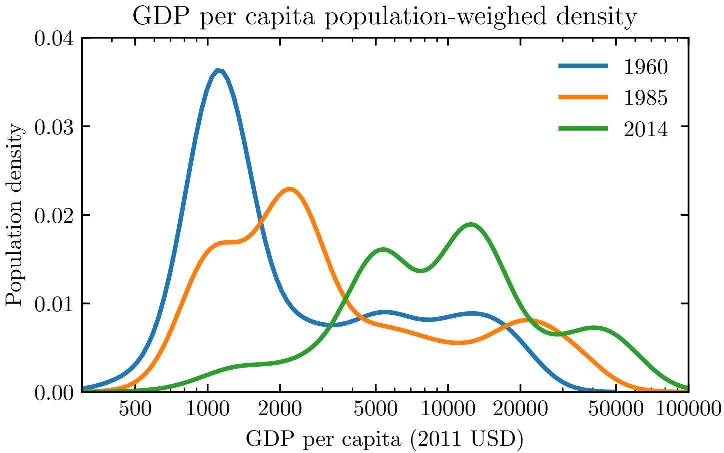
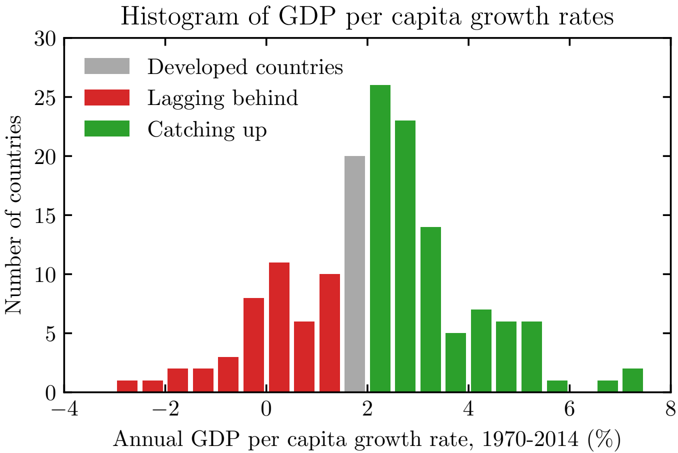


Growth fact 2

Rates of economic growth vary substantially across countries



Growth fact 2

Small differences in rates of growth translate to big differences in incomes over time:

Rate	Initial	Income after … years			
of growth	income	25	50	70	100
1.0%	100	128	164	201	270
1.5%	100	145	211	284	443
2.0%	100	164	269	400	724
2.5%	100	185	344	563	1181
3.0%	100	209	438	792	1922

Rule of 70

A way to estimate the number of years it takes for a certain variable to double

Find T for which $x_T = 2x_0$, assuming constant (annual) growth rate g

$$\begin{aligned} x_T &= x_0 \cdot (1+g)^T \\ 2x_0 &= x_0 \cdot (1+g)^T \\ 2 &= (1+g)^T \quad \mid \quad \ln \\ \ln 2 &= T \cdot \ln(1+g) \\ 0.7 &\approx T \cdot g \\ T &\approx \frac{0.7}{g} = \frac{70}{100 \cdot g} \end{aligned}$$

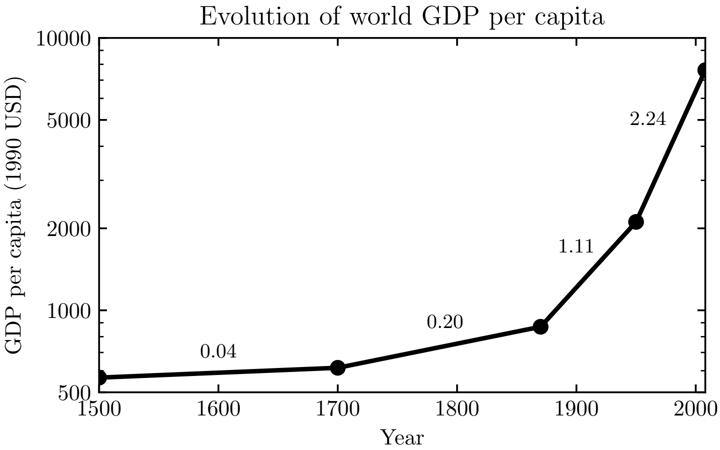
The number of years for a variable to double is approximately equal to 70 divided by g (expressed in percentage points)

There are both “growth miracles” and “growth disasters”

Country	GDP per capita 2014	GDP per worker 2014	Emp. rate 2014	GDP per capita 1970	Avg. growth (%) 1970 - 2014	Years to double
“Rich” countries						
United States	52 292	112 517	0.46	23 608	1.8	38
United Kingdom	40 242	83 612	0.48	15 176	2.2	31
France	39 374	95 498	0.41	16 436	2.0	35
Japan	35 358	68 989	0.51	12 956	2.3	30
“Growth miracles”						
Singapore	72 583	117 472	0.62	5 814	5.9	12
Hong Kong	51 808	100 467	0.52	7 613	4.5	16
Taiwan	44 328	92 979	0.48	4 738	5.2	13
South Korea	35 104	67 247	0.52	2 100	6.6	10
“Poor” countries						
Botswana	16 175	37 637	0.43	798	7.1	10
China	12 473	21 394	0.58	1 285	5.3	13
Indonesia	9 707	21 853	0.44	995	5.3	13
India	5 224	13 261	0.39	1 282	3.2	21
“Growth disasters”						
Zimbabwe	1 869	4 384	0.43	2 429	-0.6	-117
Madagascar	1 237	2 833	0.44	1 479	-0.4	-171
Dem. Rep. of Congo	1 217	3 757	0.32	2 536	-1.7	-42
Niger	852	2 397	0.36	1 395	-1.1	-62

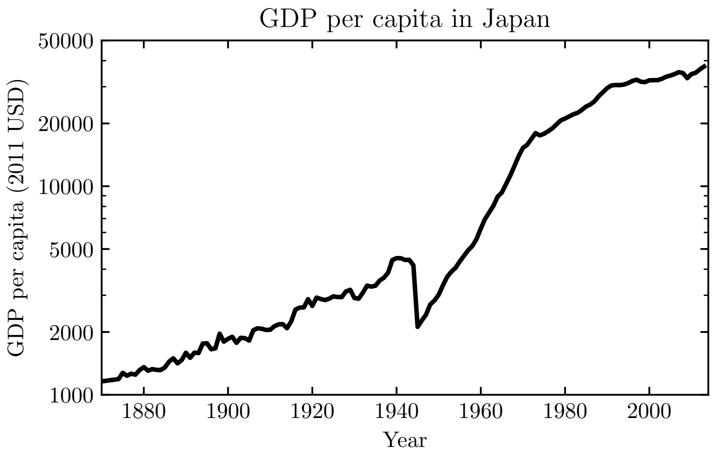
Growth fact 3

World growth rates have increased sharply in the twentieth century



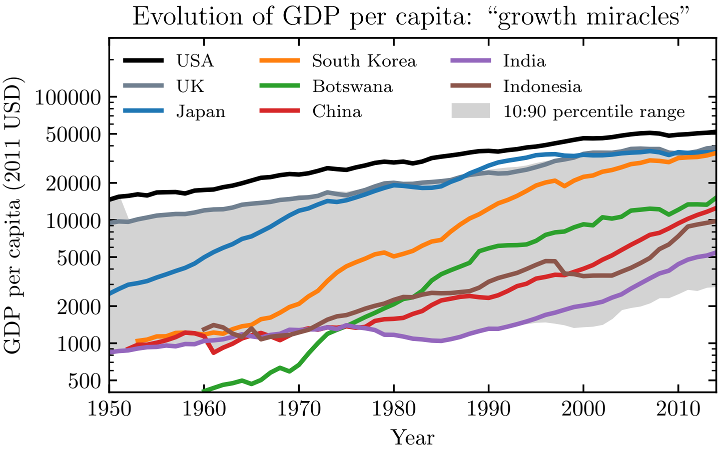
Growth fact 3

For individual countries, growth rates also change over time



Growth fact 4

Countries can go from being "poor" to being "rich"



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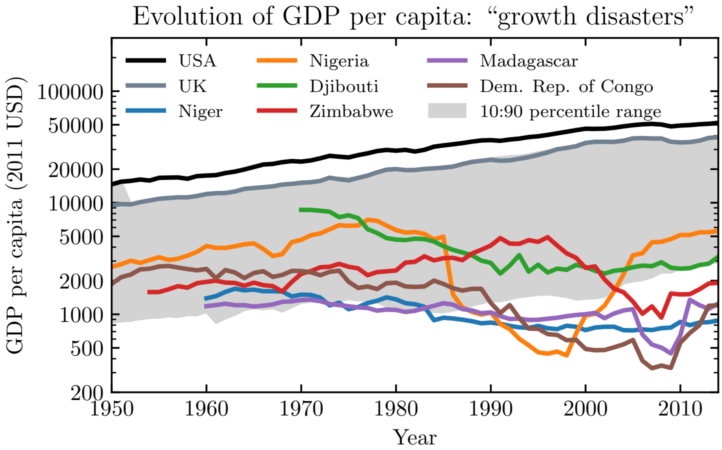


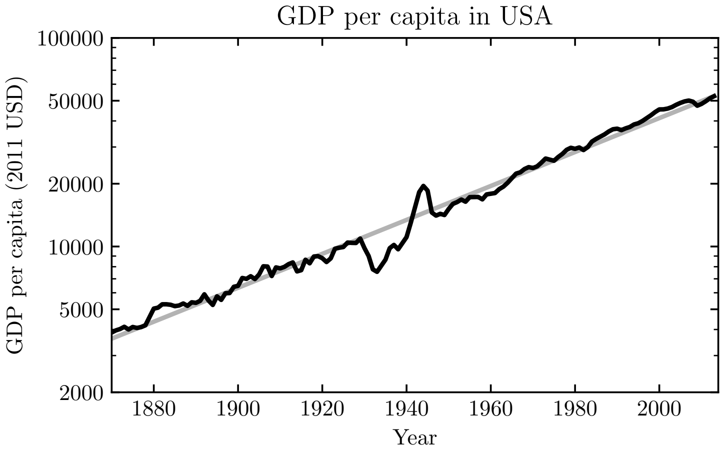
Kaldor's stylized facts

In the USA (and other developed countries):

1. Per capita output grows over time,
   and its growth rate does not tend to diminish
2. Physical capital per worker grows over time
3. The rate of return to capital is not trending
4. The ratio of physical capital to output is nearly constant
5. The shares of labor and physical capital in national income
   are nearly constant
6. Real wage grows over time

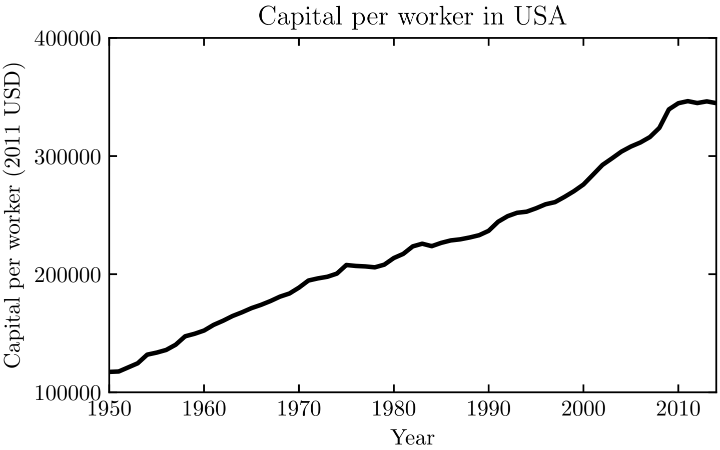
Kaldor's stylized fact 1

Per capita output grows over time, and its growth rate does not tend to diminish



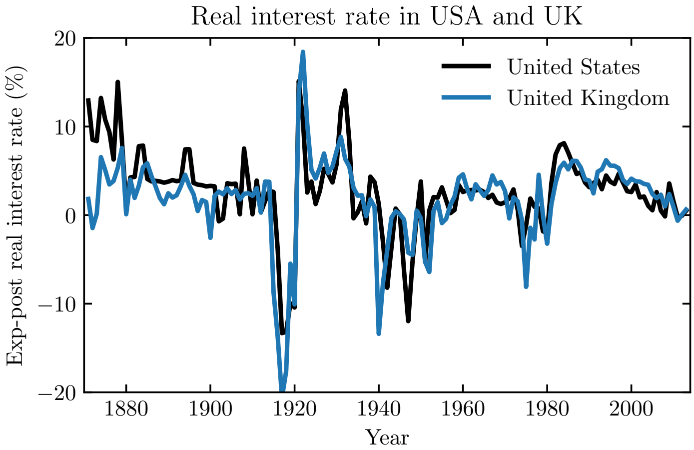
Kaldor’s stylized fact 2

Physical capital per worker grows over time



Kaldor's stylized fact 3

The rate of return to capital is not trending



Kaldor’s stylized fact 3

The rate of return to capital is not trending

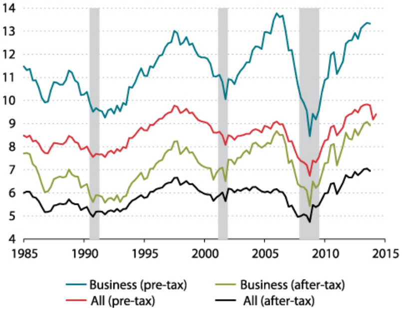
DeLong (2015)

Kaldor's stylized fact 3

The rate of return to capital is not trending

Figure 2

Real Returns on Capital (percent)

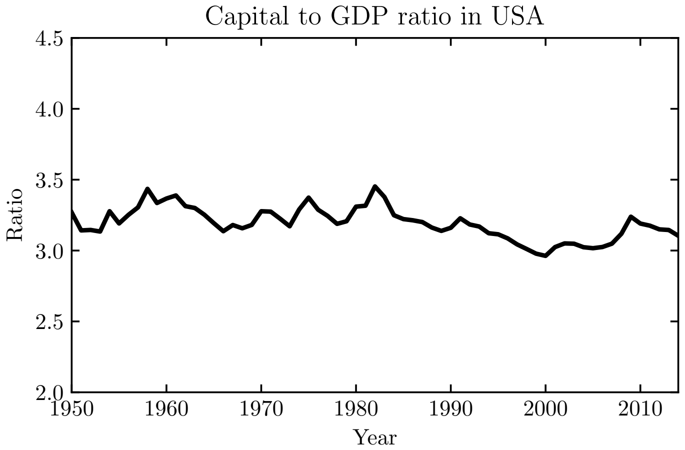


SOURCE: Authors' calculations; for details, see Gomme, Ravikumar, and Rupert (2011).

Gomme, Ravikumar and Rupert (2015)

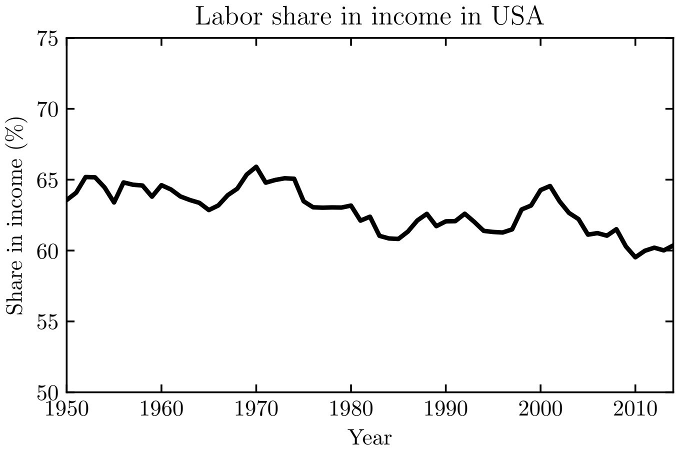
Kaldor's stylized fact 4

The ratio of physical capital to output is nearly constant



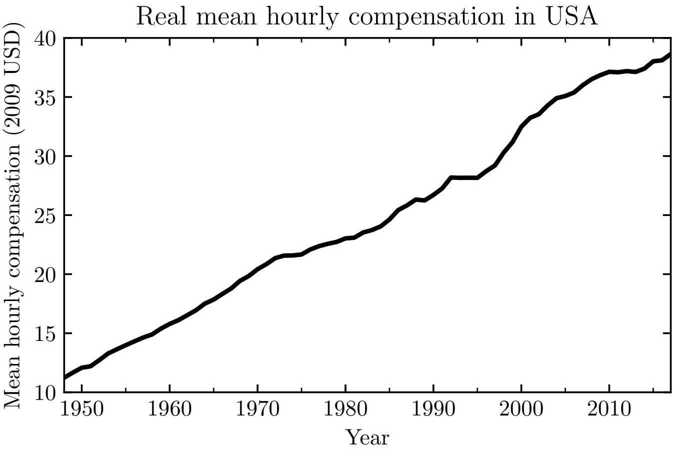
Kaldor's stylized fact 5

The shares of labor and physical capital in national income are nearly constant



Kaldor's stylized fact 6

Real wage grows over time



Explaining growth

We want to explain:

- Why some countries are poor and other rich?
- Why some countries that were previously poor became rich?
- Why not all poor countries catch up to rich countries?
- Why do rich countries still grow?

Solow-Swan model

Developed by Robert Solow (1956) and Trevor Swan (1956)

Growth in income per capita comes from two sources:

- Capital accumulation (endogenous)
- Improvements in technology (exogenous)

But capital accumulation alone cannot sustain growth in the absence of technology improvements

Does not explain "deep" sources of economic growth:

- Proximate vs fundamental causes

Departure point for growth theory

Simplifications and assumptions

- Closed economy
- No government
- Single, homogenous final good with its price normalized to 1 in each period (all variables are expressed in real terms)
- Two types of representative agents:
  - Firms
  - Households

Production

Real GDP is produced according to a neoclassical production function:

$$Y_t = F\left(K_t, A_t L_t\right)$$

where Y is real GDP, F is a neoclassical production function, K is capital stock, A is the technology level and L is the number of workers

Technology grows at a rate $g > 0$ and increases productivity of labor (otherwise Kaldor's stylized facts would be violated):

$$\dot{A}_t / A_t = g$$

Very often we use a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}$$

Like other neoclassical production functions, it exhibits constant returns to scale – doubling inputs K and L doubles the amount produced:

$$\left(zK_t\right)^{\alpha}\left(A_t \cdot zL_t\right)^{1-\alpha} = z^{\alpha} z^{1-\alpha} K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} = zY_t$$

Firms

Perfectly competitive firms maximize their profit:

$$\max_{K_t, L_t} \qquad D_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha} - r_t^K K_t - w_t L_t$$

where r^k denotes the rental rate on capital

First order conditions:

$$\begin{aligned} K_t \quad &: \quad \alpha K_t^{\alpha-1} \left(A_t L_t\right)^{1-\alpha} - r_t^K = 0 \qquad &\rightarrow \qquad r_t^K = \alpha \frac{Y_t}{K_t} \\ L_t \quad &: \quad \left(1-\alpha\right) K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha} - w_t = 0 \qquad &\rightarrow \qquad w_t = \left(1-\alpha\right) \frac{Y_t}{L_t} \end{aligned}$$

Total factor payments are equal to GDP:

$$r_t^K K_t + w_t L_t = \alpha \frac{Y_t}{K_t} K_t + \left(1-\alpha\right) \frac{Y_t}{L_t} L_t = \alpha Y_t + \left(1-\alpha\right) Y_t = Y_t$$

Factor shares

Calculate the fraction of GDP that is paid to each factor:

$$\frac{w_t L_t}{Y_t} = \frac{(1-\alpha)\,\dfrac{Y_t}{L_t} \cdot L_t}{Y_t} = (1-\alpha) \quad \text{and} \quad \frac{r_t^K K_t}{Y_t} = \frac{\alpha \dfrac{Y_t}{K_t} \cdot K_t}{Y_t} = \alpha$$

Cobb-Douglas function implies constant shares of labor and physical capital in income

Confronting with the US data, we can obtain $\alpha \approx \frac{1}{3}$ and $(1-\alpha) \approx \frac{2}{3}$

Households

Own factors of production (K and L) and earn income from renting them to firms

Each households supplies one unit of labor: $L_t = N_t$ and population grows at a rate n:

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{N}_t}{N_t} = n$$

Capital accumulates from investment I_t and depreciates at rate δ:

$$\dot{K}_t = I_t - \delta K_t$$

Income of households is consumed or saved (invested):

$$Y_t = w_t L_t + r_t^K K_t = C_t + S_t = C_t + I_t$$

Households **don't optimize**, save a constant fraction s of income:

$$I_t = sY_t \quad \text{and} \quad C_t = (1-s)\,Y_t$$

GDP per worker

Usually we are most interested in GDP per worker (or per capita), y:

$$y_t \equiv \frac{Y_t}{L_t} = \frac{K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}}{L_t} = A_t \left(\frac{K_t}{A_t L_t}\right)^{\alpha} \equiv A_t \hat{k}_t^{\alpha}$$

where $\hat{k}$ is capital K divided per effective unit of labor (AL)

Clearly, GDP per worker increases due to improvements in technology and due to capital accumulation

The production function exhibits diminishing marginal returns to capital. GDP per worker increases with $\hat{k}$, but the size of the increase falls with $\hat{k}$

It is also useful to define output per effective unit of labor $\hat{y}$:

$$\hat{y}_t = \frac{Y_t}{A_t L_t} = \frac{y_t}{A_t} = \hat{k}_t^{\alpha}$$

Time derivative of capital per effective labor

Time derivative of $\hat{k}_t$:

$$\begin{aligned}
\dot{\hat{k}}_t &= \frac{\mathrm{d}\hat{k}_t}{\mathrm{d}t} = \frac{\mathrm{d}\left(K_t / \left(A_t L_t\right)\right)}{\mathrm{d}t} = \frac{\frac{\mathrm{d}K_t}{\mathrm{d}t} \cdot \left(A_t L_t\right) - K_t \cdot \frac{\mathrm{d}\left(A_t L_t\right)}{\mathrm{d}t}}{\left(A_t L_t\right)^2} \\
&= \frac{\dot{K}_t}{A_t L_t} \cdot \frac{A_t L_t}{A_t L_t} - \frac{K_t}{A_t L_t} \cdot \frac{\frac{\mathrm{d}A_t}{\mathrm{d}t} \cdot L_t + A_t \cdot \frac{\mathrm{d}L_t}{\mathrm{d}t}}{A_t L_t} \\
&= \frac{\dot{K}_t}{A_t L_t} - \hat{k}_t \cdot \left(\frac{\dot{A}_t}{A_t} + \frac{\dot{L}_t}{L_t}\right) \\
\dot{\hat{k}}_t &= \frac{\dot{K}_t}{A_t L_t} - \hat{k}_t \cdot (g + n)
\end{aligned}$$

Therefore:

$$\frac{\dot{K}_t}{A_t L_t} = \dot{\hat{k}}_t + (g + n)\, \hat{k}_t$$

Capital accumulation

Capital accumulates according to:

$$\dot{K}_t = sY_t - \delta K_t$$

And capital per effective labor according to:

$$\begin{aligned}
\dot{K}_t &= sY_t - \delta K_t \quad | \quad : A_t L_t \\
\frac{\dot{K}_t}{A_t L_t} &= s\frac{Y_t}{A_t L_t} - \delta \frac{K_t}{A_t L_t} \\
\dot{\hat{k}}_t + (g+n)\,\hat{k}_t &= s\hat{y}_t - \delta \hat{k}_t \\
\dot{\hat{k}}_t &= s\hat{k}_t^{\alpha} - (\delta + n + g)\,\hat{k}_t
\end{aligned}$$

The growth rate of capital per effective labor equals:

$$g_{\hat{k}} \equiv \frac{\dot{\hat{k}}_t}{\hat{k}_t} = s\hat{k}_t^{\alpha-1} - (\delta + n + g)$$

Balanced growth path (steady state)

Variables per effective labor converge to their steady state values

If $\dot{\hat{k}} = 0$ (or, equivalently, $g_{\hat{k}} = 0$) then:

$$s(\hat{k}^*)^{\alpha-1} = \delta + n + g$$

$$\hat{k}^* = \left(\frac{s}{\delta + n + g}\right)^{\frac{1}{1-\alpha}}$$

$$\hat{y}^* = \left(\frac{s}{\delta + n + g}\right)^{\frac{\alpha}{1-\alpha}}$$

Along the balanced growth path (BGP) variables per worker grow together with increases in technology:

$$y_t^* = A_t \hat{y}^* \quad \rightarrow \quad g_y^* \equiv \frac{(\dot{y_t})^*}{y_t^*} = \frac{\dot{A}_t \hat{y}^*}{A_t \hat{y}^*} = \frac{\dot{A}_t}{A_t} = g$$

And aggregate variables like aggregate capital and GDP grow at the sum of rates of increase in population and technology

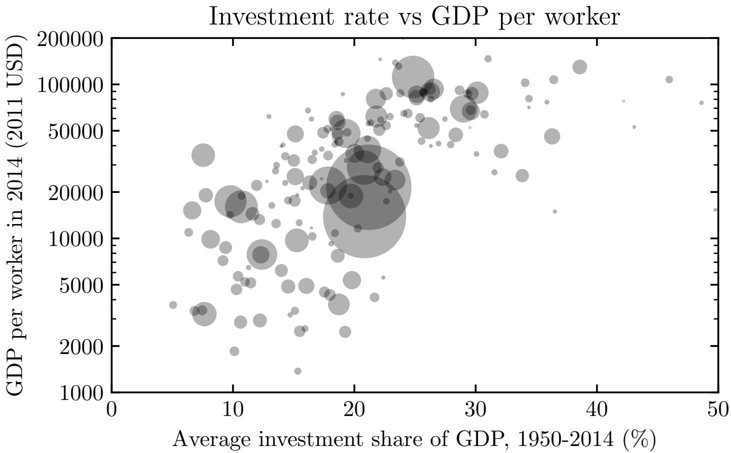
Comparative statics

Solow-Swan model predicts that the BGP level of GDP per worker:

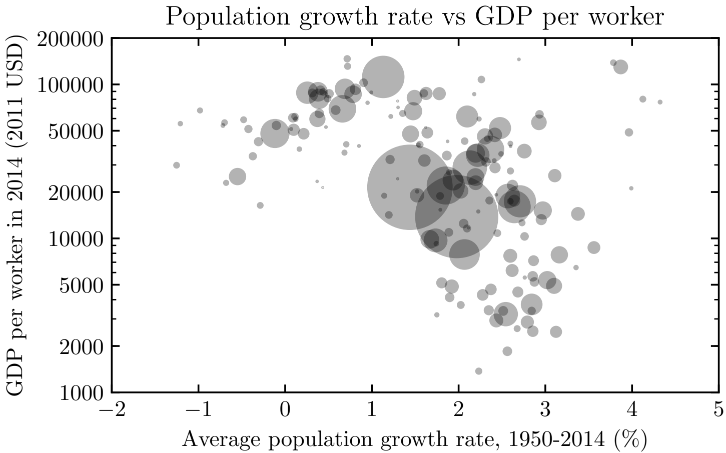
$$y_t^* = A_t \left(\frac{s}{\delta + n + g} \right)^{\frac{\alpha}{1-\alpha}}$$

is higher in countries with higher technology level A
and higher investment share of GDP s,
and lower in countries with higher population growth rate n

Investment share of GDP s vs GDP per worker y



Population growth rate n vs GDP per worker y



Transitional dynamics

We are also interested in the behavior of growth rates in GDP per worker outside the BGP

Start with growth rates of GDP per effective labor:

$$\hat{y}_t = \hat{k}_t^{\alpha} \quad \rightarrow \quad \ln \hat{y}_t = \alpha \ln \hat{k}_t \quad \rightarrow \quad g_{\hat{y}} = \alpha g_{\hat{k}}$$

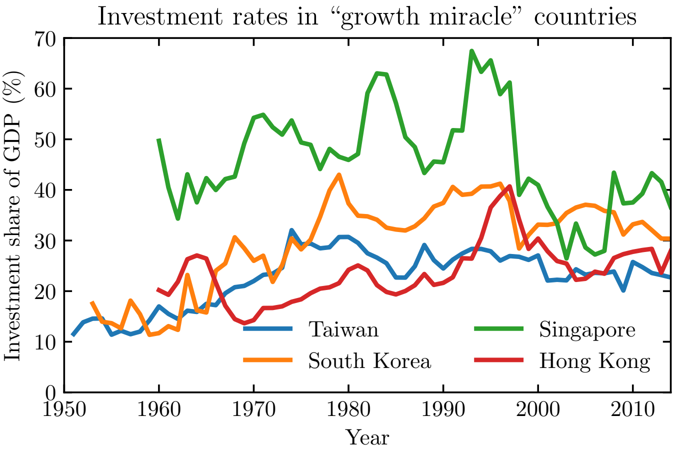
$$g_{\hat{y}} = \alpha \left[s\hat{k}_t^{\alpha-1} - (\delta + n + g) \right]$$

To obtain growth rate of GDP per worker, add the growth rate of technology g:

$$\begin{aligned} g_y &= \alpha \left[s\hat{k}_t^{\alpha-1} - (\delta + n + g) \right] + g \\ &= \alpha \left[s\hat{k}_t^{\alpha-1} - (\delta + n) \right] + (1 - \alpha)\, g \end{aligned}$$

An increase in s or a decrease in n **temporarily** increases the growth rate of GDP per worker. Note that even if higher g decreases $\hat{k}^*$, it increases the rate of growth of GDP per worker.

Investment share of GDP s in "growth miracle" countries

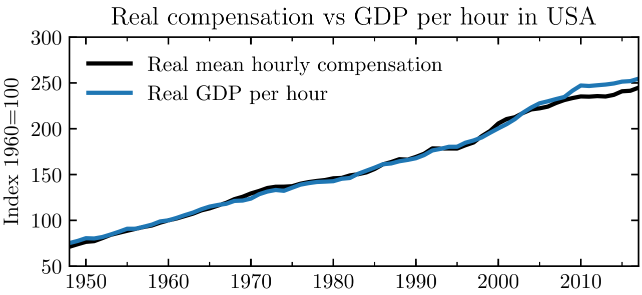


Factor payments once again

Using $\hat{k}^*$ as capital per effective labor along the BGP, let us revisit factor prices:

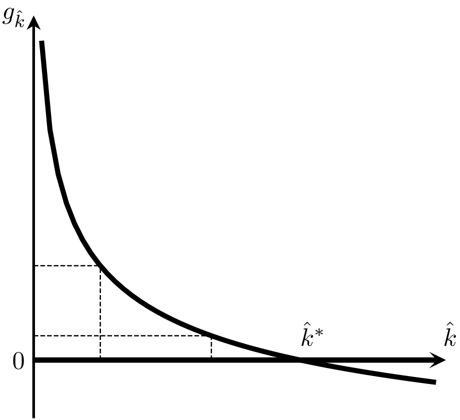
$$(r_t^K)^* = \alpha K_t^{\alpha-1} (A_t L_t)^{1-\alpha} = \alpha(\hat{k}^*)^{\alpha-1}$$
$$w_t^* = (1-\alpha) K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha} = (1-\alpha) A_t (\hat{k}^*)^{\alpha}$$

The model predicts that along the BGP the rate of return to capital is constant, while hourly wages grow at the same rate as GDP per hour:



Convergence

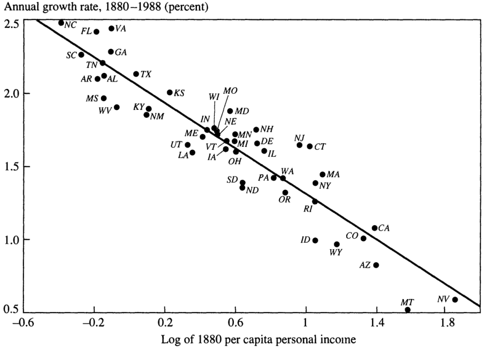
Solow-Swan model predicts that if countries have access to the same technology and share the same steady state, then ones that are initially poorer should grow faster:



Convergence: USA

We can observe convergence across individual states in USA:

Figure 1. Convergence of Personal Income across U.S. States: 1880 Income and Income Growth from 1880 to 1988

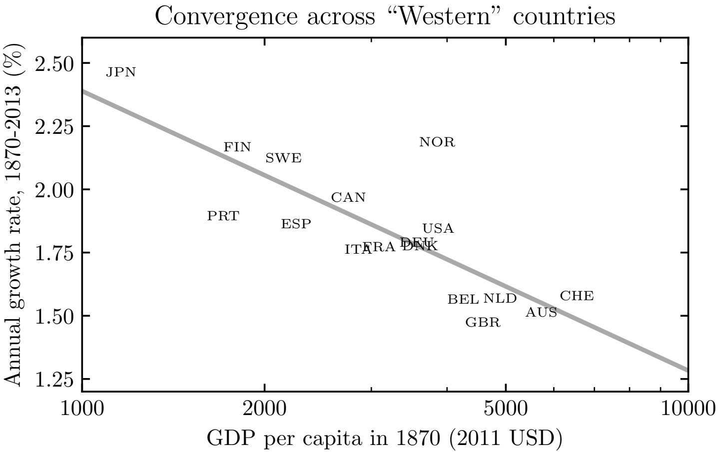


Sources: Bureau of Economic Analysis (1984), Easterlin (1960a, 1960b), and *Survey of Current Business*, various issues. The postal abbreviation for each state is used to plot the figure. Oklahoma, Alaska, and Hawaii are excluded from the analysis.

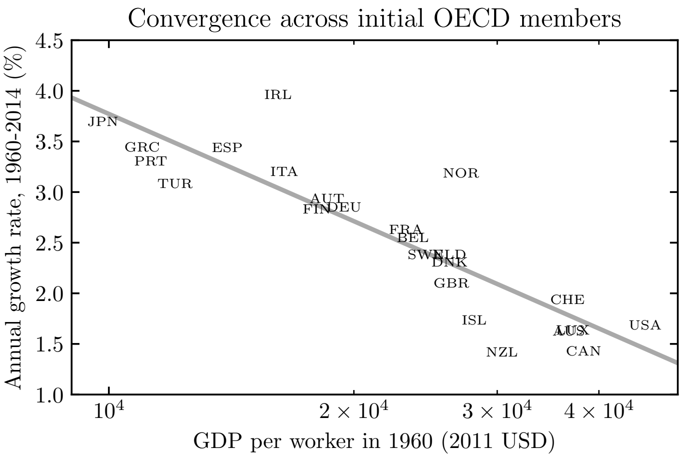
Barro and Sala-i-Martin (1991) *Convergence across States and Regions*

Convergence: "West"

We can observe convergence across "Western" countries (+ Japan):

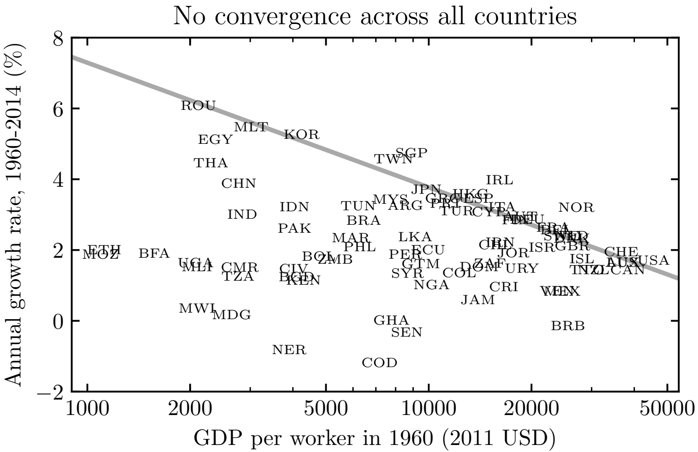


We can observe convergence across initial OECD members:

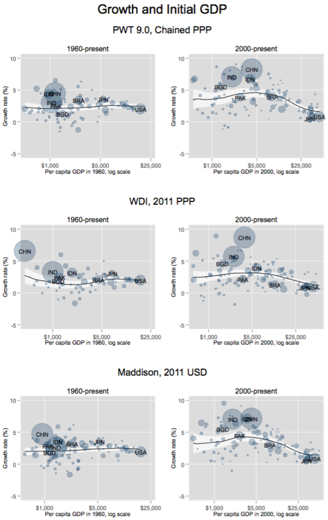


Convergence: conditional/club, but not absolute

In general it is not true that poorer countries grow faster:



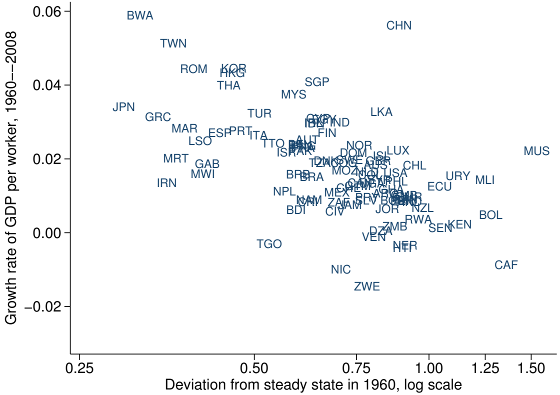
… although trends may have changed recently



Patel et al. (2018) *Everything You Know about Cross-Country Convergence Is (Now) Wrong*

Conditional convergence

But countries grow faster the further away they are from their own steady state:



Jones and Vollrath (2013) *Introduction to Economic Growth*

Speed of convergence

The model implies a relationship between the distance from steady state and the current rate of growth:

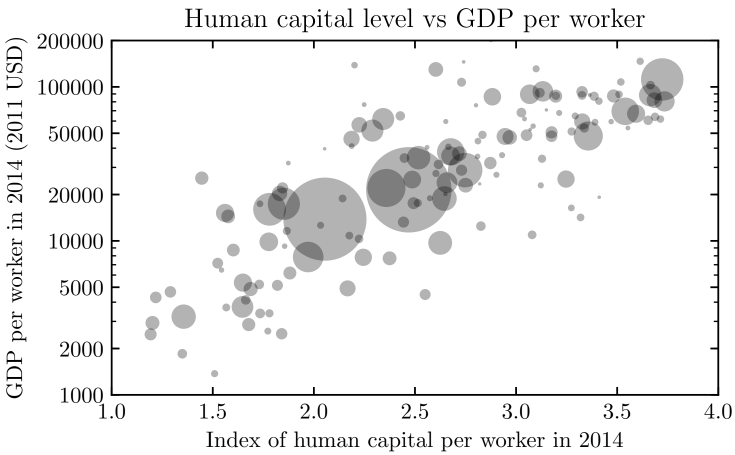
$$g_y \approx g + \underbrace{(1-\alpha)(\delta+n+g)}_{\gamma}(\log y_t^* - \log y_t)$$

Econometric studies both on individual countries and states within USA find that $\gamma \approx 0.02$, meaning that it takes about 35 years to close half of the gap between the current income and the steady state.

Given sensible parameter values: $\alpha = 0.33$, $\delta = 0.05$, $n = 0.01$, $g = 0.02$, the model generates $\gamma = 0.053$, implying that it would take about 13 years to close half of the gap, a very unrealistic number.

Adding human capital allows the model to assign lower weight to raw labor and be consistent with slow convergence.

Human capital per capita h vs real GDP per worker y



Human capital augmented Solow model

The production function that accounts for human capital:

$$Y_t = K_t^{\alpha} H_t^{\beta} \left(A_t L_t\right)^{1-\alpha-\beta}$$
$$H_t = h\left(u_t\right) L_t$$

where β is the human capital share of income and u are average years of schooling. Benchmark empirical estimates on returns to schooling are expressed via the h function:

$$\log h\left(u\right) = \begin{cases} 0.134 \cdot u & \text{if } u \leq 4 \\ 0.134 \cdot 4 + 0.101 \cdot (u-4) & \text{if } 4 < u \leq 8 \\ 0.134 \cdot 4 + 0.101 \cdot 4 + 0.068 \cdot (u-8) & \text{if } u > 8 \end{cases}$$

The estimates capture the empirical regularity that schooling boosts individuals' wages. Wages contain not only rewards to raw labor, but also to human capital.

Human capital augmented Solow model

The level of GDP per worker along the BGP:

$$y_t^* = A_t \left(\frac{s_k}{\delta + n + g} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{s_h}{\delta + n + g} \right)^{\frac{\beta}{1-\alpha-\beta}}$$

where s_k and s_h denote saving/investment rates in physical and human capital, assuming that human capital accumulates according to:

$$\dot{H}_t = s_h Y_t - \delta H_t$$

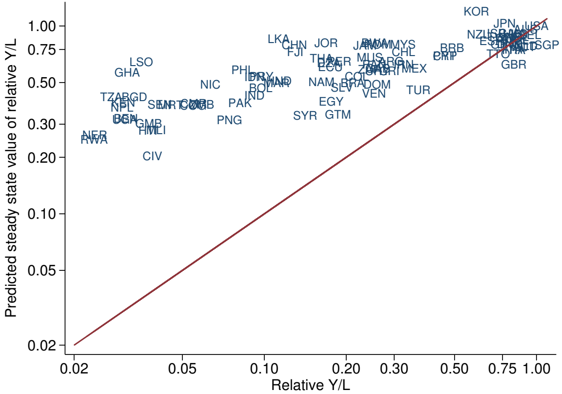
The growth rate of GDP per worker can now be expressed as:

$$g_y \approx g + (1 - \alpha - \beta)(\delta + n + g)(\log y_t^* - \log y_t)$$

Empirical estimates of the income share of human capital are consistent with rate of convergence $\gamma \approx 0.02$.

Fit of human capital-augmented Solow model

Suggests that poor countries “should” be richer:



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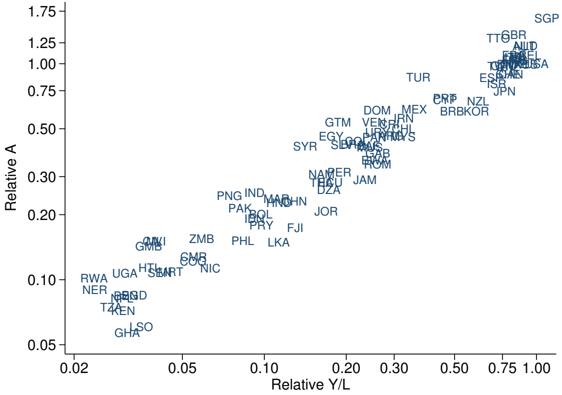
Solow residual: accounting for technology differences

There are also significant differences in technology across countries:



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Takeaway

- Countries can achieve higher balanced growth path level of y if they accumulate more physical and human capital
- Just as important as accumulation is technology adoption
- Long run growth stems from improvements in technology
- Did not touch on "deep" causes of growth
  – we treated many choice variables as exogenous parameters

In search for fundamental causes of growth

What dictates the investment rate in new capital s_k?
The investment rate in human capital s_h / years of schooling u?
The adoption/discovery of new technologies?

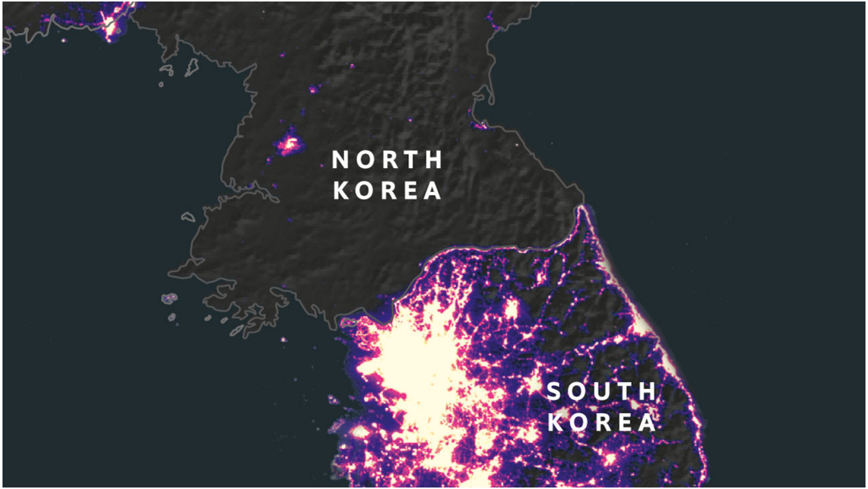
- Geography: easy access to certain resources
- Culture: certain cultures value savings or education more
- Institutions: rules of the economic game

Olson (1996) compares places identical in geography and culture:

- North vs. South Korea
- East vs. West Germany
- China vs. Taiwan and Hong Kong

They differ in institutions

North and South Korea at night



Economist (2019)

Institutions

How do we think of institutions?

- Property rights: the ability to keep what you earn in profits, savings, wages
- Transactions: the ability to easily trade assets, sign contracts
- Enforcement: contracts and laws are consistently enforced over time

“Good” institutions will encourage people to make long-run investments because they can keep what they earn and the rules won’t arbitrarily change over time

In most poor countries it is not trivial to start a new firm, invest in new equipment, adopt a new technology

Doing Business

World Bank data on how long it takes to set up businesses, and various associated costs (licenses, fees, etc.):

- USA: 6 days and equivalent 1.4% of average income
- India: 29 days and equivalent 50% of average income
- Honduras: 14 days and equivalent 63% of average income
- Nigeria: 34 days and equivalent 70% of average income

Corruption

Shleifer and Vishny (1993):

To invest in a Russian company, a foreigner must bribe every agency involved in foreign investment, including the foreign investment office, the relevant industrial ministry, the finance ministry, the executive branch of the local government, the legislative branch, the central bank, the state property bureau, and so on. The obvious result is that foreigners do not invest in Russia. Such competing bureaucracies, each of which can stop a project from proceeding, hamper investment and growth around the world, but especially in countries with weak governments.

Social infrastructure

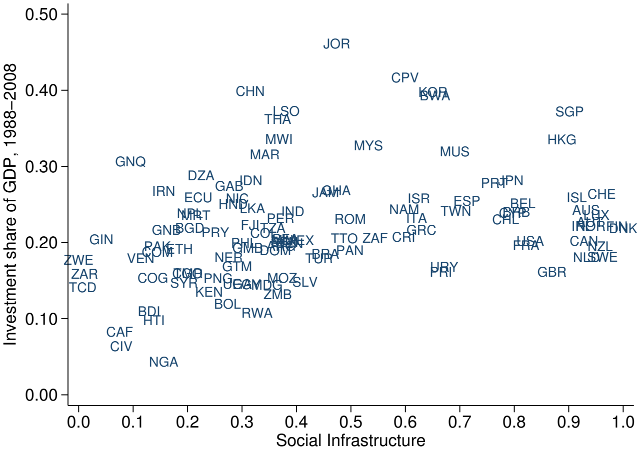
Not possible to measure institutions' quality directly

A measure of "social infrastructure" that captures six dimensions of governance from the World Bank:

- Accountability of politicians
- Political stability
- Government effectiveness
- Regulatory quality
- Rule of law
- Control of corruption

Overall index runs from 0 (worst) to 1 (best)

Social infrastructure and investment



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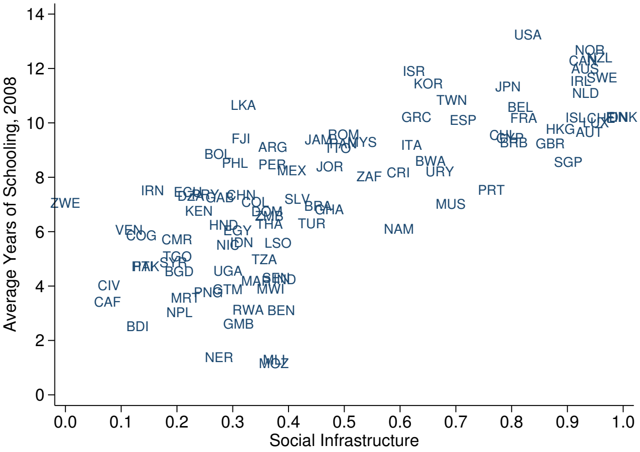
Social infrastructure and human capital



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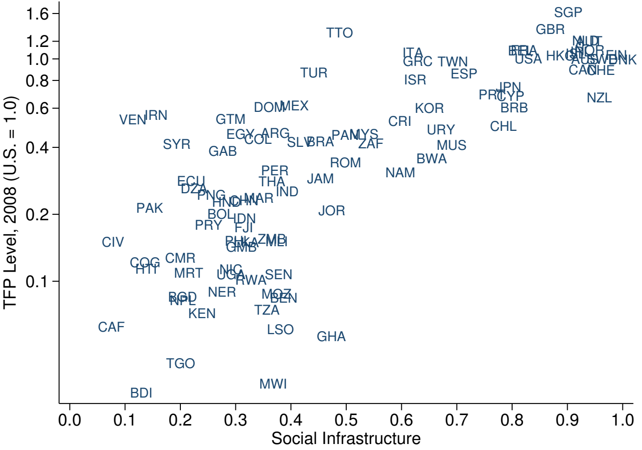
Social infrastructure and technology



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Choosing institutions

If good institutions generate big economic gains, why don't all countries have them?

- Institutions are human-designed and malleable
- Can't we bargain with each other to get good institutions?
- Can't elites take smaller slice of a bigger pie?

Example: offer beauracrats higher salaries in exchange for not taking bribes.
Acemoglu and Robinson (2012): this won't work because of commitment problems:

- The beauracrats will take the salary, then still ask for a bribe
- Elites cannot credibly promise to take smaller slice
- Non-elites cannot credibly promise not to replace elites

Institutions appear to be very persistent, and historically contingent

A depressing, but instructive CGP Grey video The Rules for Rulers