Marcin Bielecki, Advanced Macroeconomics IE, Spring 2025
Homework 7, deadline: June 3, 4:45 PM

Problem 1

Consider the special case of the RBC model. Households maximize expected utility subject to the standard budget constraint:

$$\max_{\{c_{t+j},\, h_{t+j},\, k_{t+j+1}\}_{j=0}^{\infty}} \quad U = \mathrm{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\ln c_{t+j} - \psi \frac{h_{t+j}^{1+\varphi}}{1+\varphi} \right) \right]$$

$$\text{subject to} \quad c_{t+j} + k_{t+j+1} = w_{t+j} h_{t+j} + (1 + r_{t+j}) \, k_{t+j} + d_{t+j}$$

Firms maximize profits subject to the Cobb-Douglas production function:

$$\max \quad d_t = y_t - w_t h_t - (r_t + \delta) \, k_t$$

$$\text{subject to} \quad y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$$

where $\delta = 1$ (capital deprecates fully). The technology variable z evolves according to the process:

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}$$

(a) Derive the first order conditions of the households and their optimality conditions.

(b) Derive the first order conditions of the firm and expressions for prices in equilibrium.

(c) Find the steady state of the system, assuming that $h^* = 1$. Find the corresponding value of ψ.

(d) Assuming that household behavior can be expressed as $c_t = (1 - s) \, y_t$ where s is a constant, find the value of s as a function of model parameters.

(e) Show that $h_t = h^*$. Find the expression for k_{t+1} as a function of variables at time t.

Problem 2

Consider the search model of unemployment presented in the lecture. Using comparative statics analysis, explain how each of the following events affect changes in real wages, labor market tightness (vacancies to unemployment ratio) and unemployment rate.

(a) An increase in the separation rate

(b) An increase in the marginal product of an employee

(c) An increase in the real interest rate

(d) An improvement in the matching efficiency

Problem 3

In class we considered a model where permanent changes to marginal product of an employee reduced the unemployment rate. This would imply that with trend productivity growth unemployment would disappear over time. Let's modify our model so that it is consistent with stationary unemployment in face of trend productivity growth. Suppose that the flow cost of a vacancy κ and the imputed value of free time b are functions of the trend wage rate w (instead being exogenous). In particular, assume that $\kappa_t = \kappa_0 w_t$ and $b_t = b_0 w_t$.

(a) Determine the formula for job creation and wage setting along the balanced growth path (steady state).

(b) How do labor market tightness and wages along the balanced growth path react to productivity changes?

(c) Does a continuous growth of productivity lead to a decrease in the long run unemployment rate?