Marcin Bielecki, Advanced Macroeconomics IE, Spring 2025
Homework 2, deadline: March 11, 4:45 PM

Problem 1

In this problem we will show how changes in taxation can change consumption of an agent if she is borrowing-constrained. The agent's income in the first period is 1/2 of her income in the second period. Assume that $\beta(1+r) = 1$ and consider the following utility maximization problem:

$$\max_{c_1,\, c_2,\, a} \quad U = \ln c_1 + \beta \ln c_2$$
$$\text{subject to} \quad c_1 + a = y/2$$
$$c_2 = y + (1+r)\, a$$

(a) Using the Lagrangian method find optimal c_1, c_2 and a. Are the assets at end of period 1 positive or negative?

(b) Assume now that the agent cannot borrow and faces an additional non-borrowing constraint: $a \geq 0$. What are the optimal values of c_1, c_2 and a in this situation?

(c) Show graphically in the (c_1, c_2) space the problem of the agent and especially show that the agent would be on a higher indifference curve were she allowed to borrow.

(d) Suppose that the government arranges a transfer v to this agent by issuing bonds. In the future, the government will tax the agent to be able to buy back the bonds. The new constraints of the agent are:

$$c_1 + a_2 = y/2 + v \qquad c_2 = y + (1+r)\, a - (1+r)\, v \qquad a \geq 0$$

What is the impact of the government transfer on the agent's first period consumption?

(e) Show graphically in the (c_1, c_2) space the effects of the transfer scheme from (d).

Problem 2

Consider a simplified version of the two asset model by Kaplan et al. (2014). The solution of the household problem is split into two steps: in the first step the household chooses holdings of the liquid asset between periods 1 and 2, and in the second step the household makes its initial asset allocation choice between the liquid asset held at the beginning of period 1 and the illiquid asset with positive real return $r > 0$ that can only be accessed in period 2.

(a) Solve for the optimal holdings of liquid asset m_2, as well as optimal consumption expenditures c_1 and c_2, while taking m_1 and a as given. Under which condition the household chooses $m_2 > 0$?

$$\max_{c_1,\, c_2,\, m_2} \quad U = \ln c_1 + \ln c_2 \qquad \text{subject to} \quad c_1 + m_2 = y_1 + m_1 \quad \text{and} \quad c_2 = y_2 + m_2 + (1+r)\, a$$

(b) Consider the case of $m_2 > 0$. Use your results from (a) for optimal c_1 and c_2 and solve the problem of allocating initial wealth W between illiquid asset a and liquid m_1.

$$\max_{m_1,\, a} \quad U = \ln c_1 + \ln c_2 \qquad \text{subject to} \quad m_1 + a = W$$

(c) Consider the case when condition for $m_2 > 0$ is not satisfied and so the household sets $m_2 = 0$. Solve the following asset allocation problem. Under which condition the household chooses $a > 0$ (this household is "Wealthy Hand-to-Mouth", W-HtM)? What are its consumption expenditures?

$$\max_{m_1,\, a} \quad U = \ln(y_1 + m_1) + \ln(y_2 + (1+r)\, a) \qquad \text{subject to} \quad m_1 + a = W$$

(d) Consider the last case when the household sets $m_2 = 0$ and $a = 0$ (this household is "Poor Hand-to-Mouth", P-HtM). What are its consumption expenditures?

(e) Assume now that after the initial asset allocation is made, the household receives an unexpected government transfer v in period 1 and once again solves for optimal liquid assets m_2. Under which condition the P-HtM household will spend the entire transfer (i.e. would choose $m_2 < 0$)? Under which condition the W-HtM household will spend the entire transfer?

$$\max_{c_1,\, m_2} \quad U = \ln c_1 + \ln\left(y_2 + m_2 + (1+r)\, a^*\right) \qquad \text{subject to} \quad c_1 + m_2 = y_1 + v + (1 - a^*)$$

Problem 3

Suppose that we have an agent that lives for T periods. Her lifetime utility is:

$$U = \sum_{t=0}^{T-1} \beta^t \ln c_t$$

The agent starts with no wealth, $a_0 = 0$, and plans to die with no wealth as well, $a_T = 0$.
Over her lifetime, the agent faces a sequence of budget constraints:

$$c_t + a_{t+1} = y_t + (1+r)\, a_t \qquad \text{for all } t = 0, 1, 2, \ldots, T-1$$

(a) Using the Lagrangian method find the optimal initial level of consumption c_0.

(b) Assume that $r = 0$, $\beta = 1$, and that the agent's labor income $y_t = y$ for $t = 0, 1, 2, \ldots, R-1$ and 0 afterwards. Determine the level of consumption in all periods.

(c) Find the level of agent's assets at the beginning of period R, a_R.

(d) Imagine now that the agent expects to live longer, $T' > T$. How does it affect c_0 and a_R?

(e) Imagine now that the agent will retire later, $R' > R$. How does it affect c_0 and assets at the beginning of period R'?

Problem 4

Suppose an economy with households who live for two time periods. At the same time, there are people who are young and old (this is the overlapping generations setup that we'll encounter later in the course). There is no production; each agent receives an endowment in each period. However, agents are able to buy government bonds when they are young and redeem them when they are old. Let superscript y denote the value of a variable when an agent is young and superscript o denote the value of a variable when an agent is old. There are equally many young and old agents. Young agents born at time t solve the following problem:

$$\begin{aligned}
\max_{c_t,\, c_{t+1},\, b_{t+1}} \quad & U_t = \ln c_t^y + \beta \ln c_{t+1}^o \\
\text{subject to} \quad & c_t^y + b_{t+1} = y^y - \tau_t \\
& c_{t+1}^o = y^o - \tau_{t+1} + (1+r)\, b_{t+1}
\end{aligned}$$

where $\beta = 1/(1+r)$ and $y^y - \tau_t > y^o - \tau_{t+1}$. Old agents in period t (who were young in period $t-1$) consume all resources they have at hand:

$$c_t^o = y^o - \tau_t^o + (1+r)\, b_t$$

Suppose the government reduces the taxes in period t by issuing bonds, and then raises taxes in period $t+1$ to be able to redeem the bonds: $\Delta\tau_t = -\Delta T$ and $\Delta\tau_{t+1} = (1+r)\,\Delta T$.

(a) Find the optimal levels of consumption of agents who are young in period t.

(b) What would be the value of aggregate period t consumption $C_t = c_t^y + c_t^o$, if the tax change was not implemented?

(c) What happens to aggregate period t consumption if taxes are changed at the start of period t? Calculate $\Delta C_t / \Delta T$.

(d) Given (b) and (c), does the Ricardian equivalence hold? Why or why not?