University of Warsaw
Faculty of Economic Sciences

Growth facts

Advanced Macroeconomics

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Brief review on GDP

Gross domestic product (GDP) is the current dollar value of all final goods and services that are produced within a country within a given period of time

Real GDP: Nominal GDP adjusted for inflation
(and differences in relative prices across countries via PPP adjustment)

(Real) GDP measurement:

(1) Expenditure approach: $Y_t = C_t + I_t + G_t + NX_t$

(2) Income approach: $Y_t = w_t L_t + (r_t + \delta) K_t + D_t + T_t$

(3) Value added approach: $Y_t = Y_{1t} + Y_{2t} + \ldots + Y_{Mt} = F(K_t, L_t, A_t)$

Real GDP per person: Y_t/N_t

GDP per person and welfare

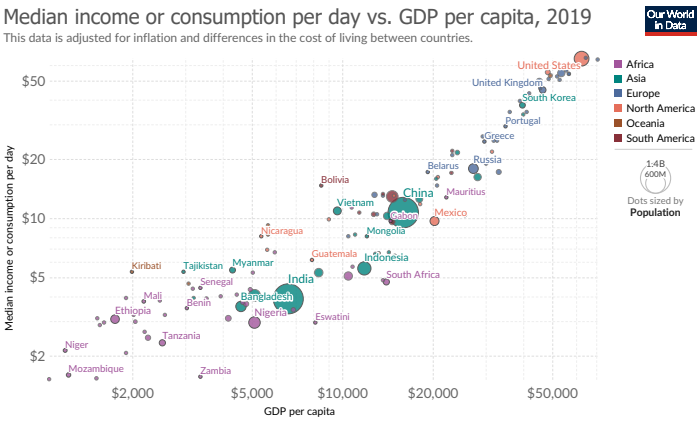
GDP per person is not designed to measure welfare, but it's a useful summary statistic

GDP per person ignores distribution of income within a country

Comparing GDP per person across countries is not trivial in practice:

- You have to convert between currencies
- Countries have different relative prices for goods
- Large uncertainty in comparing real GDP across countries and over time: Johnson et al. (2013) *Is newer better? Penn World Table Revisions and their impact on growth estimates*

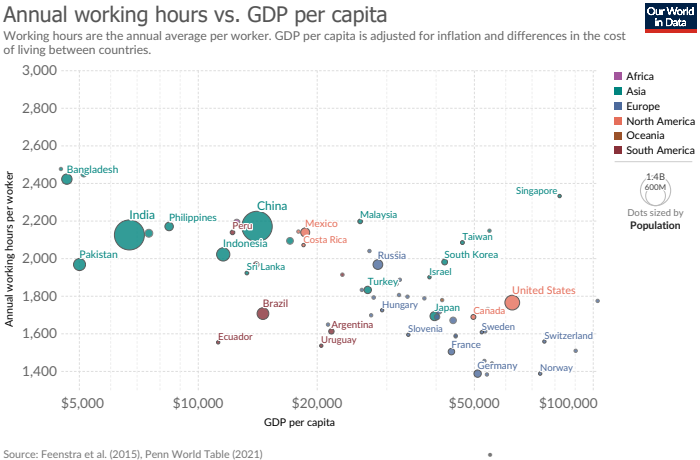
GDP per person and welfare: consumption



Source: World Bank Poverty and Inequality Platform (2022), Data compiled from multiple sources by World Bank
Note: This data is expressed in international-$ at 2017 prices. Depending on the country and year, median data relates to income measured after taxes and benefits, or to consumption, per capita.

Our World in Data

GDP per person and welfare: hours worked

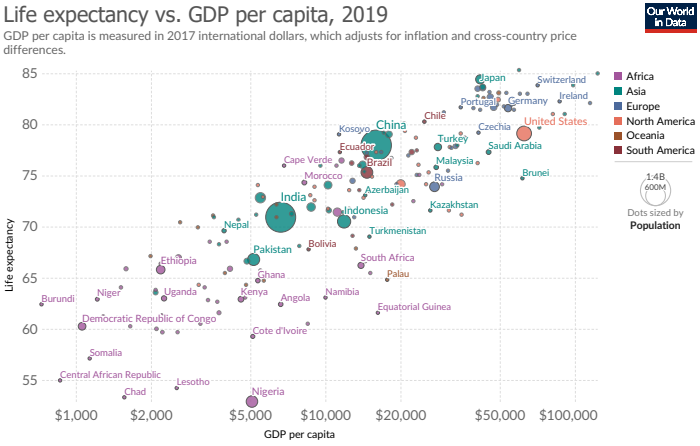


Source: Feenstra et al. (2015), Penn World Table (2021)

Note: This data is expressed in international-$ at 2017 prices, using multiple benchmark years to adjust for differences in the cost of living between countries over time.

Our World in Data

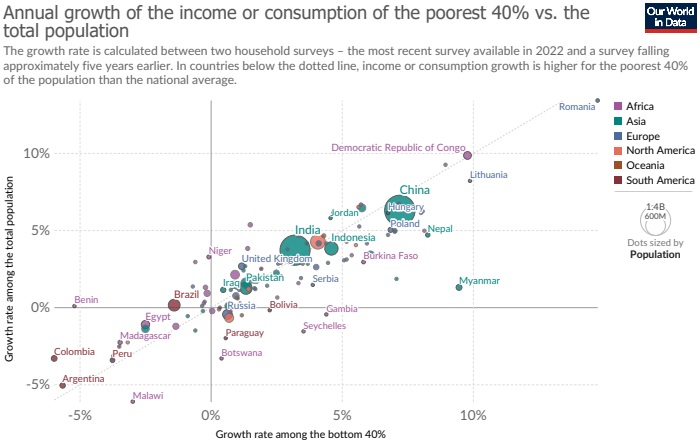
GDP per person and welfare: life expectancy



Source: UN, World Population Prospects (2022); Data compiled from multiple sources by World Bank

Our World in Data

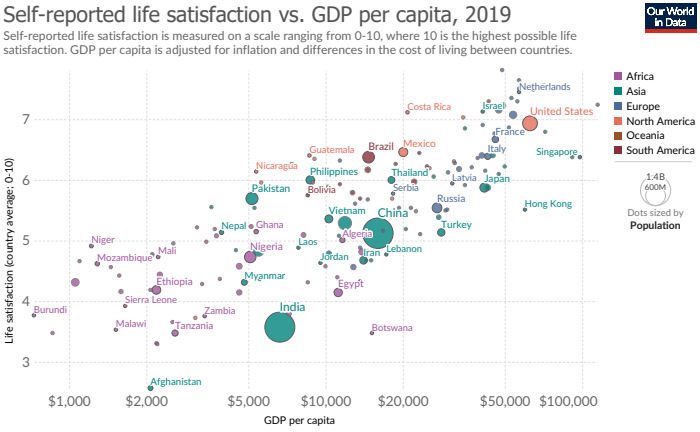
Growth in average income vs income of bottom 40%



Source: World Bank
Note: Depending on the country and year, the data relates to income measured after taxes and benefits, or to consumption, per capita.

Our World in Data

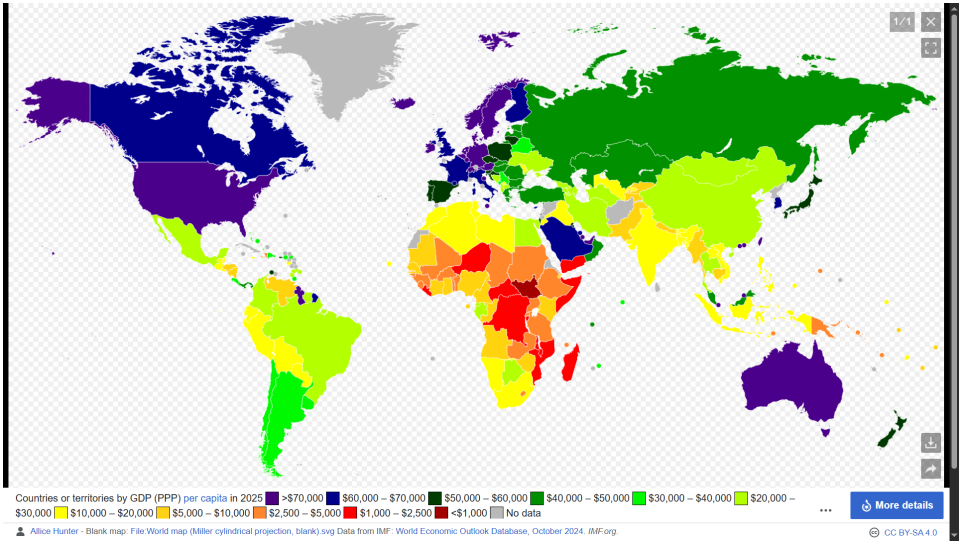
GDP per person and welfare: life satisfaction



Source: World Happiness Report (2023), Data compiled from multiple sources by World Bank
Note: GDP per capita is expressed in international-$ at 2017 prices.

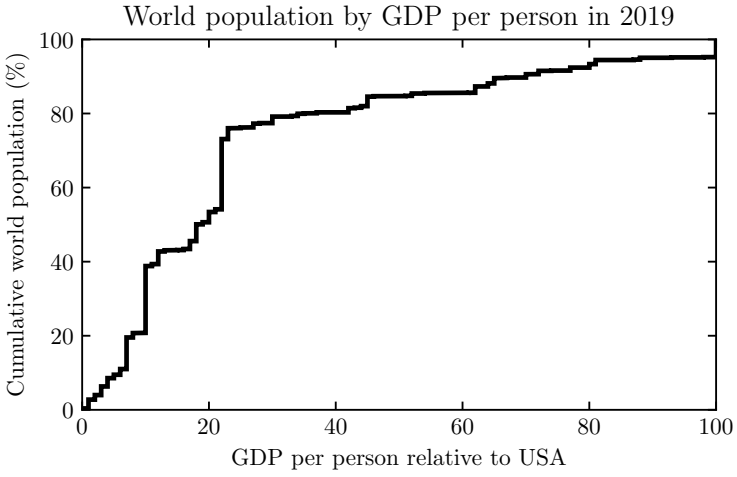
Our World in Data

There is enormous variation in GDP per person across economies

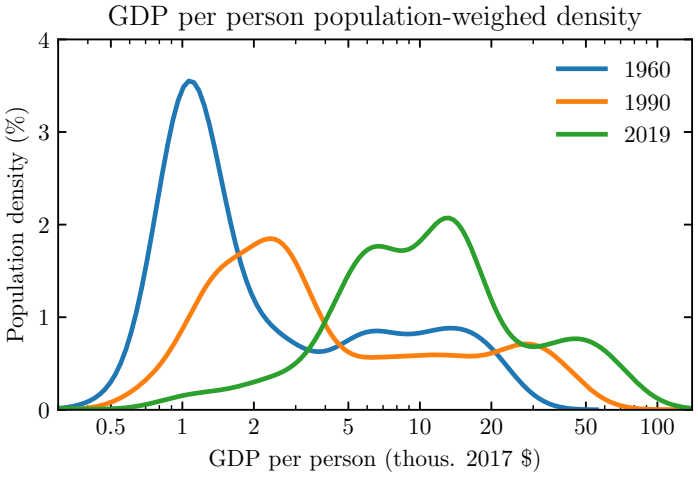


Wikipedia, IMF

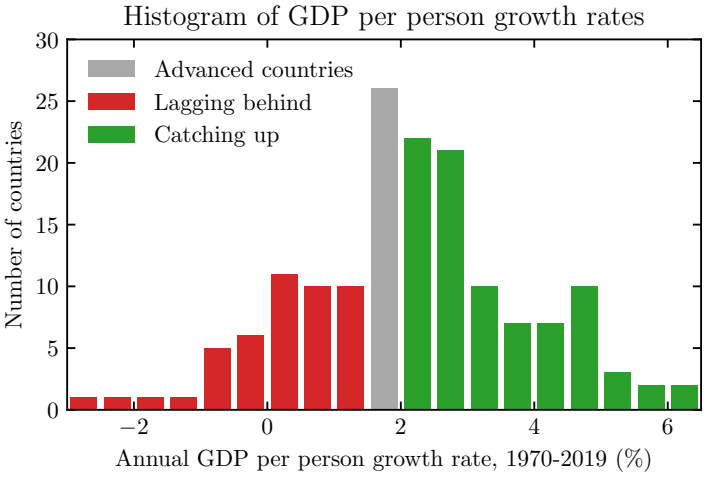
There is enormous variation in GDP per person across economies



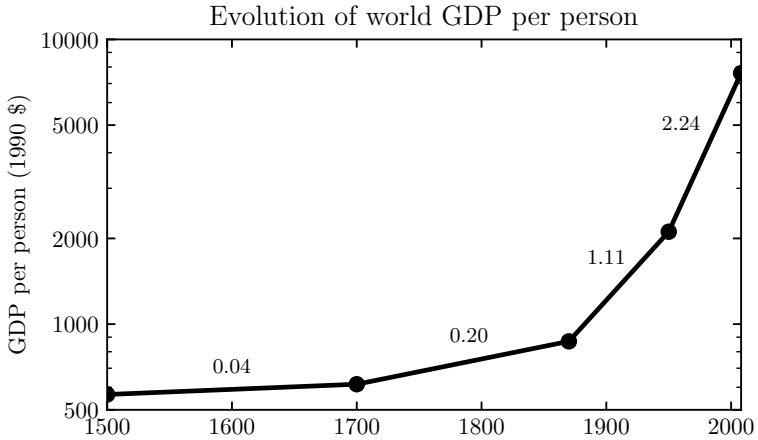
Less variation now than in the previous decades



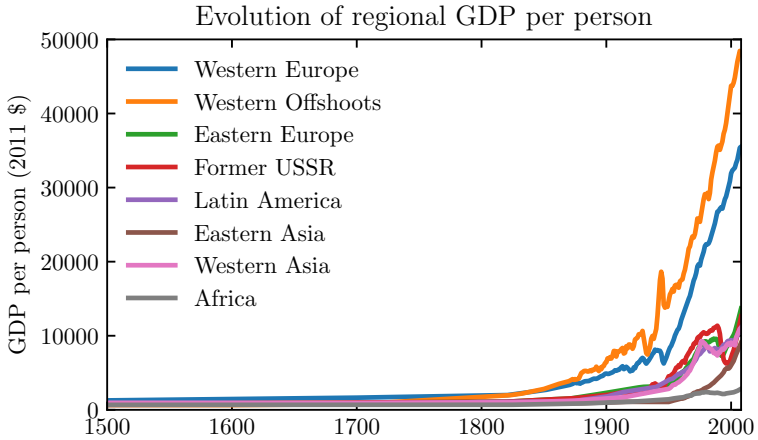
Rates of economic growth vary substantially across countries



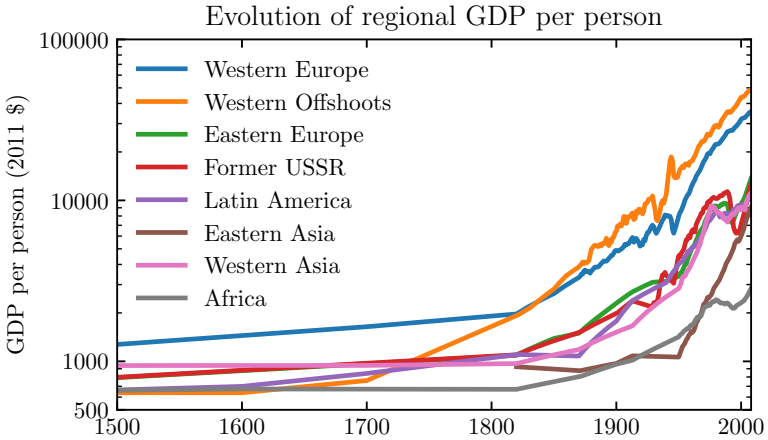
Economic growth is a “recent” phenomenon



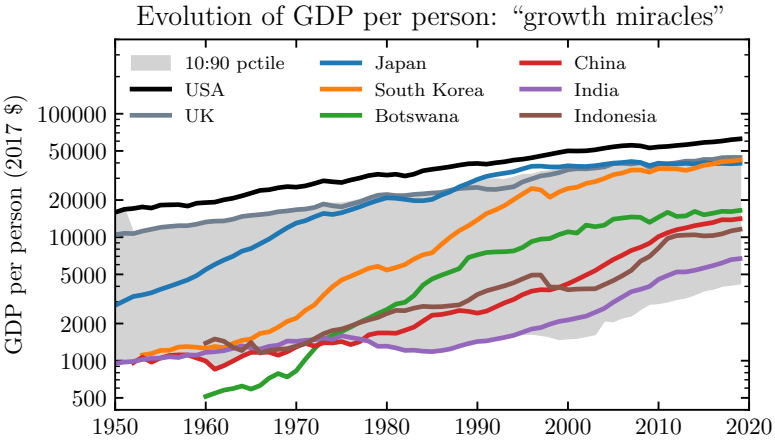
Until the 19th century everyone was similarily poor



Growth took off with different timing across world regions



Countries can go from being “poor” to being “rich”

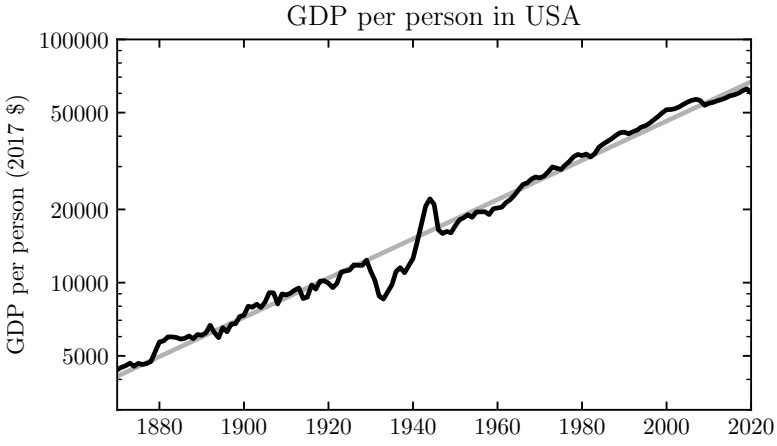


Kaldor's stylized facts

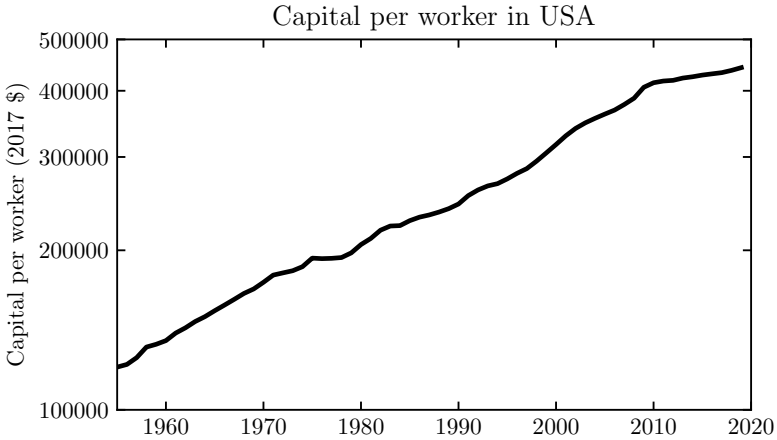
Kaldor (1957, 1961)**: In the USA (and other developed countries):**

1. GDP per person sustainably grows at positive rate
2. Physical capital per worker grows over time
3. The rate of return to capital is not trending
4. The ratio of physical capital to output is nearly constant
5. The shares of labor and physical capital in national income are nearly constant
6. Real wages grow over time

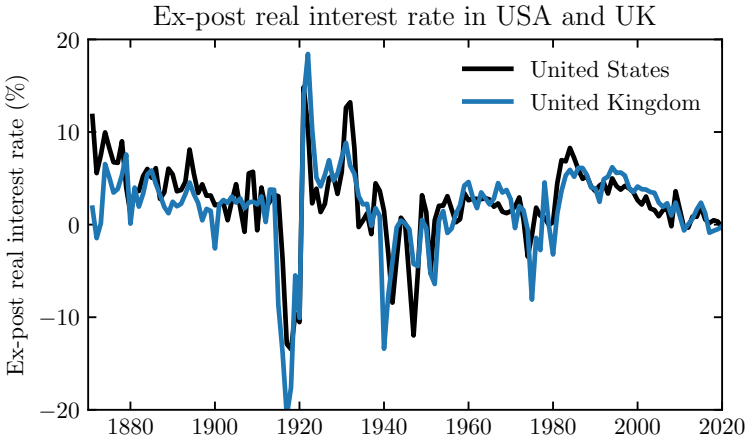
K1: GDP per person sustainably grows at positive rate



K2: Physical capital per worker grows over time



K3: The rate of return to capital is not trending



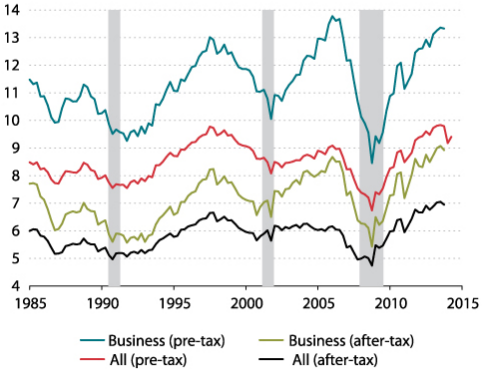
K3: The rate of return to capital is not trending

DeLong (2015)

K3: The rate of return to capital is not trending

Figure 2

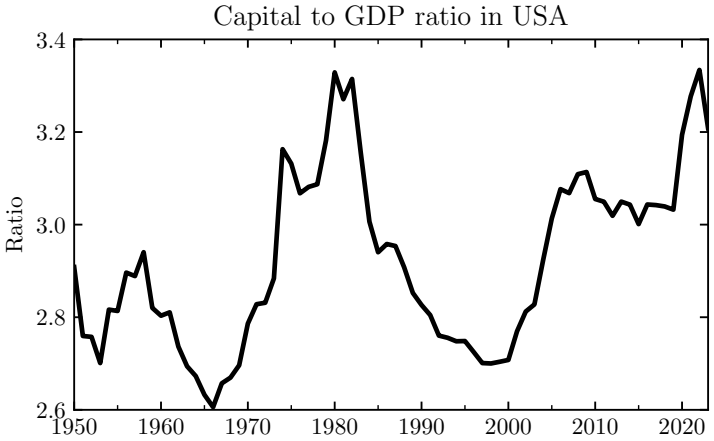
Real Returns on Capital (percent)



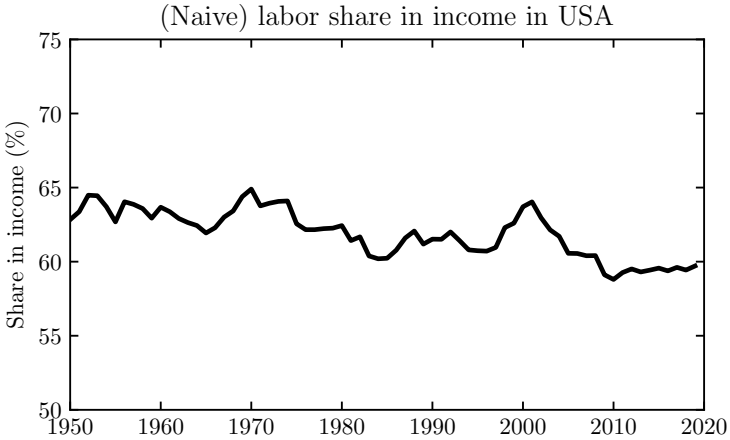
SOURCE: Authors' calculations; for details, see Gomme, Ravikumar, and Rupert (2011).

Gomme, Ravikumar and Rupert (2015) *Secular Stagnation and Returns on Capital*

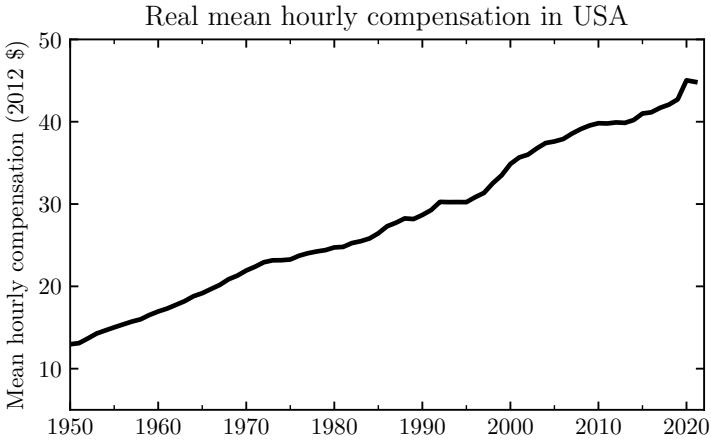
K4: The ratio of physical capital to output is nearly constant



K5: The labor share of national income is nearly constant



K6: Real wages grow over time



Explaining growth

We want to explain:

- Why some countries are “poor” and other “rich”?
- Why some countries that were previously “poor” became “rich”?
- Why not all “poor” countries catch up to “rich” countries?
- Why do “rich” countries still grow?

Solow-Swan model: refer to the notes

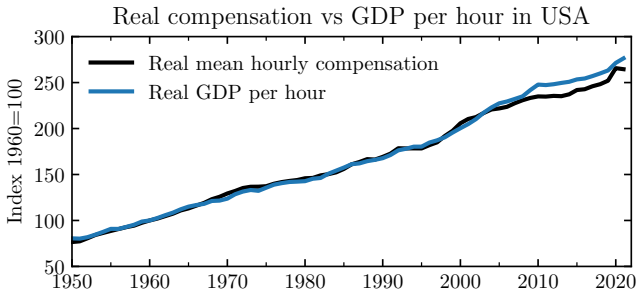
Factor payments once again

Using $\hat{k}^*$ as capital per effective labor along the BGP, let us revisit factor prices:

$$(r_t^K)^* = \alpha K_t^{\alpha-1} \left(A_t L_t\right)^{1-\alpha} = \alpha(\hat{k}^*)^{\alpha-1}$$

$$w_t^* = (1-\alpha)\, K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha} = (1-\alpha)\, A_t (\hat{k}^*)^{\alpha}$$

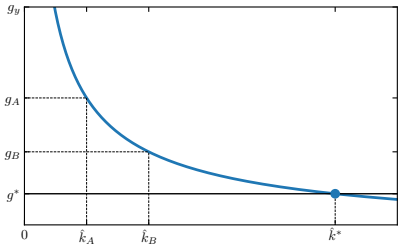
The model predicts that along the BGP the rate of return to capital is constant, while hourly wages grow at the same rate as GDP per hour:

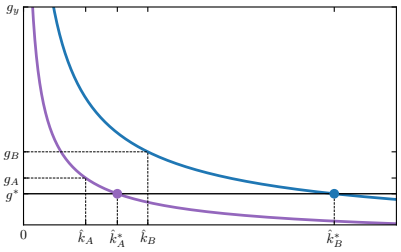


Convergence

Solow-Swan model predicts that if countries share the same steady state (balanced growth path), initially poorer countries will exhibit higher growth rates

If countries differ in their steady states, an initially richer country can still grow faster (conditional convergence)



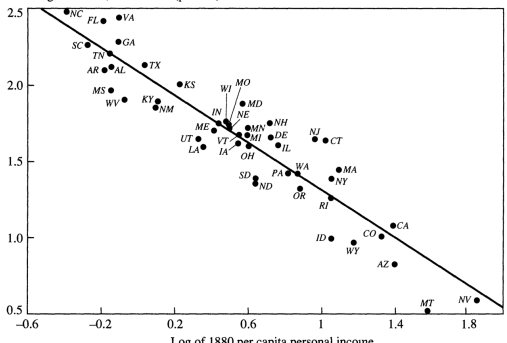


Convergence: USA

We can observe (absolute) convergence across individual states in USA:

Figure 1. Convergence of Personal Income across U.S. States: 1880 Income and Income Growth from 1880 to 1988

Annual growth rate, 1880–1988 (percent)



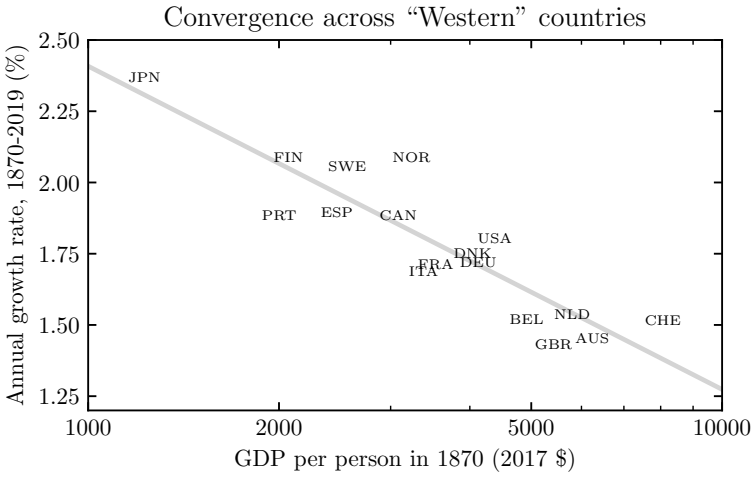
Log of 1880 per capita personal income

Sources: Bureau of Economic Analysis (1984), Easterlin (1960a, 1960b), and *Survey of Current Business*, various issues. The postal abbreviation for each state is used to plot the figure. Oklahoma, Alaska, and Hawaii are excluded from the analysis.

Barro and Sala-i-Martin (1991) *Convergence across States and Regions*

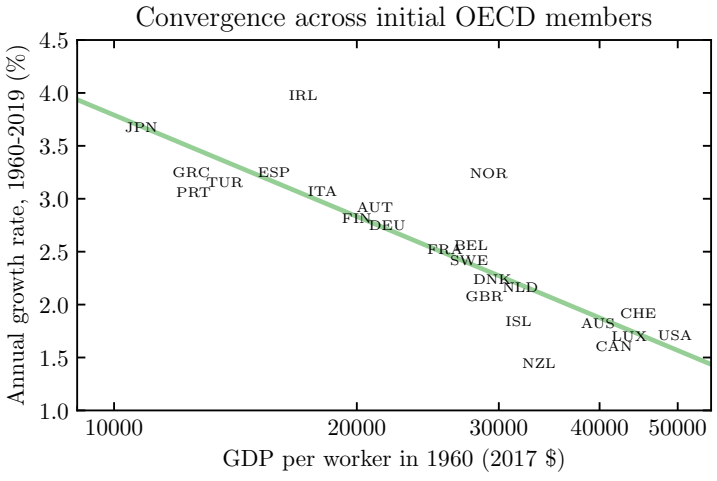
Convergence: “West”

We can observe (absolute) convergence across “Western” countries (+ Japan):



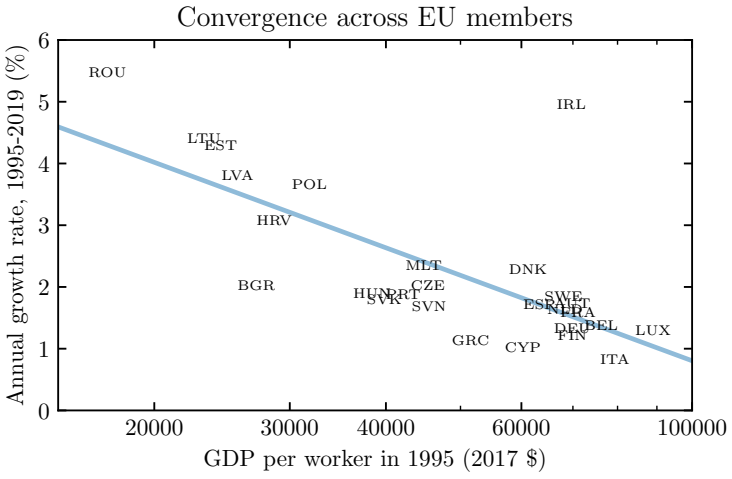
Convergence: OECD

We can observe (absolute) convergence across initial OECD members:



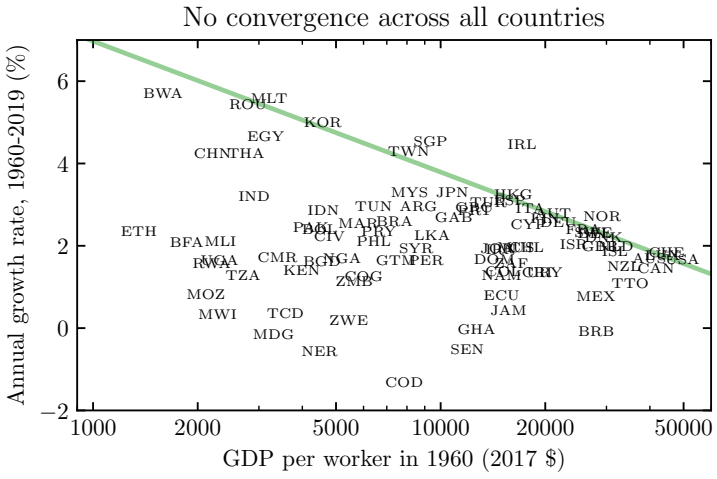
Convergence: EU

We can observe (absolute) convergence across EU countries:

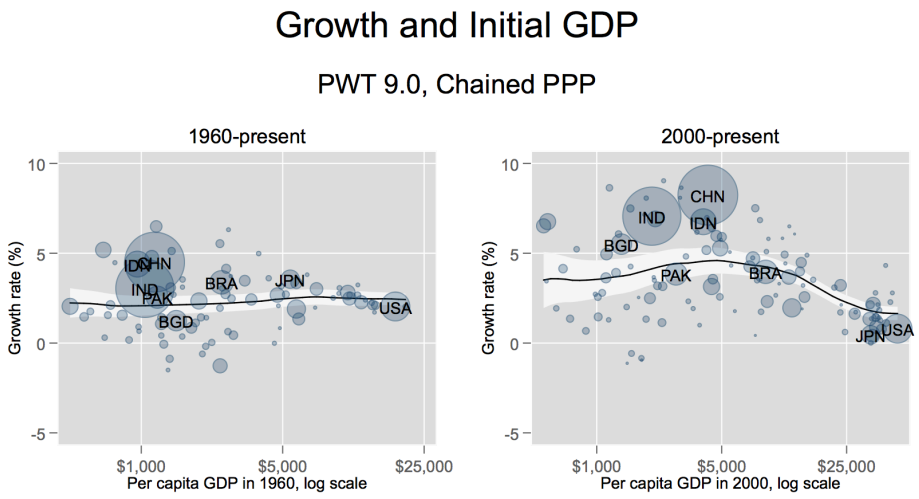


Convergence: conditional/club, but not absolute

But in general it is not true that poorer countries grow faster:



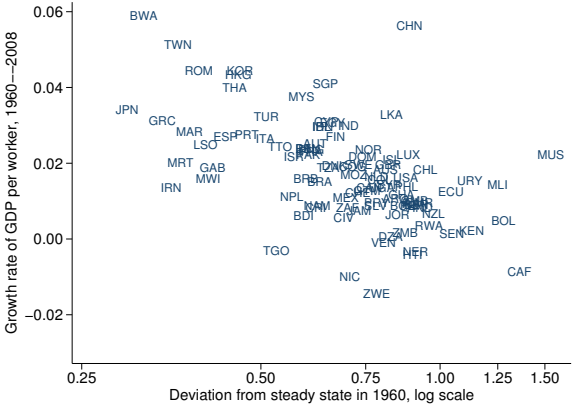
… although trends may have changed recently



Patel et al. (2018) *Everything You Know about Cross-Country Convergence Is (Now) Wrong*

Conditional convergence

Countries grow faster the further away they are from their own steady state:



Jones and Vollrath (2013) *Introduction to Economic Growth*

Mankiw, Romer and Weil (1992)

GDP per worker along the balanced growth path

$$y_t^* = A_t \left(\frac{s}{\delta + n + g} \right)^{\frac{\alpha}{1-\alpha}}$$

In logarithms

$$\ln y_t^* = \ln \left(A_0 \left(1+g\right)^t \right) + \frac{\alpha}{1-\alpha} \ln s - \frac{\alpha}{1-\alpha} \ln \left(\delta + n + g\right)$$

Econometric model

$$\ln y_{it} = \underbrace{g \cdot t}_{\text{const.}} + \frac{\alpha}{1-\alpha} \ln s_i - \frac{\alpha}{1-\alpha} \ln \left(\delta + n_i + g\right) + \underbrace{\ln A_{i0} + \left(\ln y_{it} - \ln y_{it}^*\right)}_{\text{error term}}$$

Restricted regression

$$\ln y_{it} = gt + \frac{\alpha}{1-\alpha} \left[\ln s_i - \ln \left(\delta + n_i + g\right)\right] + \varepsilon_i$$

MRW: basic Solow model estimates

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\overline{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\overline{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

Mankiw, Romer and Weil (1992) *A Contribution to the Empirics of Economic Growth*, Table I

Rate of convergence

The model implies a relationship between the distance from the balanced growth path and the current rate of growth:

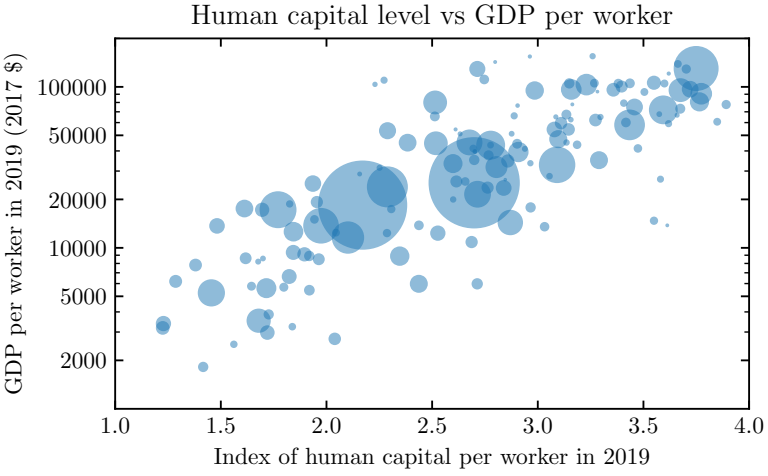
$$g_y \approx g + \underbrace{(1-\alpha)(\delta + n + g)}_{\lambda} (\log y_t^* - \log y_t)$$

Econometric studies typically find that $\lambda \approx 0.02$, meaning that it takes about 35 years to close half of the gap between the current income and its BGP value

Given sensible parameter values: $\alpha = 0.33$, $\delta = 0.05$, $n = 0.01$, $g = 0.02$, the model generates $\lambda = 0.053$, implying that it would take about 13 years to close half of the gap, a very unrealistic number

Adding human capital allows the model to assign lower weight to raw labor and be consistent with empirically estimated rate of convergence

Human capital per capita h vs real GDP per worker y



MRW: human capital augmented Solow model

The production function that accounts for human capital

$$
\begin{aligned}
Y_t &= K_t^{\alpha} H_t^{\beta} \left(A_t L_t\right)^{1-\alpha-\beta} \\
K_{t+1} &= s_k Y_t + (1-\delta) K_t \\
H_{t+1} &= s_h Y_t + (1-\delta) H_t
\end{aligned}
$$

GDP per worker along the BGP

$$
y_t^* = A_t \left(\frac{s_k^{\frac{\alpha}{\alpha+\beta}} s_h^{\frac{\beta}{\alpha+\beta}}}{\delta + n + g} \right)^{\frac{\alpha+\beta}{1-\alpha-\beta}}
$$

MRW: human capital augmented Solow model

Econometric model

$$\ln y_{it} = gt + \frac{\alpha}{1-\alpha-\beta}\ln s_{k,i} + \frac{\beta}{1-\alpha-\beta}\ln s_{h,i} - \frac{\alpha+\beta}{1-\alpha-\beta}\ln\left(\delta + n_i + g\right) + \varepsilon_i$$

Restricted regression

$$\ln y_{it} = gt + \frac{\alpha}{1-\alpha-\beta}\left[\ln s_{k,i} - \ln\left(\delta + n_i + g\right)\right] + \frac{\beta}{1-\alpha-\beta}\left[\ln s_{h,i} - \ln\left(\delta + n_i + g\right)\right] + \varepsilon_i$$

MRW: human capital augmented Solow model estimates

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\bar{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\bar{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Mankiw, Romer and Weil (1992) *A Contribution to the Empirics of Economic Growth*, Table II

Speed of convergence in the augmented Solow model

Speed of convergence in the human capital augmented Solow model

$$g_y \approx g + \underbrace{(1-\alpha-\beta)(\delta+n+g)}_{\lambda}(\ln y_t^* - \ln y_t)$$

Econometric model

$$\ln y_{iT} - \ln y_{i0} = \left(1 - e^{-\lambda T}\right)\left[\begin{array}{c} \frac{\alpha}{1-\alpha-\beta}\ln s_{k,i} + \frac{\beta}{1-\alpha-\beta}\ln s_{h,i} \\ -\frac{\alpha+\beta}{1-\alpha-\beta}\log(\delta+n_i+g) \end{array} - \ln y_{i0}\right] + \varepsilon_i$$

Restricted regression

$$\ln y_{iT} - \ln y_{i0} = \left(1 - e^{-\lambda T}\right)\left[\begin{array}{c} \frac{\alpha}{1-\alpha-\beta}\left[\ln s_{k,i} - \log(\delta+n_i+g)\right] \\ +\frac{\beta}{1-\alpha-\beta}\left[\ln s_{h,i} - \log(\delta+n_i+g)\right] \end{array} - \ln y_{i0}\right] + \varepsilon_i$$

MRW: speed of conditional convergence

Dependent variable: log difference GDP per working-age person 1960–1985

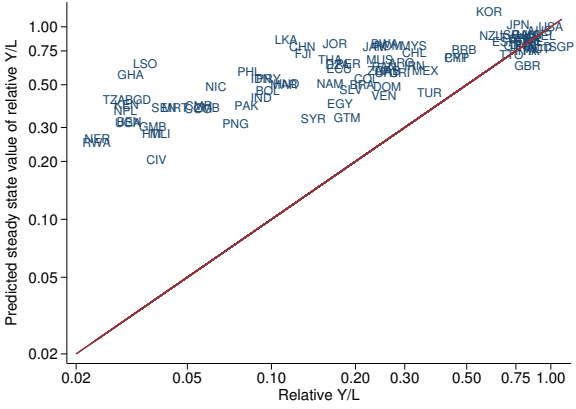
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	2.46	3.09	3.55
	(0.48)	(0.53)	(0.63)
ln(Y60)	−0.299	−0.372	−0.402
	(0.061)	(0.067)	(0.069)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.500	0.506	0.396
	(0.082)	(0.095)	(0.152)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.238	0.266	0.236
	(0.060)	(0.080)	(0.141)
$\overline{R}^2$	0.46	0.44	0.66
s.e.e.	0.33	0.30	0.15
Test of restriction:			
p-value	0.40	0.42	0.47
Implied λ	0.0142	0.0186	0.0206
	(0.0019)	(0.0019)	(0.0020)
Implied α	0.48	0.44	0.38
	(0.07)	(0.07)	(0.13)
Implied β	0.23	0.23	0.23
	(0.05)	(0.06)	(0.11)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Mankiw, Romer and Weil (1992) *A Contribution to the Empirics of Economic Growth*, Table VI

Fit of human capital-augmented Solow model

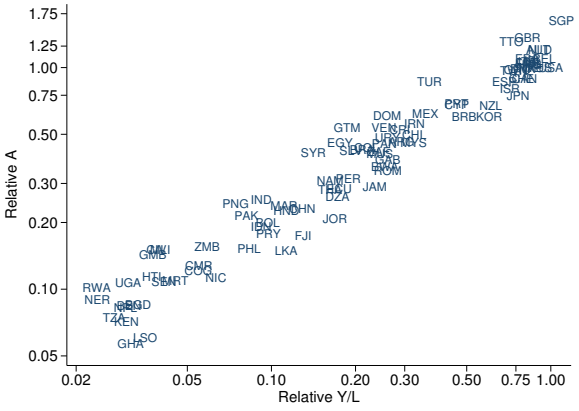
Suggests that poor countries “should” be richer



Jones and Vollrath (2013) *Introduction to Economic Growth*

Solow residual: accounting for technology differences

There are also significant differences in technology across countries



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Decomposition of GDP per worker differentials

The biggest differentials are in A and H/L, not in K/Y!

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

Hall and Jones (1999)

Why Do Some Countries Produce So Much More Output Per Worker Than Others?

In search for fundamental causes of growth

What dictates the investment rate in new capital s_k?
The investment rate in human capital s_h / years of schooling?
The adoption/discovery of new technologies?

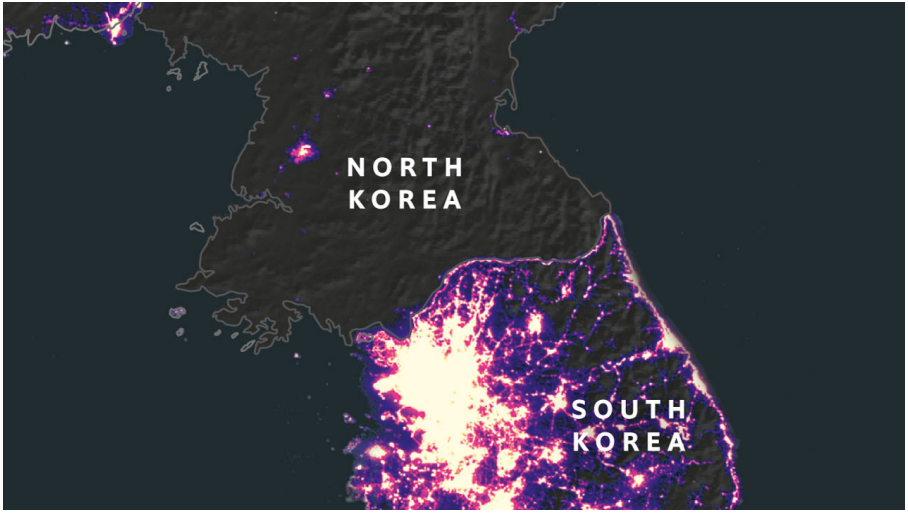
- Geography: easy access to certain resources
- Culture: certain cultures value savings or education more
- Institutions: rules of the economic game

Olson (1996) compares places almost identical in geography and culture:

- North vs. South Korea
- East vs. West Germany
- China vs. Taiwan and Hong Kong

They differ in institutions

North and South Korea at night



Economist (2019)

Institutions

How do we think of institutions?

- Property rights: the ability to keep what you earn in profits, savings, wages
- Transactions: the ability to easily trade assets, sign contracts
- Enforcement: contracts and laws are consistently enforced over time

“Good” institutions will encourage people to make long-run investments because they can keep what they earn and the rules won’t arbitrarily change over time

In most poor countries it is not trivial to start a new firm,
invest in new equipment, adopt a new technology

Social infrastructure

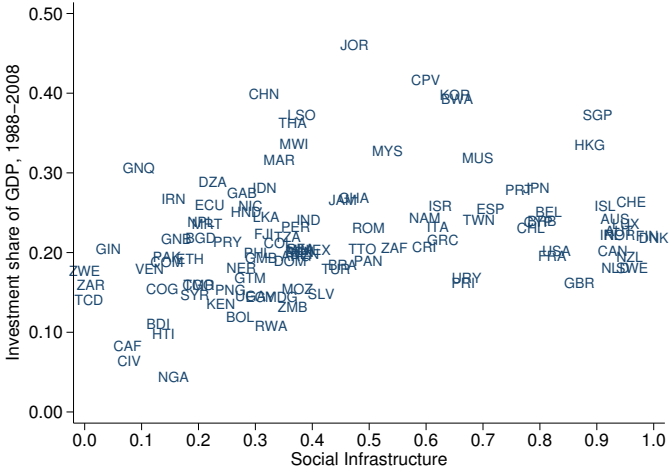
Not possible to measure institutions' quality directly

A measure of "social infrastructure" that captures six dimensions of governance from the World Bank:

- Accountability of politicians
- Political stability
- Government effectiveness
- Regulatory quality
- Rule of law
- Control of corruption

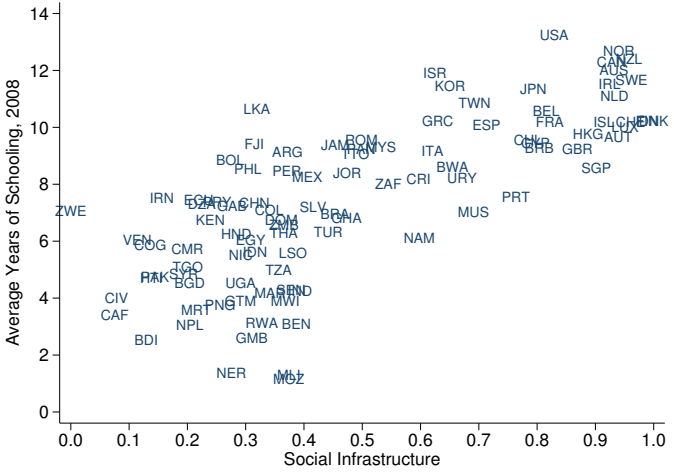
Overall index runs from 0 (worst) to 1 (best)

Social infrastructure and investment



Jones and Vollrath (2013) *Introduction to Economic Growth*

Social infrastructure and human capital



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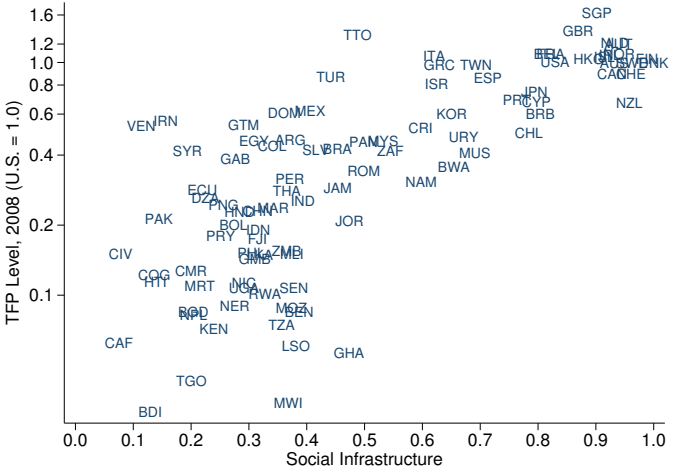
Social infrastructure and technology



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Choosing institutions

If good institutions are so useful, why don't all countries have them?

- Institutions are human-designed and malleable
- Can't we bargain with each other to get good institutions?
- Can't elites take smaller slice of a bigger pie?

Example: offer beauracrats higher salaries in exchange for not taking bribes.
Acemoglu and Robinson (2012): won't work because of commitment problems:

- The beauracrats will take the salary, then still ask for a bribe
- Elites cannot credibly promise to take smaller slice
- Non-elites cannot credibly promise not to replace elites

Institutions appear to be very persistent, and historically contingent

A depressing, but instructive CGP Grey video The Rules for Rulers