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Business cycles models catalog

Government budget is always balanced and public debt is 0, unless explicitly stated otherwise

Public expenditure is financed via lump-sum (neutral) taxes, unless explicitly stated otherwise

Investment adjustment cost parameter Ψ set at 1, unless explicitly stated otherwise

In all models steady state labor supply is normalized to 1 (ψ is adjusted accordingly)

Shock persistence set at 0.9, unless explicitly stated otherwise

Calibrated parameters: $\sigma = 2$, $\varphi = 0.25$, $(G/Y)_{ss} = 0.2$

All models feature constant population growth at 1% and technological progress at 2% (both annually), but the equations below omit the relevant parts of expressions for clarity of exposition

Common model targets (postwar US averages) pin down α , β and δ (potentially model-specific)

$$I/Y = 0.25$$

$$K/Y = 10$$

$$Ret_t^K = 7/400$$

Real Business Cycles

Households

$$\begin{aligned}\lambda_t &= \mathbf{D}_t \cdot C_t^{-\sigma} \\ \lambda_t &= \beta E_t [(1 + r_{t+1}) \lambda_{t+1}] \\ L_t &= [w_t \lambda_t / \psi \cdot \mathbf{H}_t]^{1/\varphi}\end{aligned}$$

Firms

$$\begin{aligned}Y_t &= \mathbf{Z}_t \cdot K_t^\alpha L_t^{1-\alpha} \\ w_t &= (1 - \alpha) Y_t / L_t \\ r_t^K &= \alpha Y_t / K_t\end{aligned}$$

Capital and investment

$$\begin{aligned}K_{t+1} &= I_t + (1 - \delta) K_t \\ 1 + r_t &= \frac{r_t^K + (1 - \delta) q_t}{q_{t-1}} \\ 1 &= q_t \left[1 - \frac{\Psi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Psi \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + E_t \left[\frac{q_{t+1}}{1 + r_{t+1}} \Psi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]\end{aligned}$$

Fiscal policy

$$\begin{aligned}G_t &= \gamma_t \cdot (G/Y)_{ss} Y_t \\ \ln \gamma_t &= \rho_G \ln \gamma_{t-1} + \epsilon_{G,t}\end{aligned}$$

National accounting

$$Y_t = C_t + I_t + G_t$$

Non-policy shocks

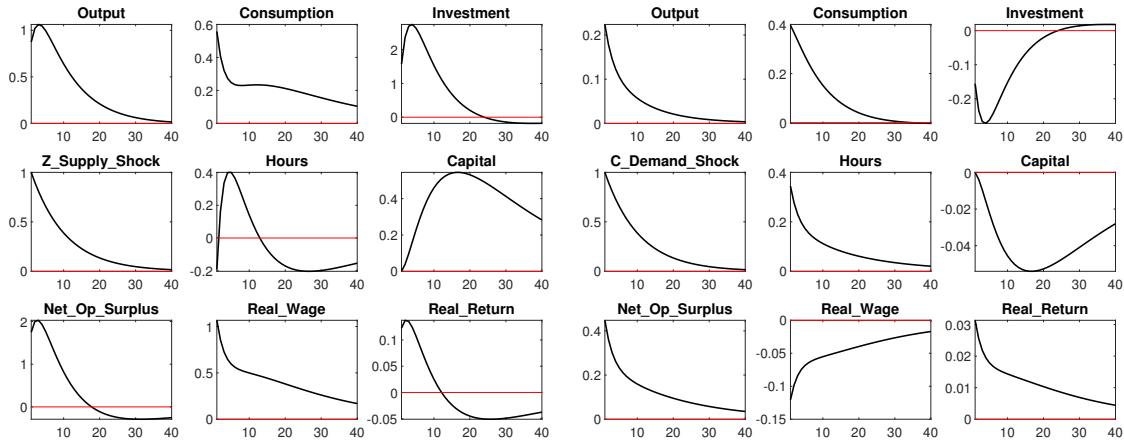
$$\begin{aligned}\ln Z_t &= \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} \\ \ln D_t &= \rho_D \ln D_{t-1} + \epsilon_{D,t} \\ \ln H_t &= \rho_H \ln H_{t-1} + \epsilon_{H,t}\end{aligned}$$

Reporting

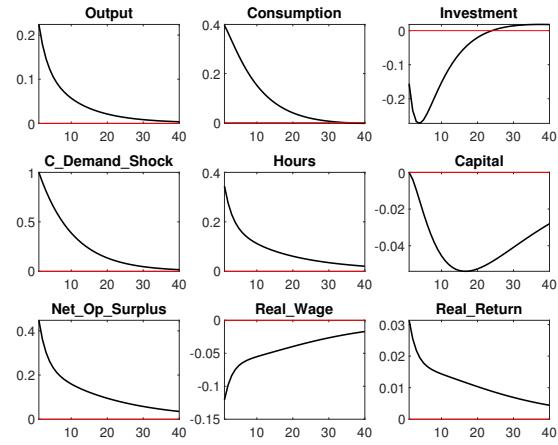
$$\begin{aligned}NOS_t &= Y_t - w_t L_t - \delta K_t \\ Ret_t^K &= NOS_t / K_t\end{aligned}$$

Real Business Cycles: Impulse Response Functions

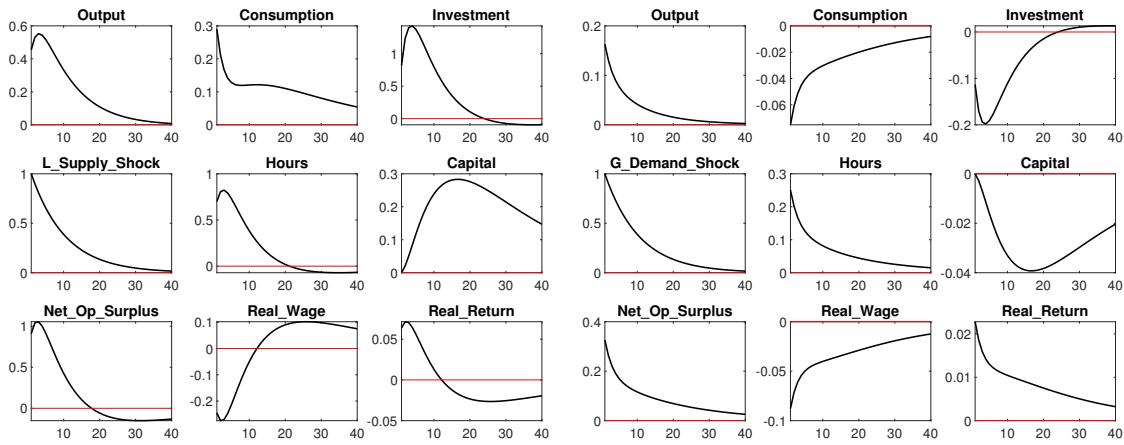
Total Factor Productivity shock ($Z \uparrow$)



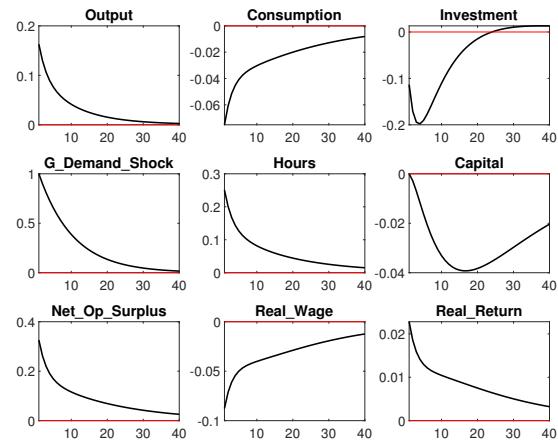
Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)



Public expenditure shock ($\gamma \uparrow$)



All variables except for real return on capital expressed in % deviations from steady state
 Real return on capital (annualized) expressed in percentage points deviations from steady state

Real Business Cycles + Money In Utility

Households

$$\begin{aligned}\lambda_t &= \mathbf{D}_t \cdot C_t^{-\sigma} \\ \lambda_t &= \beta \mathbb{E}_t [(1 + r_{t+1}) \lambda_{t+1}] \\ L_t &= [w_t \lambda_t / \psi \cdot \mathbf{H}_t]^{1/\varphi}\end{aligned}$$

Firms

$$\begin{aligned}Y_t &= \mathbf{Z}_t \cdot K_t^\alpha L_t^{1-\alpha} \\ w_t &= (1 - \alpha) Y_t / L_t \\ r_t^K &= \alpha Y_t / K_t\end{aligned}$$

Capital and investment

$$\begin{aligned}K_{t+1} &= I_t + (1 - \delta) K_t \\ 1 + r_t &= \frac{r_t^K + (1 - \delta) q_t}{q_{t-1}} \\ 1 = q_t &\left[1 - \frac{\Psi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Psi \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + \mathbb{E}_t \left[\frac{q_{t+1}}{1 + r_{t+1}} \Psi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]\end{aligned}$$

Fiscal policy

$$\begin{aligned}G_t &= \gamma_t \cdot (G/Y)_{ss} Y_t \\ \ln \gamma_t &= \rho_G \ln \gamma_{t-1} + \epsilon_{G,t}\end{aligned}$$

National accounting

$$Y_t = C_t + I_t + G_t$$

Non-policy shocks

$$\begin{aligned}\ln Z_t &= \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} \\ \ln D_t &= \rho_D \ln D_{t-1} + \epsilon_{D,t} \\ \ln H_t &= \rho_H \ln H_{t-1} + \epsilon_{H,t}\end{aligned}$$

Money In Utility: new equations

Nominal variables (do not influence real ones), with ν_{ss} such that $M^* = 1$ and $P^* = 1$

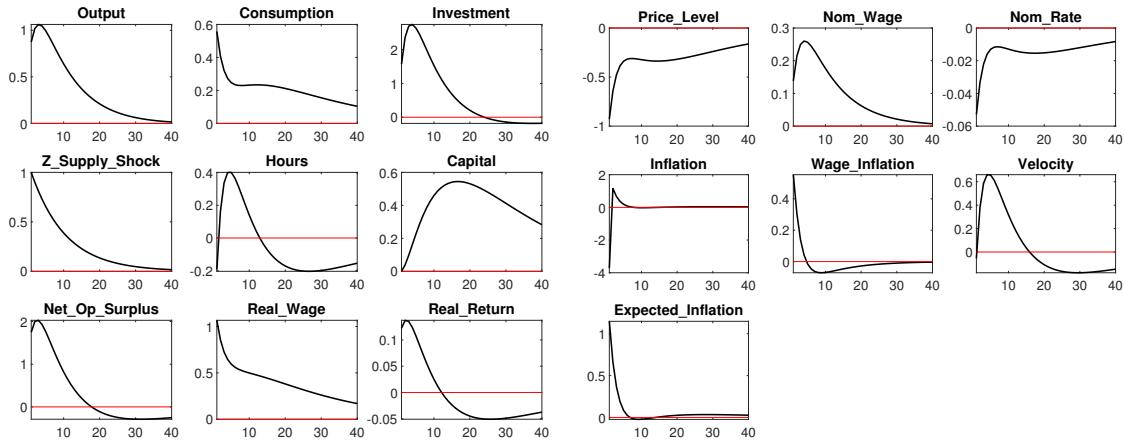
$$\begin{aligned}M_t^d &= (\nu_{ss} \cdot \nu_t)^{1/\sigma} P_t C_t [(1 + i_t) / i_t]^{1/\sigma} \\ M_t^d &= \mathbf{M}_t \\ V_t &= P_t Y_t / M_t \\ W_t &= w_t P_t \\ 1 + \pi_t &= P_t / P_{t-1} \\ 1 + \pi_t^W &= W_t / W_{t-1} \\ 1 + \pi_{t+1}^e &= \mathbb{E}_t [1 + \pi_{t+1}] \cdot \mathbf{P}_t \\ \mathbb{E}_t [1 + r_{t+1}] &= (1 + i_t) / (1 + \pi_{t+1}^e)\end{aligned}$$

Nominal shocks

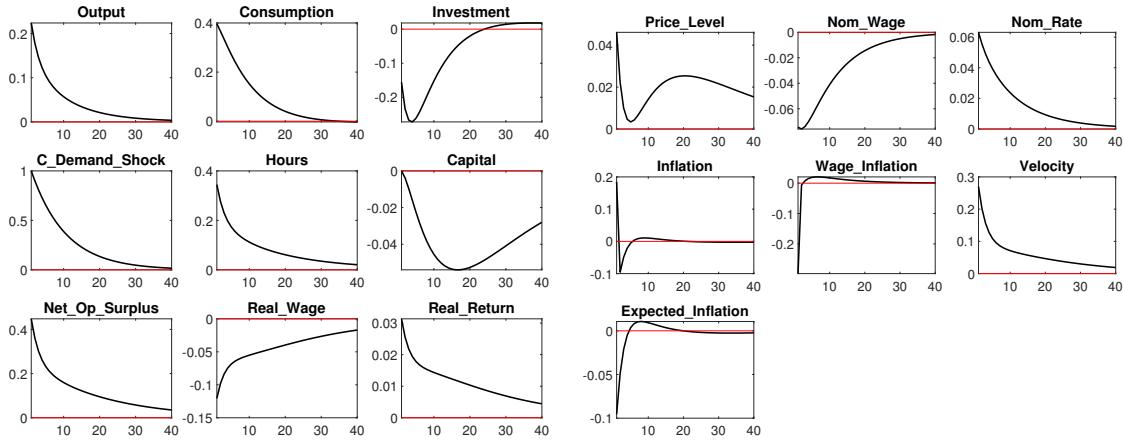
$$\begin{aligned}\ln \nu_t &= \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \\ \ln M_t &= \rho_M \ln M_{t-1} + \epsilon_{M,t} \\ \ln PP_t &= \rho_P \ln PP_{t-1} + \epsilon_{P,t}\end{aligned}$$

Real Business Cycles + Money In Utility: Impulse Response Functions

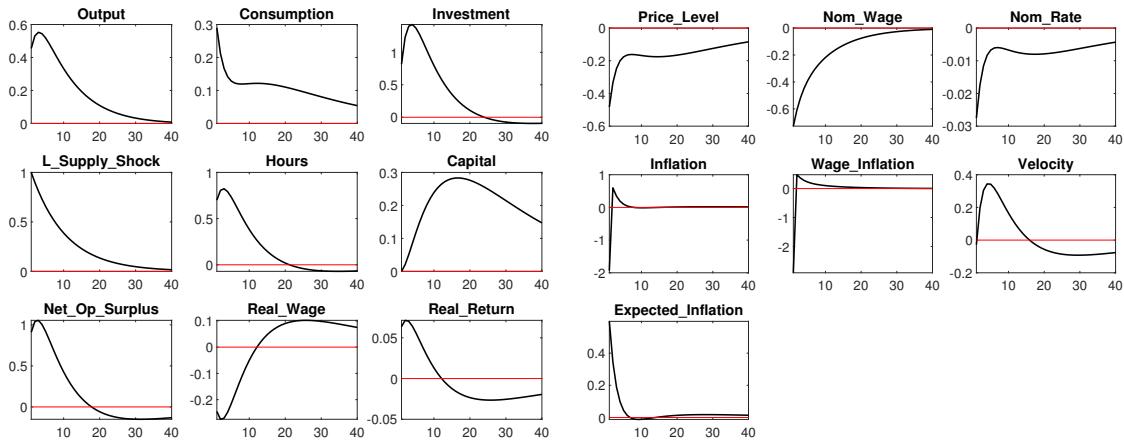
Total Factor Productivity shock ($Z \uparrow$)



Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)

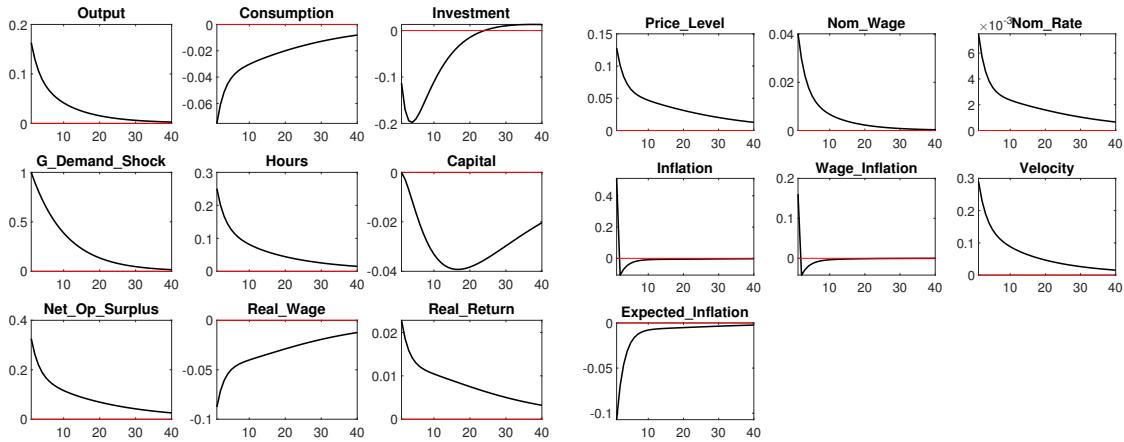


All variables except for rates expressed in % deviations from steady state

Rate variables are annualized and expressed in percentage points deviations from steady state

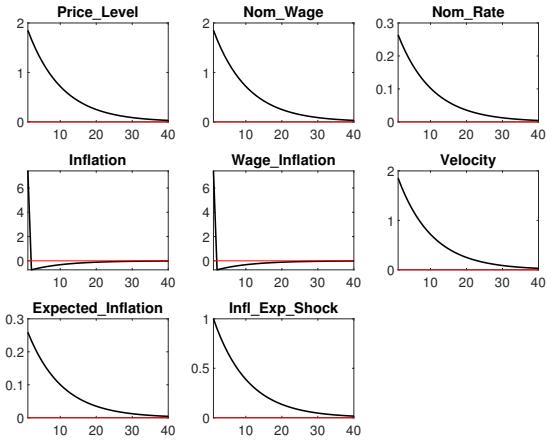
Real Business Cycles + Money In Utility: Impulse Response Functions

Public expenditure shock ($\gamma \uparrow$)



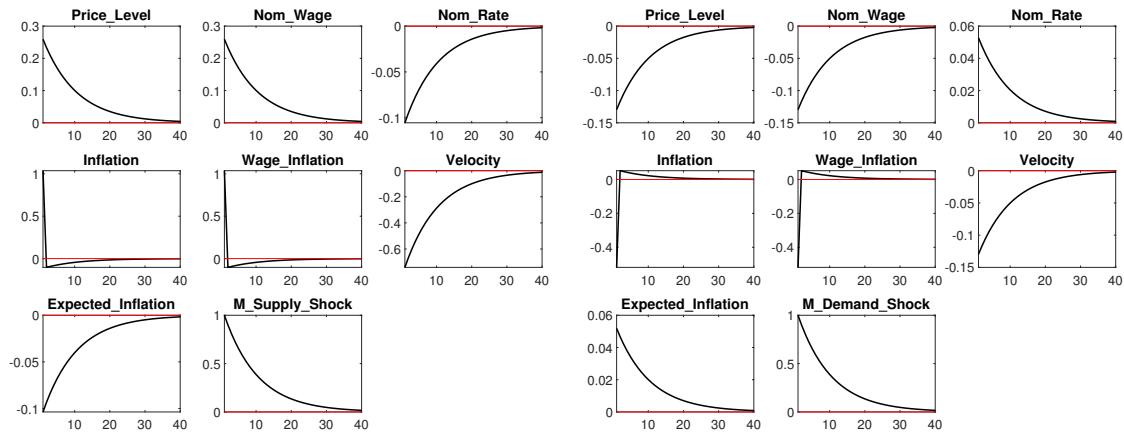
No reaction of real variables
to nominal shocks (PP, M, ν)

Inflation expectations shock ($PP \uparrow$)



Money supply shock ($M \uparrow$)

Money demand shock ($\nu \uparrow$)



All variables except for rates expressed in % deviations from steady state

Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Money In Utility

Households

$$\begin{aligned}\lambda_t &= \mathbf{D}_t \cdot C_t^{-\sigma} \\ \lambda_t &= \beta \mathbb{E}_t \left[\frac{\mathbf{1} + i_t}{\mathbf{1} + \pi_{t+1}^e} \lambda_{t+1} \right] \\ L_t &= [w_t \lambda_t / \psi \cdot \mathbf{H}_t]^{1/\varphi} \\ M_t^d &= (\nu_{ss} \cdot \mathbf{v}_t)^{1/\sigma} P_t C_t [(1 + i_t) / i_t]^{1/\sigma}\end{aligned}$$

Firms

$$\begin{aligned}Y_t &= Z_t K_t^\alpha L_t^{1-\alpha} / \Delta_t \\ w_t &= \mathbf{m}_t \cdot (1 - \alpha) Y_t / L_t \\ r_t^K &= \mathbf{m}_t \cdot \alpha Y_t / K_t\end{aligned}$$

Capital and investment

$$\begin{aligned}K_{t+1} &= I_t + (1 - \delta) K_t \\ 1 + r_t &= \frac{r_t^K + (1 - \delta) q_t}{q_{t-1}} \\ 1 &= q_t \left[1 - \frac{\Psi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Psi \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + \mathbb{E}_t \left[\frac{q_{t+1}}{1 + r_{t+1}} \Psi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]\end{aligned}$$

Fiscal policy

$$\begin{aligned}G_t &= \gamma_t \cdot (G/Y)_{ss} Y_t \\ \ln \gamma_t &= \rho_G \ln \gamma_{t-1} + \epsilon_{G,t}\end{aligned}$$

Monetary policy

$$\begin{aligned}M_t^d &= \mathbf{M}_t \\ \ln M_t &= \rho_M \ln M_{t-1} + \epsilon_{M,t}\end{aligned}$$

National accounting

$$Y_t = C_t + I_t + G_t$$

Nominal variables

$$\begin{aligned}V_t &= P_t Y_t / M_t \\ W_t &= w_t P_t \\ 1 + \pi_t &= P_t / P_{t-1} \\ 1 + \pi_t^W &= W_t / W_{t-1} \\ 1 + \pi_{t+1}^e &= \mathbb{E}_t [1 + \pi_{t+1}] \cdot \mathbf{P}_t \\ \mathbb{E}_t [1 + r_{t+1}] &= (1 + i_t) / (1 + \pi_{t+1}^e)\end{aligned}$$

Non-policy shocks

$$\begin{aligned}\ln Z_t &= \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} \\ \ln D_t &= \rho_D \ln D_{t-1} + \epsilon_{D,t} \\ \ln H_t &= \rho_H \ln H_{t-1} + \epsilon_{H,t} \\ \ln \nu_t &= \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \\ \ln PP_t &= \rho_P \ln PP_{t-1} + \epsilon_{P,t}\end{aligned}$$

Reporting

$$\begin{aligned} NOS_t &= Y_t - w_t L_t - \delta K_t \\ Ret_t^K &= NOS_t / K_t \\ LS_t &= w_t L_t / Y_t \end{aligned}$$

New Keynesian model: new equations

Firm price setting, with elasticity of substitution $\varepsilon = 10$ and $\theta = 0.75$

$$\begin{aligned} \tilde{p}_t &= \frac{\varepsilon}{\varepsilon - 1} \frac{X_{1,t}}{X_{2,t}} \\ X_{1,t} &= \lambda_t Y_t mc_t + \theta \beta E_t \left[(1 + \pi_{t+1}^e)^\varepsilon X_{1,t+1} \right] \\ X_{2,t} &= \lambda_t Y_t + \theta \beta E_t \left[(1 + \pi_{t+1}^e)^{\varepsilon-1} X_{2,t+1} \right] \end{aligned}$$

Inflation dynamics and price dispersion

$$\begin{aligned} (1 + \pi_t)^{1-\varepsilon} &= \theta + (1 - \theta) (\tilde{p}_t (1 + \pi_t))^{1-\varepsilon} \\ \Delta_t &= \theta \Delta_{t-1} (1 + \pi_t)^\varepsilon + (1 - \theta) \tilde{p}_t^{-\varepsilon} \end{aligned}$$

Reporting

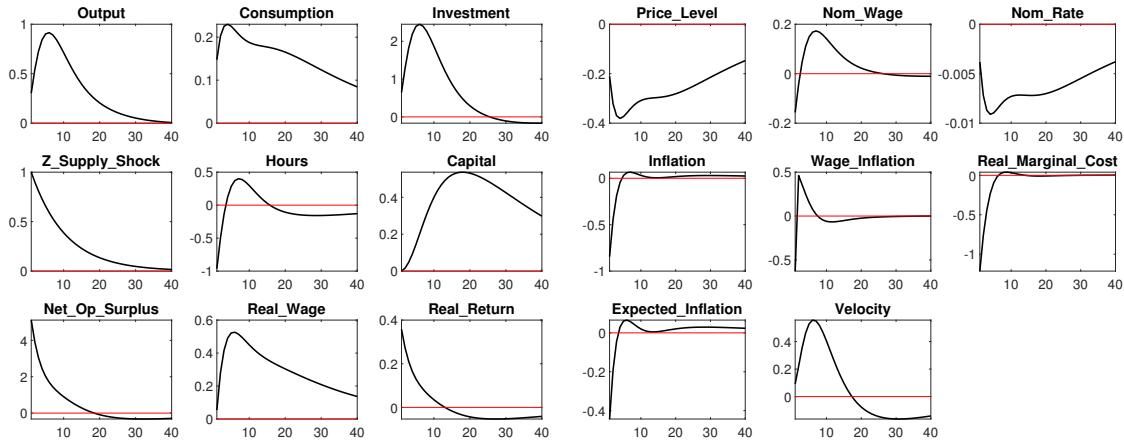
$$\begin{aligned} mc_t &= \frac{w_t^{1-\alpha}}{1-\alpha} \frac{(r_t^K)^\alpha}{\alpha} \frac{1}{Z_t} \\ mc_{ss} &= \frac{\varepsilon - 1}{\varepsilon} \end{aligned}$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \ln (mc_t / mc_{ss})$$

New Keynesian + Money In Utility: Impulse Response Functions

Total Factor Productivity shock ($Z \uparrow$)

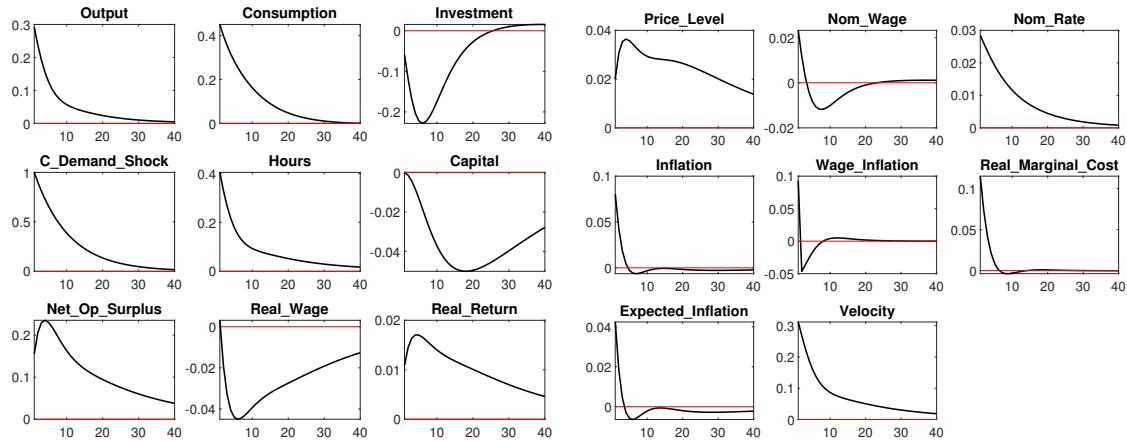


All variables except for rates expressed in % deviations from steady state

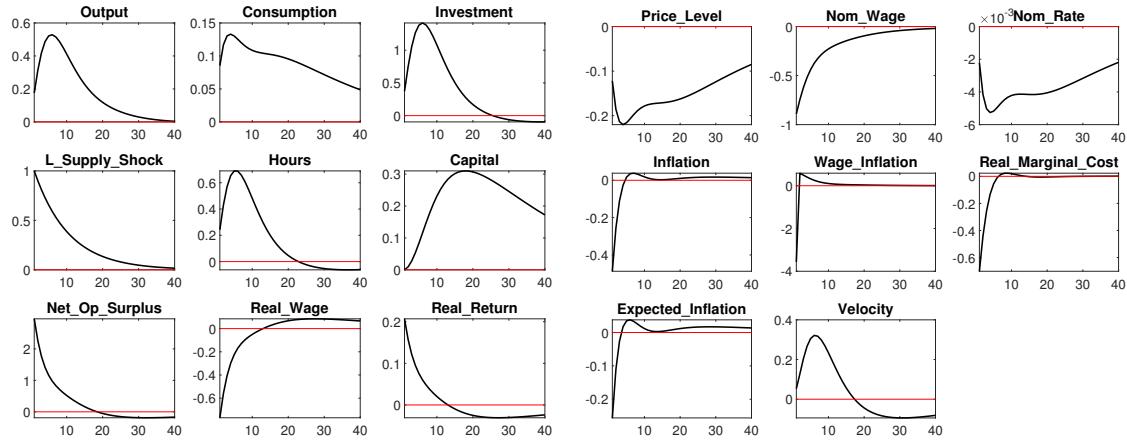
Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Money In Utility: Impulse Response Functions

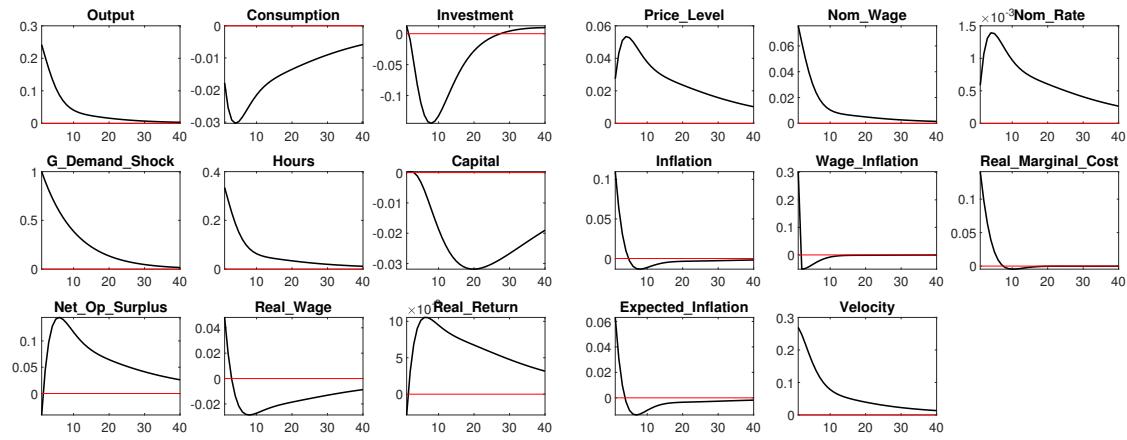
Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)



Public expenditure shock ($\gamma \uparrow$)

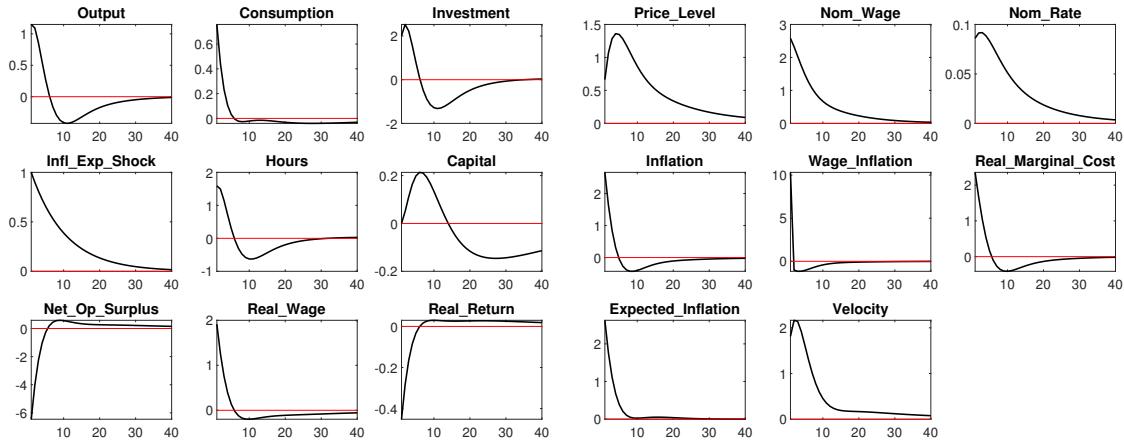


All variables except for rates expressed in % deviations from steady state

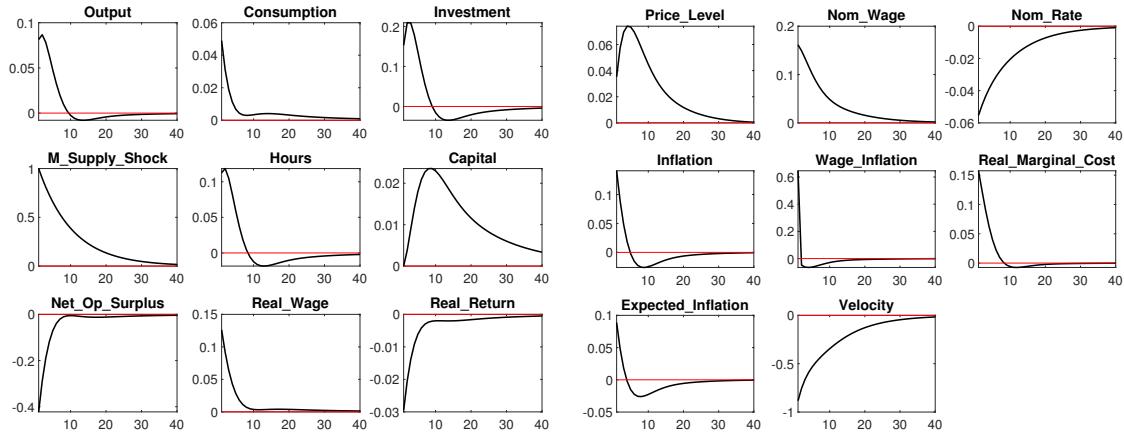
Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Money In Utility: Impulse Response Functions

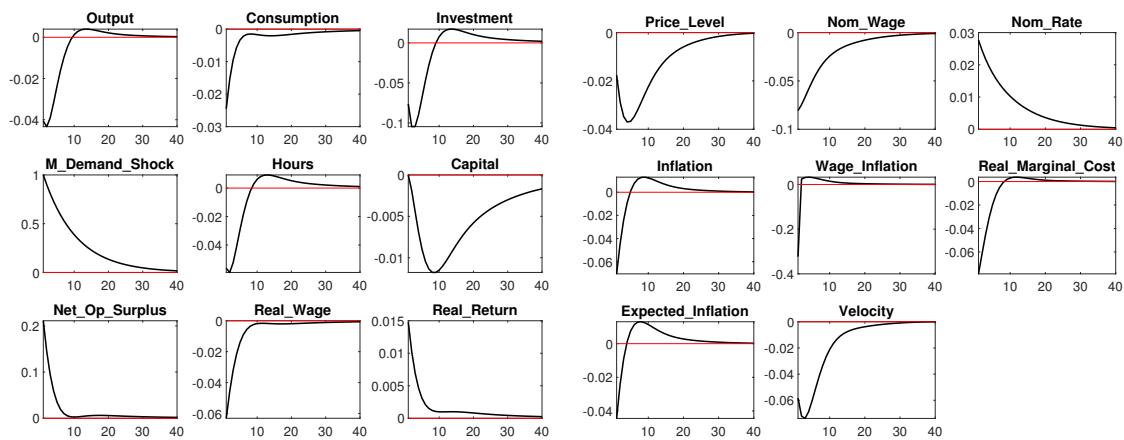
Inflation expectations shock ($PP \uparrow$)



Money supply shock ($M \uparrow$)



Money demand shock ($\nu \uparrow$)



All variables except for rates expressed in % deviations from steady state

Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Taylor Rule

Households

$$\begin{aligned}\lambda_t &= \mathbf{D}_t \cdot C_t^{-\sigma} \\ \lambda_t &= \beta \mathbb{E}_t \left[\frac{1+i_t}{1+\pi_{t+1}^e} \lambda_{t+1} \right] \\ M_t^d &= (\nu_{ss} \cdot \mathbf{v}_t)^{1/\sigma} P_t C_t [(1+i_t)/i_t]^{1/\sigma}\end{aligned}$$

Firms

$$\begin{aligned}Y_t &= Z_t K_t^\alpha L_t^{1-\alpha} / \Delta_t \\ w_t &= mc_t (1-\alpha) Y_t / L_t \\ r_t^K &= mc_t \alpha Y_t / K_t\end{aligned}$$

Firm price setting

$$\begin{aligned}\tilde{p}_t &= \frac{\varepsilon}{\varepsilon-1} \frac{X_{1,t}}{X_{2,t}} \\ X_{1,t} &= \lambda_t Y_t mc_t + \theta \beta \mathbb{E}_t \left[(1+\pi_{t+1}^e)^\varepsilon X_{1,t+1} \right] \\ X_{2,t} &= \lambda_t Y_t + \theta \beta \mathbb{E}_t \left[(1+\pi_{t+1}^e)^{\varepsilon-1} X_{2,t+1} \right]\end{aligned}$$

Inflation dynamics and price dispersion

$$\begin{aligned}(1+\pi_t)^{1-\varepsilon} &= \theta + (1-\theta) (\tilde{p}_t (1+\pi_t))^{1-\varepsilon} \\ \Delta_t &= \theta \Delta_{t-1} (1+\pi_t)^\varepsilon + (1-\theta) \tilde{p}_t^{-\varepsilon}\end{aligned}$$

Capital and investment

$$\begin{aligned}K_{t+1} &= I_t + (1-\delta) K_t \\ 1+r_t &= \frac{r_t^K + (1-\delta) q_t}{q_{t-1}} \\ 1 &= q_t \left[1 - \frac{\Psi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Psi \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + \mathbb{E}_t \left[\frac{q_{t+1}}{1+r_{t+1}} \Psi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]\end{aligned}$$

Fiscal policy

$$\begin{aligned}G_t &= \gamma_t \cdot (G/Y)_{ss} Y_t \\ \ln \gamma_t &= \rho_G \ln \gamma_{t-1} + \epsilon_{G,t}\end{aligned}$$

Monetary policy, with $\rho_i = 0.8$, $\gamma_\pi = 1.5$ and 0% steady state inflation target

$$\begin{aligned}i_t &= \rho_i i_{t-1} + \rho_i (i_{ss} + \gamma_\pi \pi_t) + \epsilon_{i,t} \\ M_t &= M_t^d\end{aligned}$$

National accounting

$$Y_t = C_t + I_t + G_t$$

Nominal variables

$$\begin{aligned}V_t &= P_t Y_t / M_t \\ W_t &= w_t P_t \\ 1+\pi_t &= P_t / P_{t-1} \\ 1+\pi_t^W &= W_t / W_{t-1} \\ 1+\pi_{t+1}^e &= \mathbb{E}_t [1+\pi_{t+1}] \cdot \mathbf{P}_t \\ \mathbb{E}_t [1+r_{t+1}] &= (1+i_t) / (1+\pi_{t+1}^e)\end{aligned}$$

Non-policy shocks

$$\begin{aligned}\ln Z_t &= \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} \\ \ln D_t &= \rho_D \ln D_{t-1} + \epsilon_{D,t} \\ \ln H_t &= \rho_H \ln H_{t-1} + \epsilon_{H,t} \\ \ln \nu_t &= \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \\ \ln PP_t &= \rho_P \ln PP_{t-1} + \epsilon_{P,t}\end{aligned}$$

Reporting

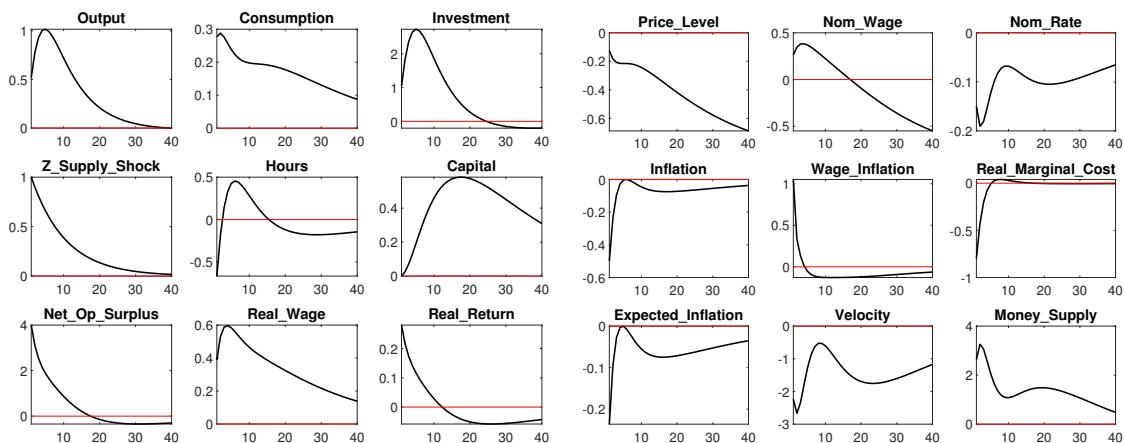
$$\begin{aligned}NOS_t &= Y_t - w_t L_t - \delta K_t \\ Ret_t^K &= NOS_t / K_t \\ LS_t &= w_t L_t / Y_t \\ mc_t &= \frac{w_t^{1-\alpha}}{1-\alpha} \frac{(r_t^K)^\alpha}{\alpha} \frac{1}{Z_t} \\ mc_{ss} &= \frac{\varepsilon - 1}{\varepsilon}\end{aligned}$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \ln (mc_t/mc_{ss})$$

New Keynesian + Taylor Rule: Impulse Response Functions

Total Factor Productivity shock ($Z \uparrow$)

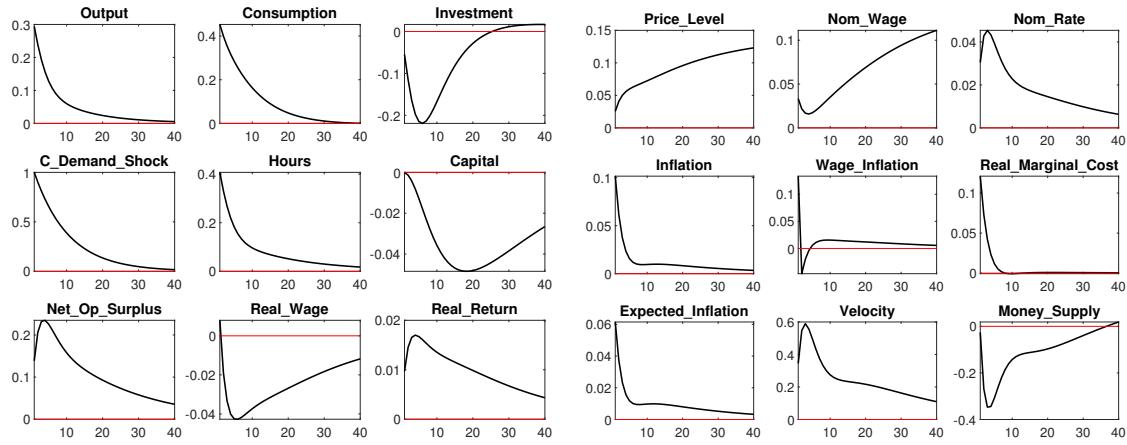


All variables except for rates expressed in % deviations from steady state

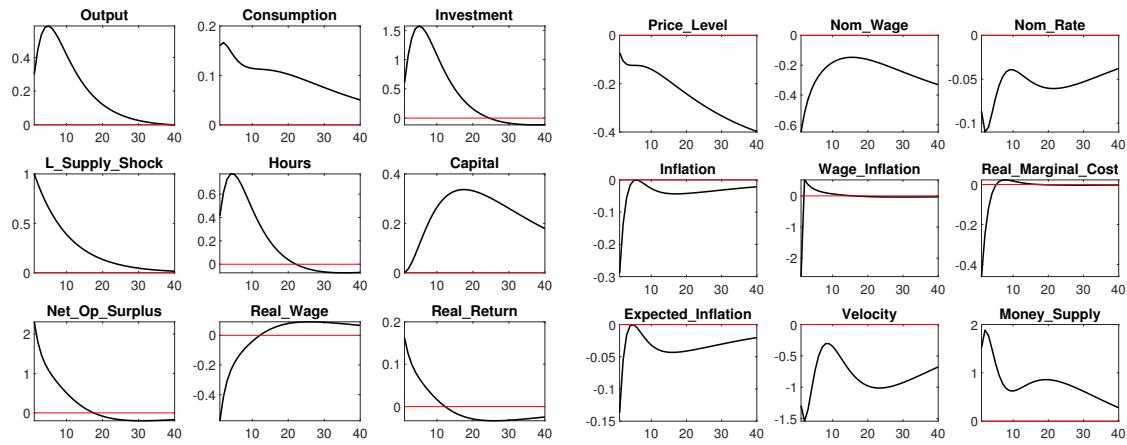
Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Taylor Rule: Impulse Response Functions

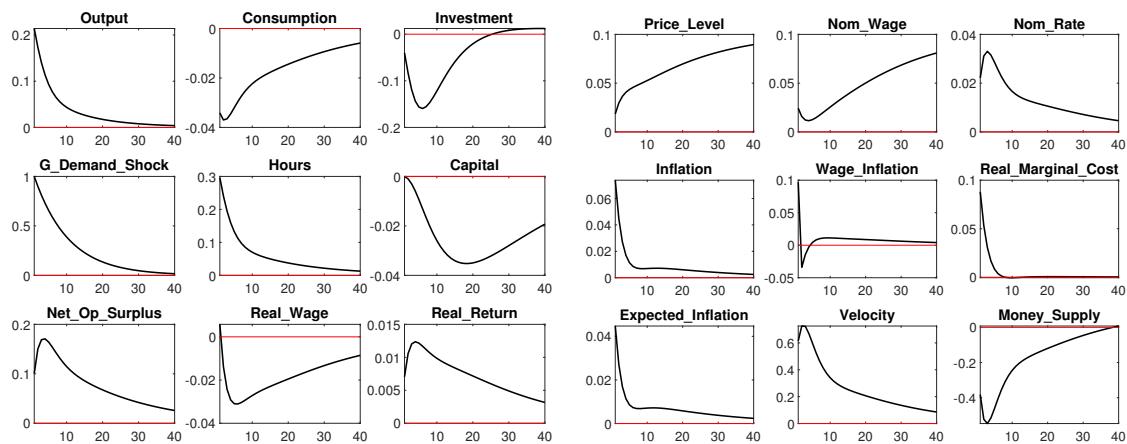
Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)



Public expenditure shock ($\gamma \uparrow$)

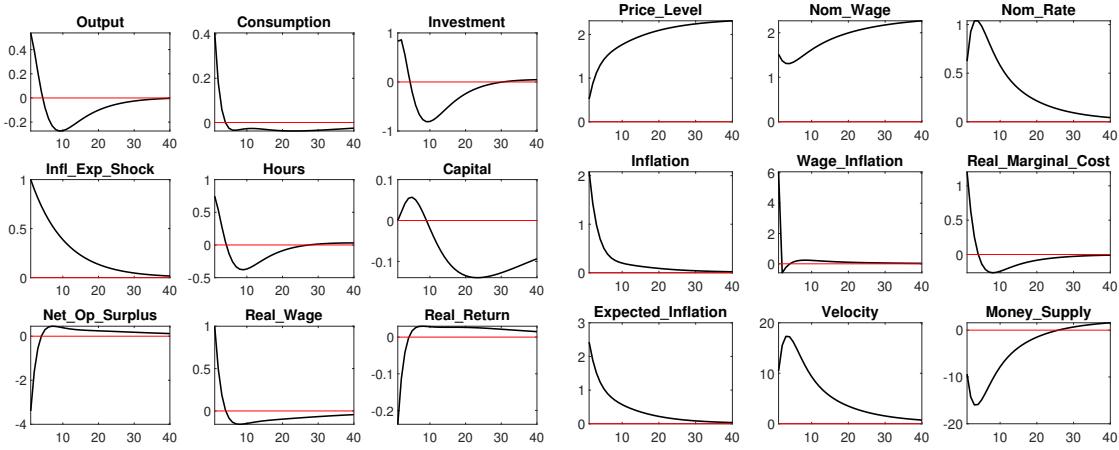


All variables except for rates expressed in % deviations from steady state

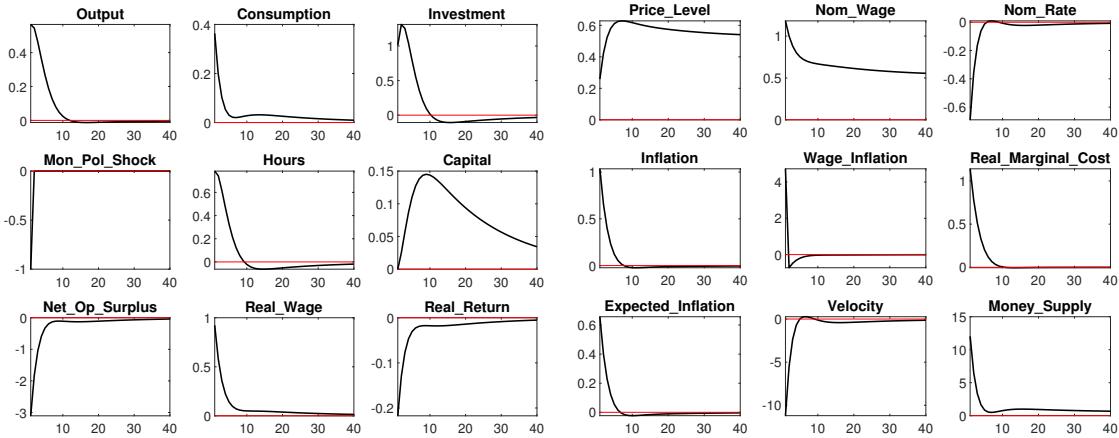
Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Taylor Rule: Impulse Response Functions

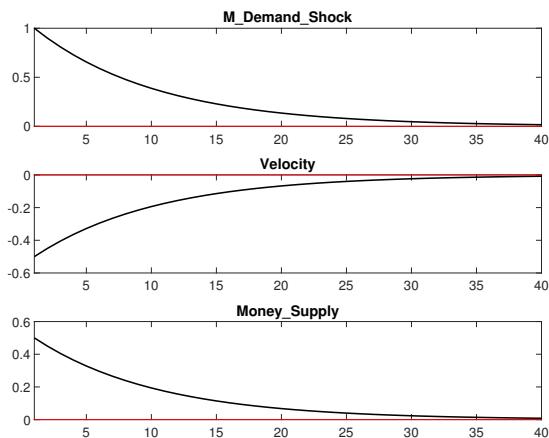
Inflation expectations shock ($PP \uparrow$)



Monetary policy shock ($i \downarrow$)



Money demand shock ($\nu \uparrow$)



No reaction of other variables

All variables except for rates expressed in % deviations from steady state

Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Money In Utility + Wage Stickiness

Households

$$\begin{aligned}\lambda_t &= \mathbf{D}_t \cdot C_t^{-\sigma} \\ \lambda_t &= \beta \mathbb{E}_t \left[\frac{1+i_t}{1+\pi_{t+1}^e} \lambda_{t+1} \right] \\ M_t^d &= (\nu_{ss} \cdot \mathbf{v}_t)^{1/\sigma} P_t C_t [(1+i_t)/i_t]^{1/\sigma}\end{aligned}$$

Firms

$$\begin{aligned}Y_t &= Z_t K_t^\alpha L_t^{1-\alpha} / \Delta_t \\ w_t &= mc_t (1-\alpha) Y_t / L_t \\ r_t^K &= mc_t \alpha Y_t / K_t\end{aligned}$$

Firm price setting

$$\begin{aligned}\tilde{p}_t &= \frac{\varepsilon}{\varepsilon-1} \frac{X_{1,t}}{X_{2,t}} \\ X_{1,t} &= \lambda_t Y_t mc_t + \theta \beta \mathbb{E}_t \left[(1+\pi_{t+1}^e)^\varepsilon X_{1,t+1} \right] \\ X_{2,t} &= \lambda_t Y_t + \theta \beta \mathbb{E}_t \left[(1+\pi_{t+1}^e)^{\varepsilon-1} X_{2,t+1} \right]\end{aligned}$$

Inflation dynamics and price dispersion

$$\begin{aligned}(1+\pi_t)^{1-\varepsilon} &= \theta + (1-\theta) (\tilde{p}_t (1+\pi_t))^{1-\varepsilon} \\ \Delta_t &= \theta \Delta_{t-1} (1+\pi_t)^\varepsilon + (1-\theta) \tilde{p}_t^{-\varepsilon}\end{aligned}$$

Capital and investment

$$\begin{aligned}K_{t+1} &= I_t + (1-\delta) K_t \\ 1+r_t &= \frac{r_t^K + (1-\delta) q_t}{q_{t-1}} \\ 1 = q_t \left[1 - \frac{\Psi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Psi \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + \mathbb{E}_t \left[\frac{q_{t+1}}{1+r_{t+1}} \Psi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]\end{aligned}$$

Fiscal policy

$$\begin{aligned}G_t &= \gamma_t \cdot (G/Y)_{ss} Y_t \\ \ln \gamma_t &= \rho_G \ln \gamma_{t-1} + \epsilon_{G,t}\end{aligned}$$

Monetary policy

$$\begin{aligned}M_t^d &= \mathbf{M}_t \\ \ln M_t &= \rho_M \ln M_{t-1} + \epsilon_{M,t}\end{aligned}$$

National accounting

$$Y_t = C_t + I_t + G_t$$

Nominal variables

$$\begin{aligned}V_t &= P_t Y_t / M_t \\ W_t &= w_t P_t \\ 1+\pi_t &= P_t / P_{t-1} \\ 1+\pi_t^W &= W_t / W_{t-1} \\ 1+\pi_{t+1}^e &= \mathbb{E}_t [1+\pi_{t+1}] \cdot \mathbf{P}_t \\ \mathbb{E}_t [1+r_{t+1}] &= (1+i_t) / (1+\pi_{t+1}^e)\end{aligned}$$

Non-policy shocks

$$\begin{aligned}\ln Z_t &= \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} \\ \ln D_t &= \rho_D \ln D_{t-1} + \epsilon_{D,t} \\ \ln H_t &= \rho_H \ln H_{t-1} + \epsilon_{H,t} \\ \ln \nu_t &= \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \\ \ln PP_t &= \rho_P \ln PP_{t-1} + \epsilon_{P,t}\end{aligned}$$

Reporting

$$\begin{aligned}NOS_t &= Y_t - w_t L_t - \delta K_t \\ Ret_t^K &= NOS_t / K_t \\ LS_t &= w_t L_t / Y_t \\ mc_t &= \frac{w_t^{1-\alpha}}{1-\alpha} \frac{(r_t^K)^\alpha}{\alpha} \frac{1}{Z_t} \\ mc_{ss} &= \frac{\varepsilon - 1}{\varepsilon}\end{aligned}$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \ln (mc_t/mc_{ss})$$

Wage Stickiness: new equations

Instead of

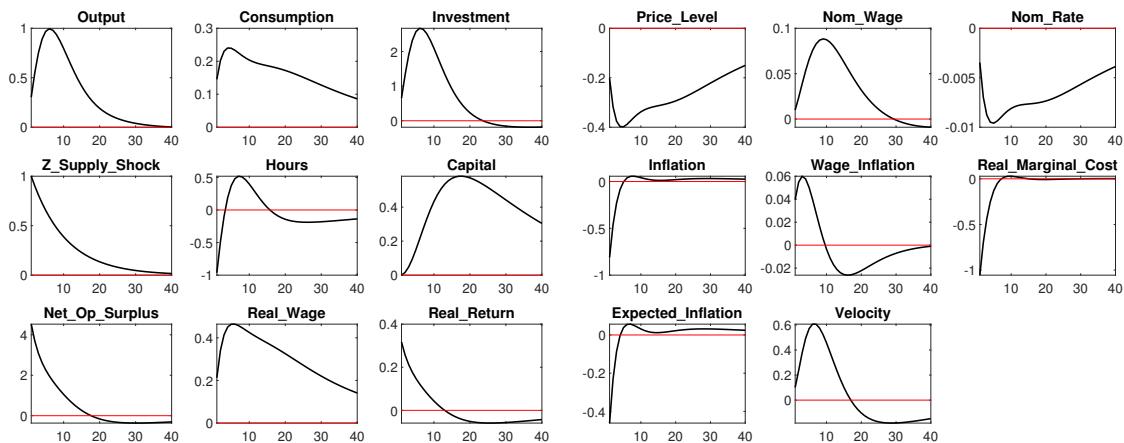
$$L_t = [w_t \lambda_t / \psi \cdot H_t]^{1/\varphi}$$

Household wage setting, with elasticity of substitution $\varepsilon^W = 10$ and $\theta^W = 0.75$

$$\begin{aligned}\tilde{w}_t &= \left[\frac{\varepsilon^W}{\varepsilon^W - 1} \frac{X_{1,t}^W}{X_{2,t}^W} \right]^{1/(1+\varepsilon^W\varphi)} \\ X_{1,t}^W &= \psi / \textcolor{red}{H_t} \cdot w_t^{\varepsilon^W(1+\varphi)} L_t^{1+\varphi} + \beta \theta^W E_t \left[(1 + \pi_{t+1}^e)^{\varepsilon^W(1+\varphi)} X_{1,t+1}^W \right] \\ X_{2,t}^W &= \lambda_t w_t^{\varepsilon^W} L_t + \beta \theta^W E_t \left[(1 + \pi_{t+1}^e)^{\varepsilon^W-1} X_{2,t+1}^W \right] \\ w_t^{1-\varepsilon^W} &= \theta^W \left(\frac{w_{t-1}}{1 + \pi_t} \right)^{1-\varepsilon^W} + (1 - \theta^W) \tilde{w}_t^{1-\varepsilon^W}\end{aligned}$$

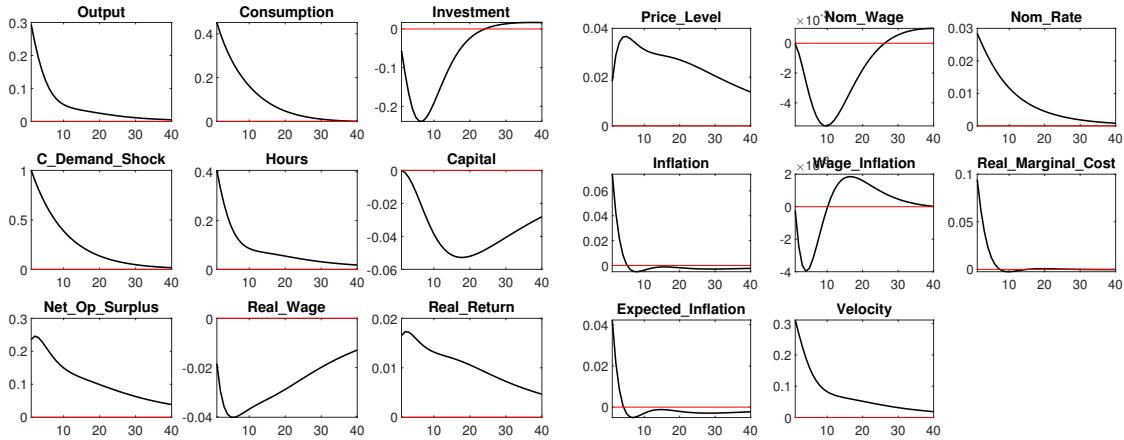
New Keynesian + Money In Utility + Wage Stickiness: Impulse Response Functions

Total Factor Productivity shock ($Z \uparrow$)

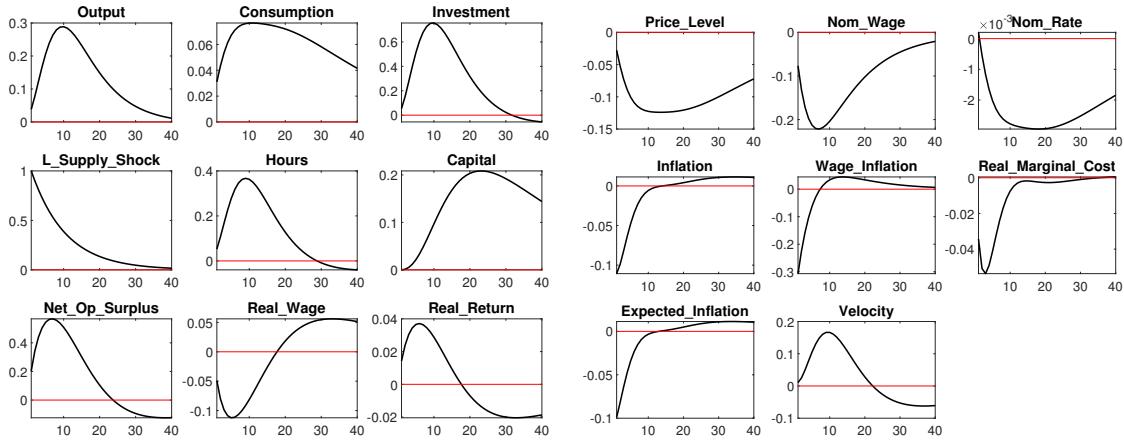


New Keynesian + Money In Utility + Wage Stickiness: Impulse Response Functions

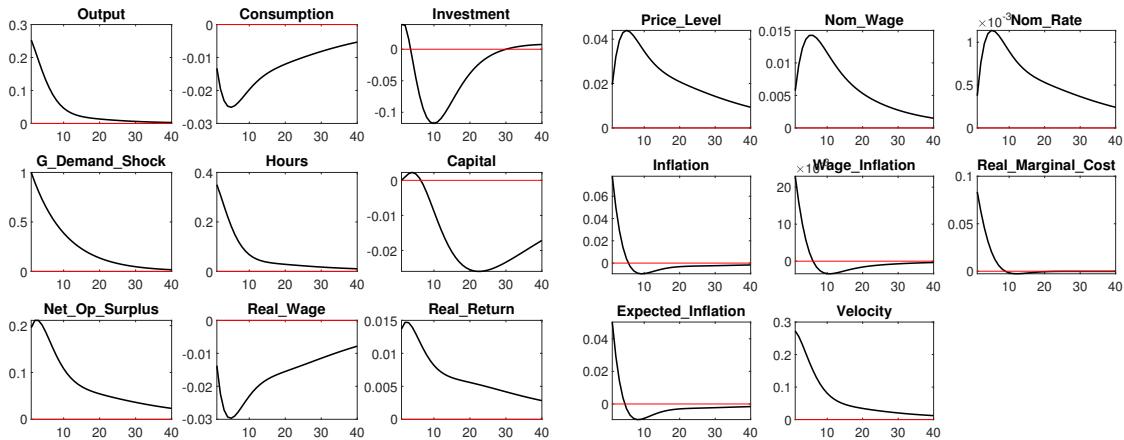
Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)



Public expenditure shock ($\gamma \uparrow$)

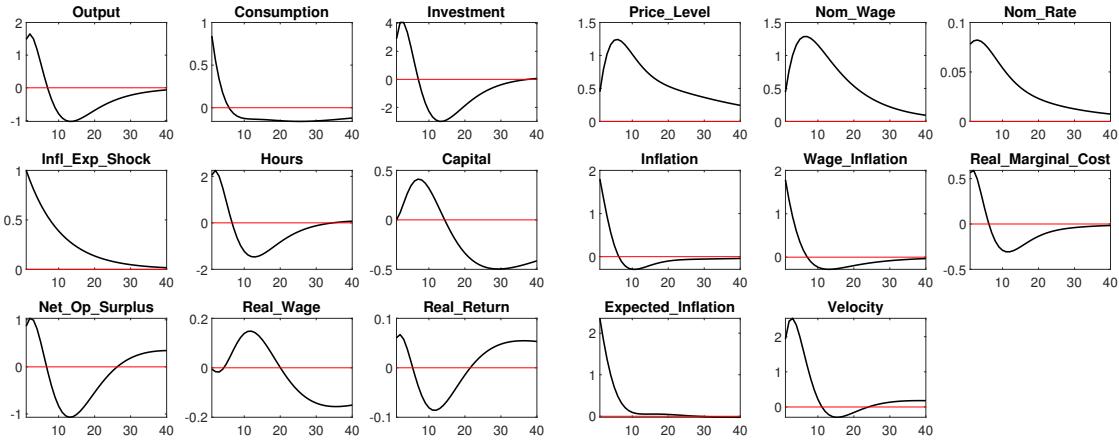


All variables except for rates expressed in % deviations from steady state

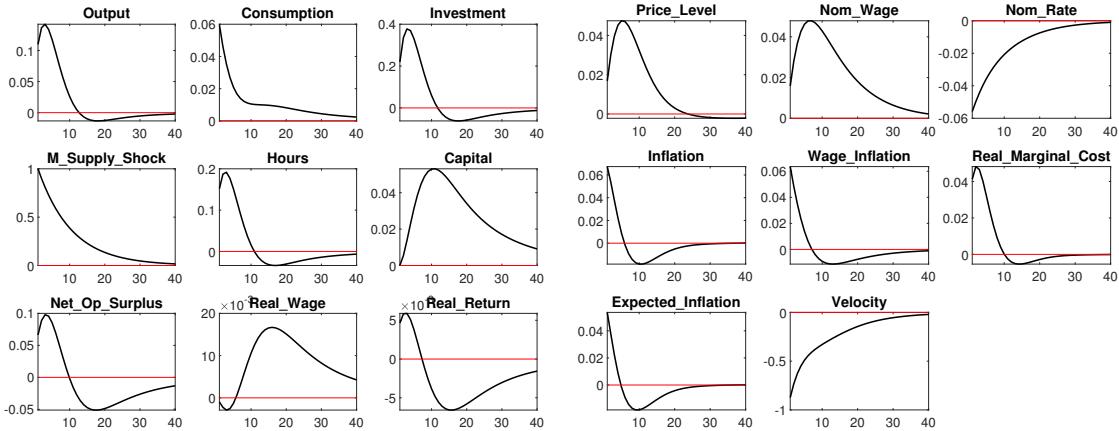
Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Money In Utility + Wage Stickiness: Impulse Response Functions

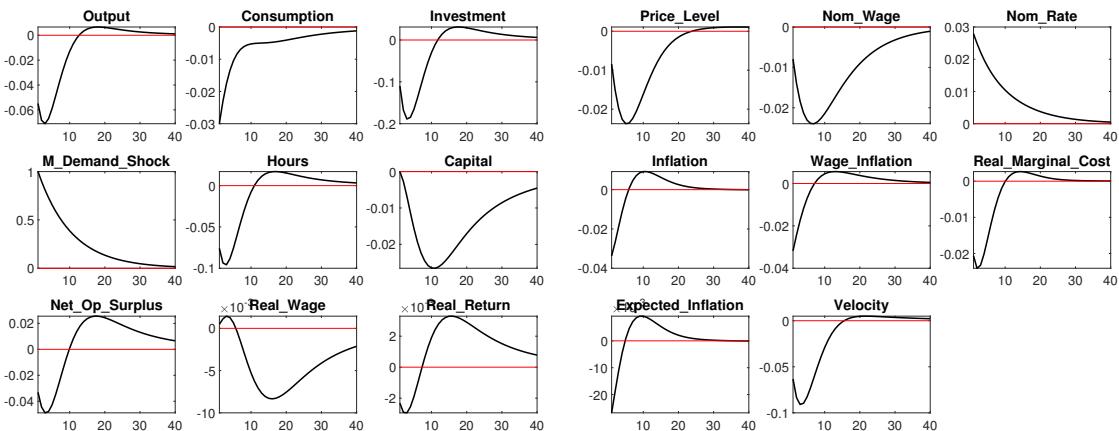
Inflation expectations shock ($PP \uparrow$)



Money supply shock ($M \uparrow$)



Money demand shock ($\nu \uparrow$)



All variables except for rates expressed in % deviations from steady state

Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Taylor Rule + Wage Stickiness

Households

$$\lambda_t = \mathbf{D}_t \cdot C_t^{-\sigma} \quad (1)$$

$$\lambda_t = \beta E_t \left[\frac{1+i_t}{1+\pi_{t+1}^e} \lambda_{t+1} \right] \quad (2)$$

$$M_t^d = (\nu_{ss} \cdot \nu_t)^{1/\sigma} P_t C_t [(1+i_t)/i_t]^{1/\sigma} \quad (3)$$

Household wage setting

$$\tilde{w}_t = \left[\frac{\varepsilon^W}{\varepsilon^W - 1} \frac{X_{1,t}^W}{X_{2,t}^W} \right]^{1/(1+\epsilon_W \varphi)} \quad (4)$$

$$X_{1,t}^W = \psi / \mathbf{H}_t \cdot w_t^{\varepsilon^W(1+\varphi)} L_t^{1+\varphi} + \beta \theta^W E_t \left[(1+\pi_{t+1}^e)^{\varepsilon^W(1+\varphi)} X_{1,t+1}^W \right] \quad (5)$$

$$X_{2,t}^W = \lambda_t w_t^{\varepsilon^W} L_t + \beta \theta^W E_t \left[(1+\pi_{t+1}^e)^{\varepsilon^W-1} X_{2,t+1}^W \right] \quad (6)$$

$$w_t^{1-\varepsilon^W} = \theta^W \left(\frac{w_{t-1}}{1+\pi_t} \right)^{1-\varepsilon^W} + (1-\theta^W) \tilde{w}_t^{1-\varepsilon^W} \quad (7)$$

Firms

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha} / \Delta_t \quad (8)$$

$$w_t = m c_t (1-\alpha) Y_t / L_t \quad (9)$$

$$r_t^K = m c_t \alpha Y_t / K_t \quad (10)$$

Firm price setting

$$\tilde{p}_t = \frac{\varepsilon}{\varepsilon-1} \frac{X_{1,t}}{X_{2,t}} \quad (11)$$

$$X_{1,t} = \lambda_t Y_t m c_t + \theta \beta E_t \left[(1+\pi_{t+1}^e)^\varepsilon X_{1,t+1} \right] \quad (12)$$

$$X_{2,t} = \lambda_t Y_t + \theta \beta E_t \left[(1+\pi_{t+1}^e)^{\varepsilon-1} X_{2,t+1} \right] \quad (13)$$

Inflation dynamics and price dispersion

$$(1+\pi_t)^{1-\varepsilon} = \theta + (1-\theta) (\tilde{p}_t (1+\pi_t))^{1-\varepsilon} \quad (14)$$

$$\Delta_t = \theta \Delta_{t-1} (1+\pi_t)^\varepsilon + (1-\theta) \tilde{p}_t^{-\varepsilon} \quad (15)$$

Capital and investment

$$K_{t+1} = I_t + (1-\delta) K_t \quad (16)$$

$$\frac{1+r_t^K}{q_{t-1}} = \frac{r_t^K + (1-\delta) q_t}{q_{t-1}} \quad (17)$$

$$1 = q_t \left[1 - \frac{\Psi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Psi \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + E_t \left[\frac{q_{t+1}}{1+r_{t+1}} \Psi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \quad (18)$$

Fiscal policy

$$G_t = \gamma_t \cdot (G/Y)_{ss} Y_t \quad (19)$$

$$\ln \gamma_t = \rho_G \ln \gamma_{t-1} + \epsilon_{G,t} \quad (20)$$

Monetary policy, with $\rho_i = 0.8$, $\gamma_\pi = 1.5$ and 0% inflation target

$$i_t = \rho_i i_{t-1} + \rho_i (i_{ss} + \gamma_\pi \pi_t) + \epsilon_{i,t} \quad (21)$$

$$M_t = M_t^d \quad (22)$$

National accounting

$$Y_t = C_t + I_t + G_t \quad (23)$$

Nominal variables

$$V_t = P_t Y_t / M_t \quad (24)$$

$$W_t = w_t P_t \quad (25)$$

$$1 + \pi_t = P_t / P_{t-1} \quad (26)$$

$$1 + \pi_t^W = W_t / W_{t-1} \quad (27)$$

$$1 + \pi_{t+1}^e = E_t [1 + \pi_{t+1}] \cdot \textcolor{red}{PP}_t \quad (28)$$

$$E_t [1 + r_{t+1}] = (1 + i_t) / (1 + \pi_{t+1}^e) \quad (29)$$

Non-policy shocks

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} \quad (30)$$

$$\ln D_t = \rho_D \ln D_{t-1} + \epsilon_{D,t} \quad (31)$$

$$\ln H_t = \rho_H \ln H_{t-1} + \epsilon_{H,t} \quad (32)$$

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \quad (33)$$

$$\ln PP_t = \rho_P \ln PP_{t-1} + \epsilon_{P,t} \quad (34)$$

Reporting

$$NOS_t = Y_t - w_t L_t - \delta K_t \quad (35)$$

$$Ret_t^K = NOS_t / K_t \quad (36)$$

$$LS_t = w_t L_t / Y_t \quad (37)$$

$$mc_t = \frac{w_t^{1-\alpha}}{1-\alpha} \frac{(r_t^K)^\alpha}{\alpha} \frac{1}{Z_t} \quad (38)$$

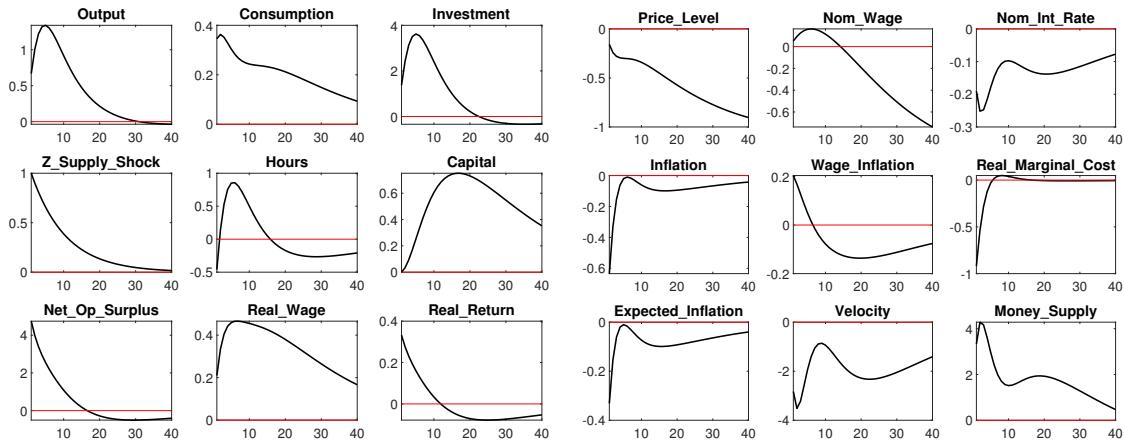
$$mc_{ss} = \frac{\varepsilon - 1}{\varepsilon} \quad (39)$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \ln (mc_t / mc_{ss}) \quad (40)$$

New Keynesian + Taylor Rule + Wage Stickiness: Impulse Response Functions

Total Factor Productivity shock ($Z \uparrow$)

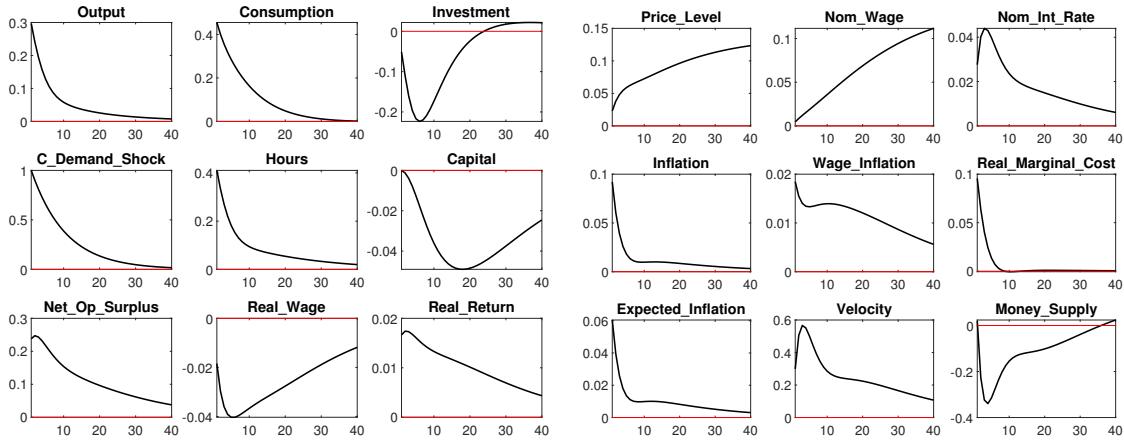


All variables except for rates expressed in % deviations from steady state

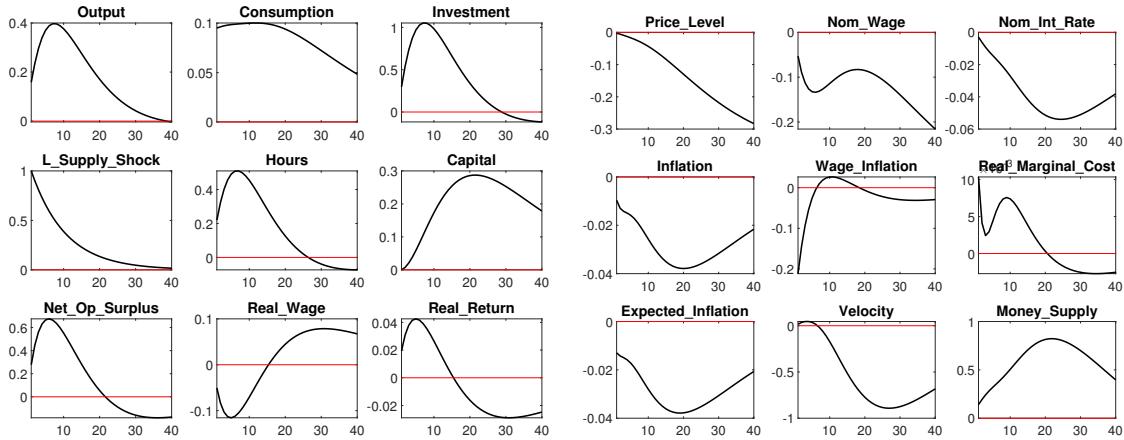
Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Taylor Rule + Wage Stickiness: Impulse Response Functions

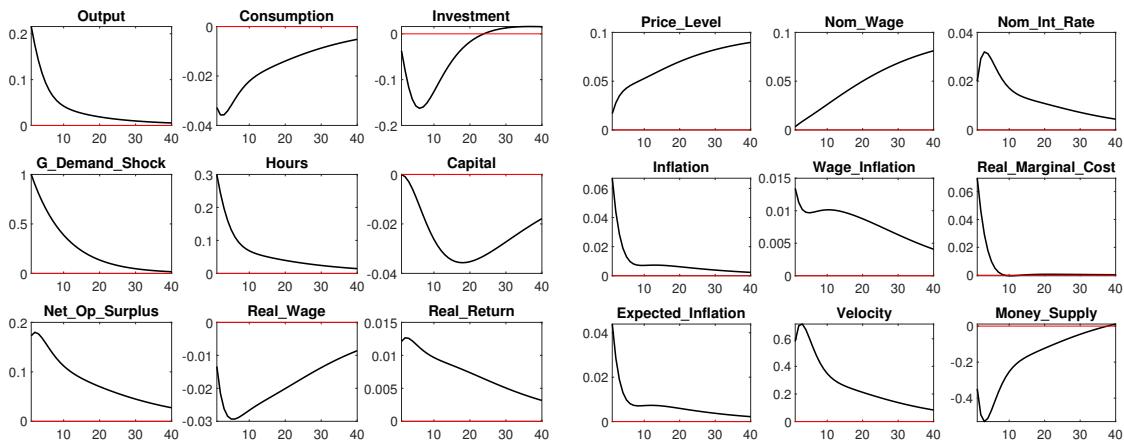
Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)



Public expenditure shock ($\gamma \uparrow$)

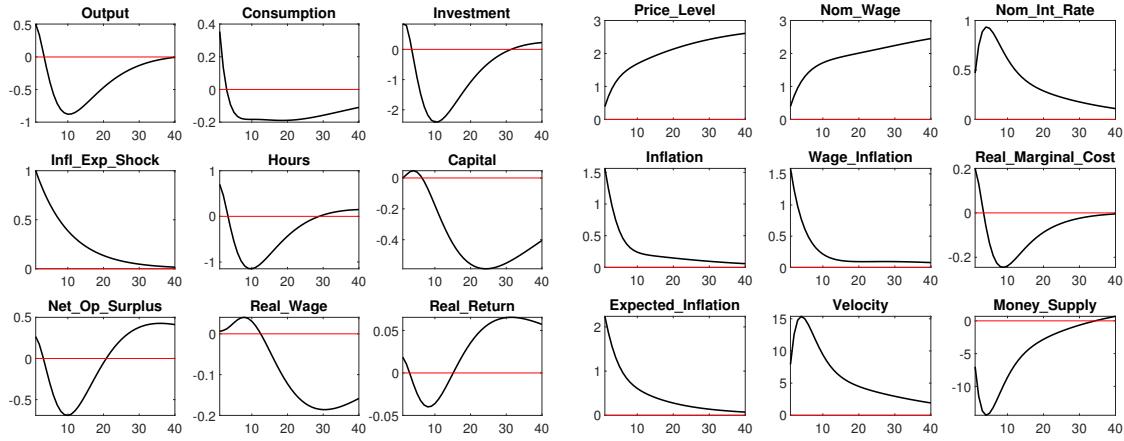


All variables except for rates expressed in % deviations from steady state

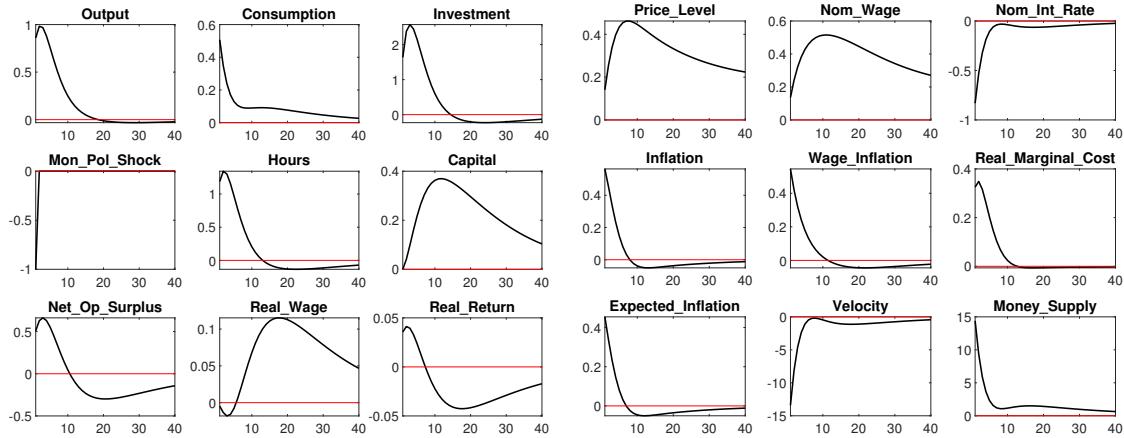
Rate variables are annualized and expressed in percentage points deviations from steady state

New Keynesian + Taylor Rule + Wage Stickiness: Impulse Response Functions

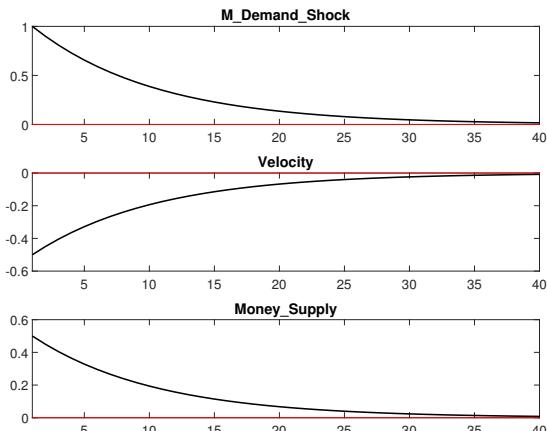
Inflation expectations shock ($PP \uparrow$)



Monetary policy shock ($i \downarrow$)



Money demand shock ($\nu \uparrow$)



No reaction of other variables

All variables except for rates expressed in % deviations from steady state

Rate variables are annualized and expressed in percentage points deviations from steady state

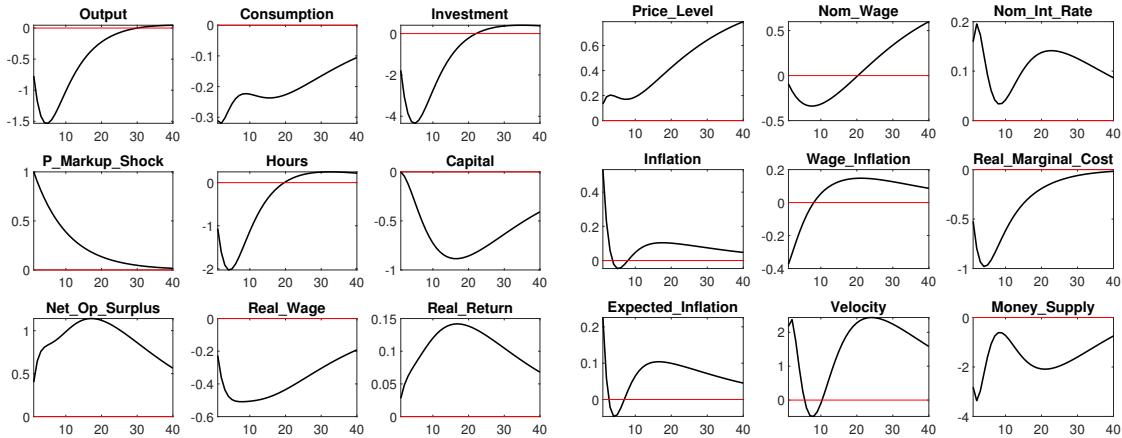
**New Keynesian + Taylor Rule + Wage Stickiness:
Reactions to markup shocks (“greedflation”)**

$$\begin{aligned}\varepsilon_t &= \rho_\varepsilon \varepsilon_{t-1} + (1 - \rho_\varepsilon) \varepsilon_{ss} + \epsilon_{\varepsilon,t} \\ \varepsilon_t^W &= \rho_\varepsilon^W \varepsilon_{t-1}^W + (1 - \rho_\varepsilon^W) \varepsilon_{ss}^W + \epsilon_{\varepsilon,t}^W\end{aligned}$$

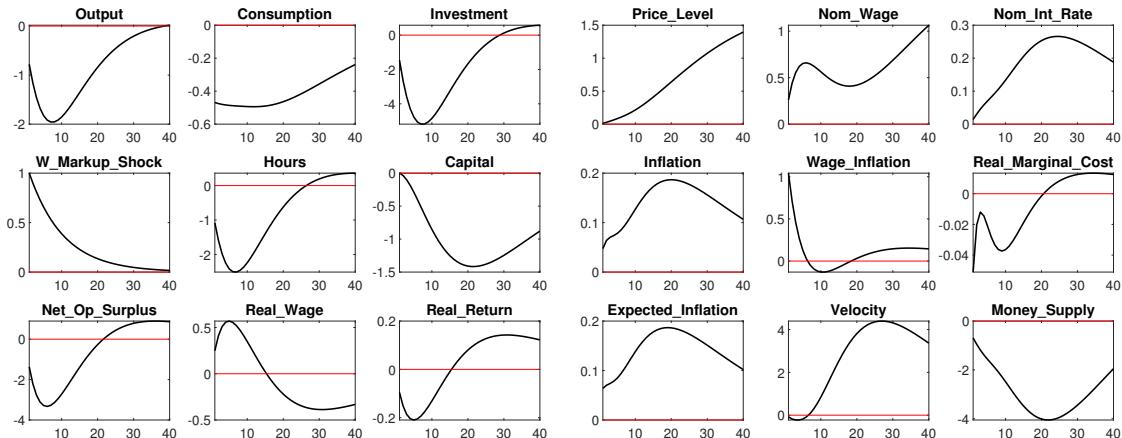
Markups

$$1 + \mu_t = \frac{\varepsilon_t}{\varepsilon_t - 1} \quad 1 + \mu_t^W = \frac{\varepsilon_t^W}{\varepsilon_t^W - 1}$$

Price markup shock ($\varepsilon \downarrow \equiv \mu \uparrow$)



Wage markup shock ($\varepsilon^W \downarrow \equiv \mu^W \uparrow$)



All variables except for rates expressed in % deviations from steady state

Rate variables are annualized and expressed in percentage points deviations from steady state

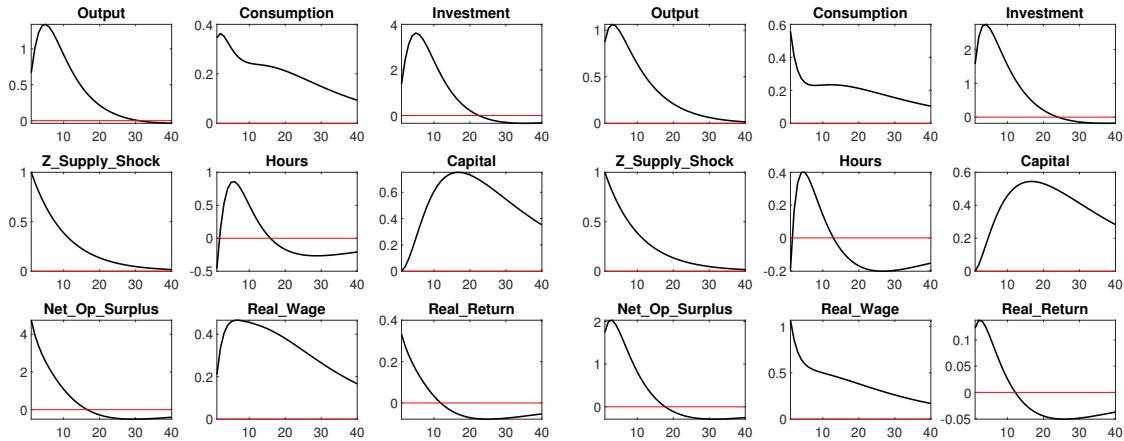
New Keynesian + Taylor Rule + Wage Stickiness vs Real Business Cycles + Money In Utility

Impulse Response Functions of real variables

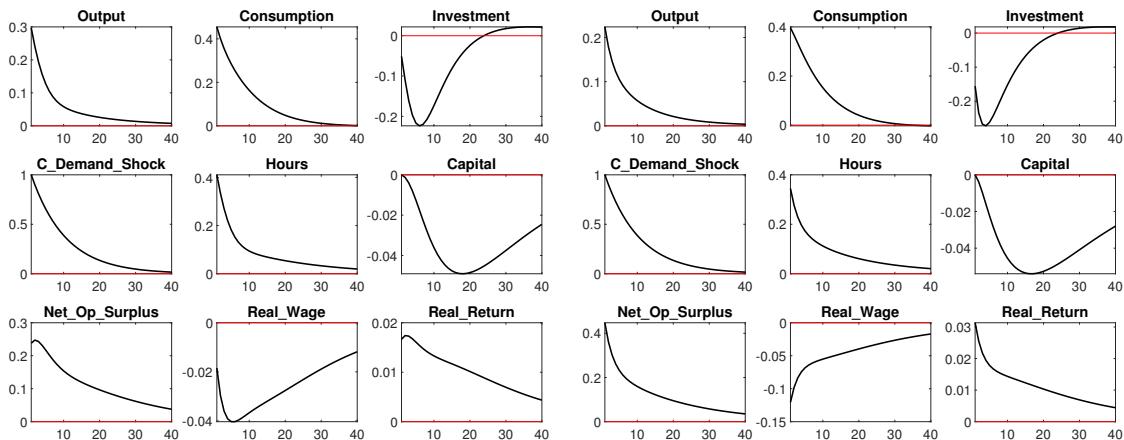
NK + TR + WS

RBC + MIU

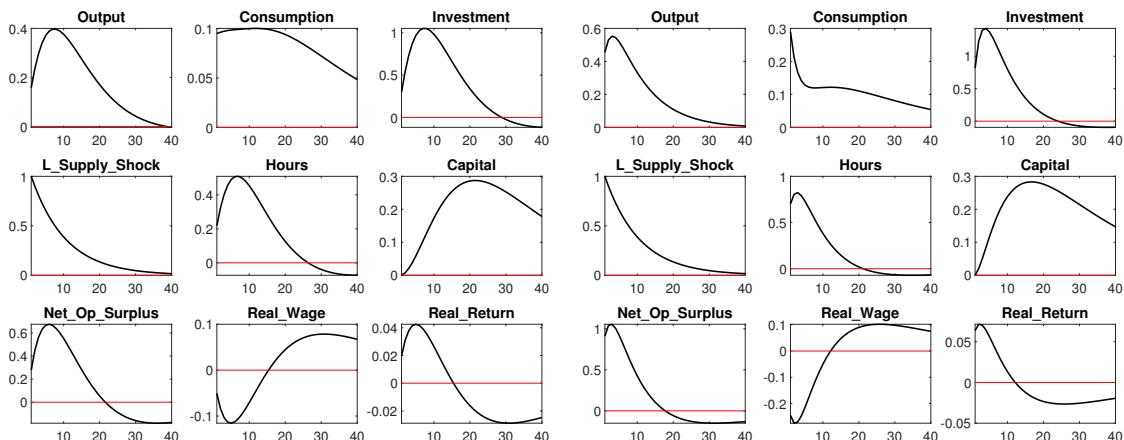
Total Factor Productivity shock ($Z \uparrow$)



Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)

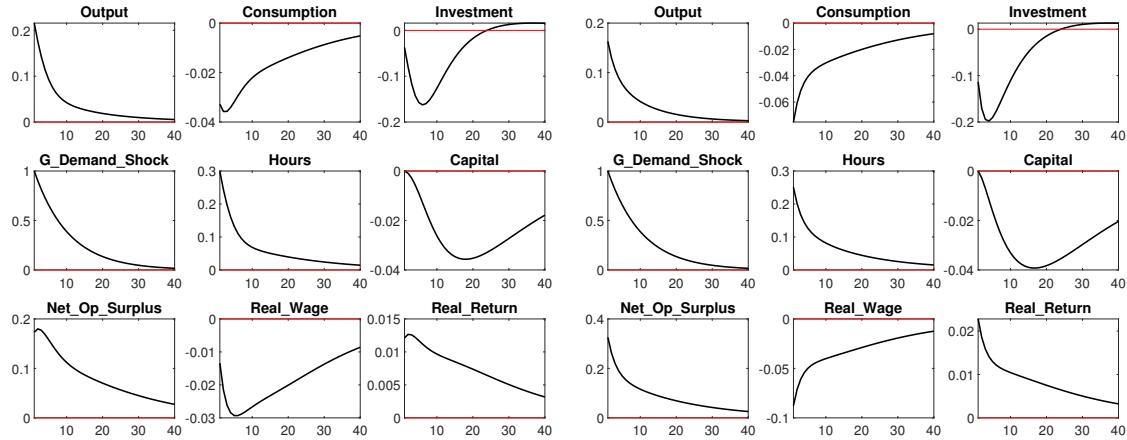


Impulse Response Functions of real variables

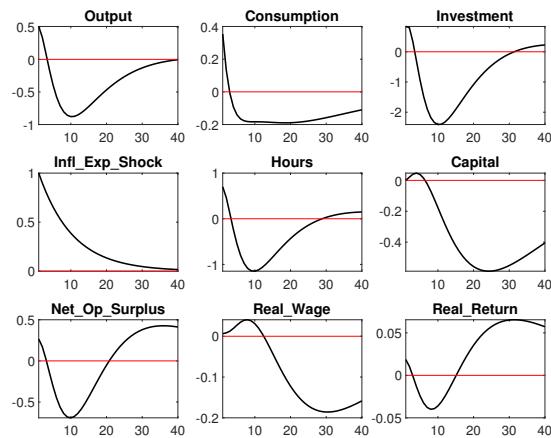
NK + TR + WS

RBC + MIU

Public expenditure shock ($\gamma \uparrow$)

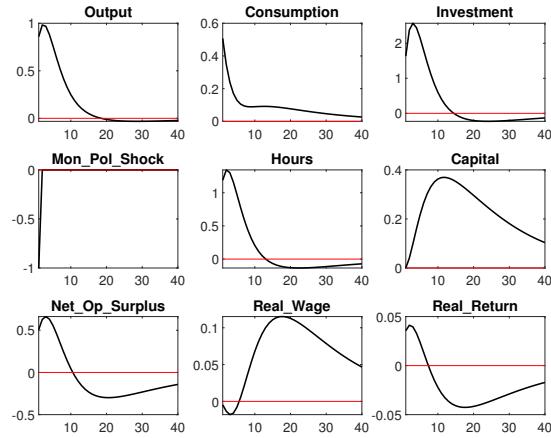


Inflation expectations shock ($PP \uparrow$)



No reaction of real variables

Monetary policy shock ($i \downarrow$)



No reaction of real variables

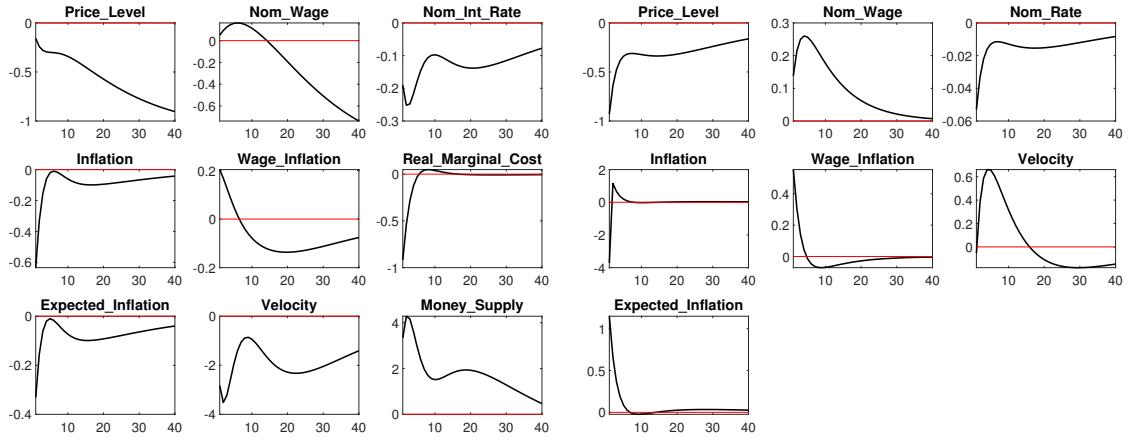
Money supply shock ($M \uparrow$)

Impulse Response Functions of nominal variables

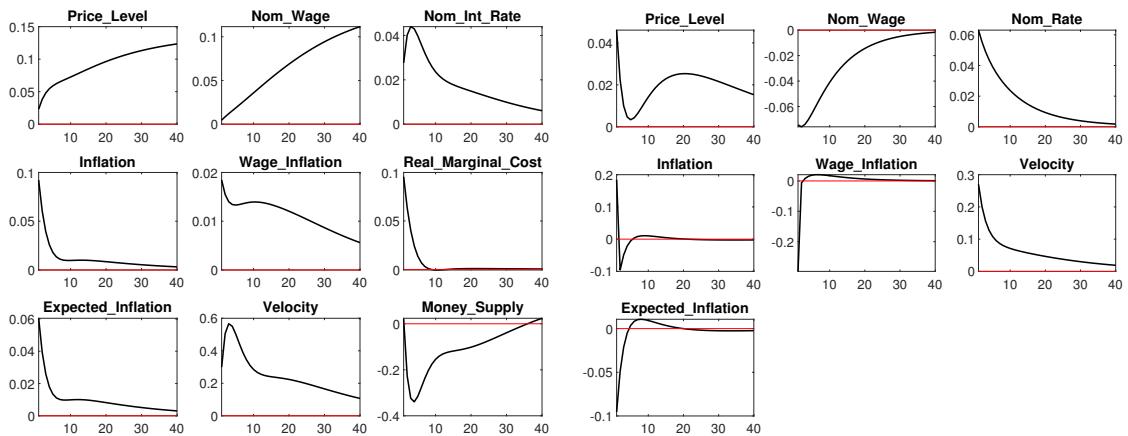
NK + TR + WS

RBC + MIU

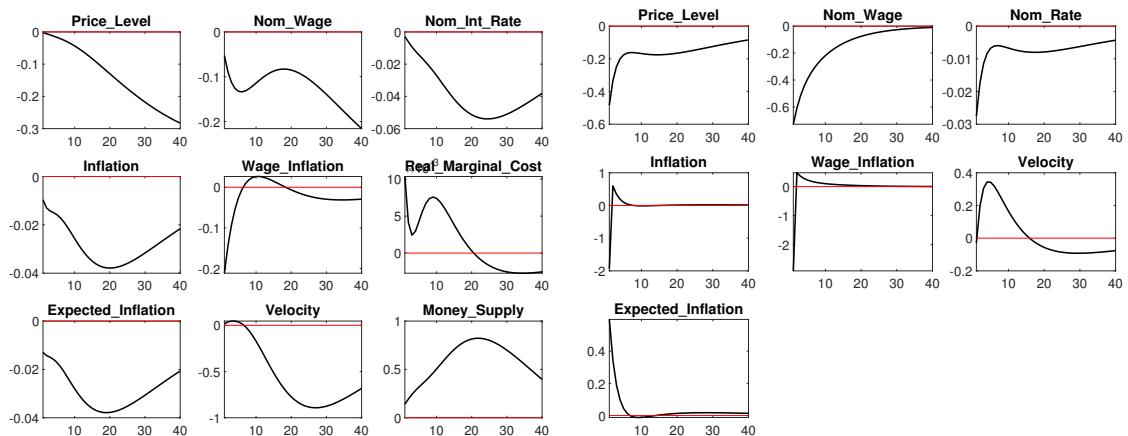
Total Factor Productivity shock ($Z \uparrow$)



Consumption demand shock ($D \uparrow$)



Labor supply shock ($H \uparrow$)

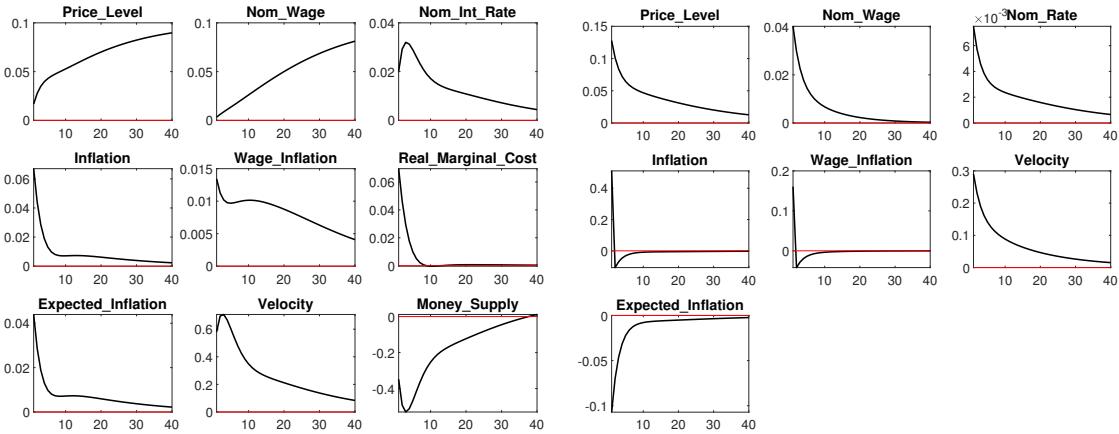


Impulse Response Functions of nominal variables

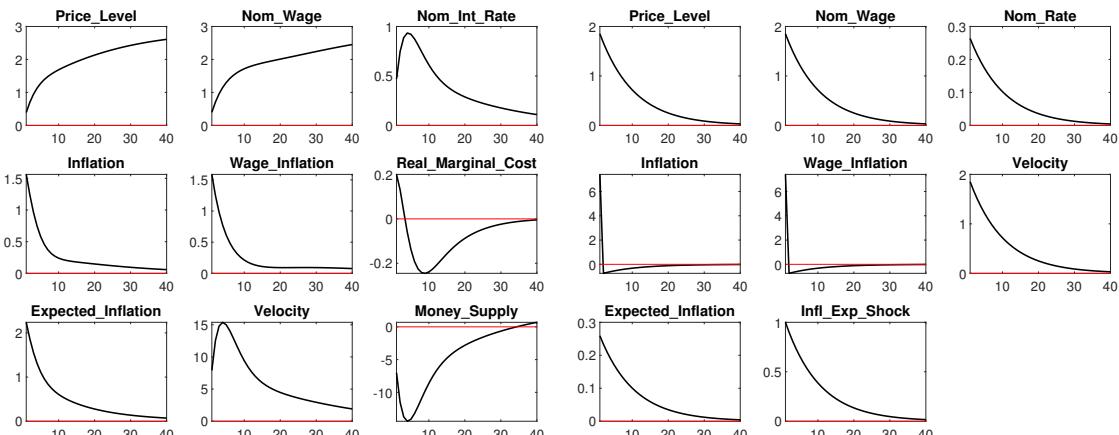
NK + TR + WS

RBC + MIU

Public expenditure shock ($\gamma \uparrow$)

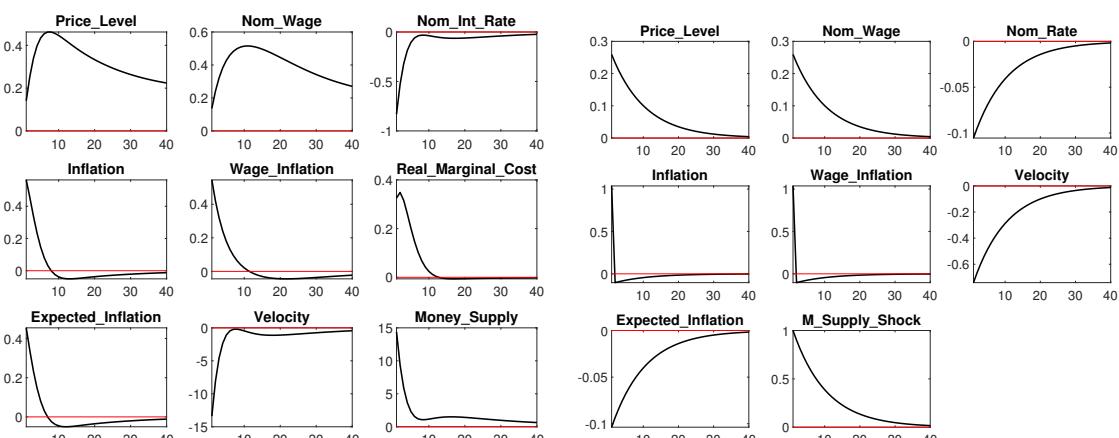


Inflation expectations shock ($PP \uparrow$)



Monetary policy shock ($i \downarrow$)

Money supply shock ($M \uparrow$)



Monetary vs fiscal dominance

TBD

Search and matching in the labor market

TBD

Heterogeneous household economies: RANK vs TANKs vs HANKs

TBD