Problem 4

a)

Sequential employment: first in first out. Let the number of all unemployed be $\bar{L} - NL$ and the job finding rate is a. Then the length of the unemployment spell for a newly unemployed is

$$T = \frac{1}{a}$$

In the steady state there is equilibrium between inflows and outflows so

$$\begin{aligned} a &= \frac{NLb}{\bar{L} - NL} = b\left(\frac{1-u}{u}\right) \\ T &= \frac{\bar{L} - NL}{NLb} = \frac{u}{b(1-u)} \end{aligned}$$

b)

$$V_U = e^{-\rho T} V_E = e^{-\frac{\rho}{a}} V_E$$

Use inequality $e^x > 1 + x$, valid for all $x \neq 0$ ($e^0 = 1$)

$$\begin{aligned} V_E &= e^{\frac{\rho}{a}} V_U > \left(1 + \frac{\rho}{a}\right) V_U \\ V_E - V_U &> \frac{\rho}{a} V_U \\ a(V_E - V_U) &> \rho V_U \\ \rho V_U &< a(V_E - V_U) \end{aligned}$$

Intuition: in this model the flow value of unemployment is lower than in the standard one -> lower wages and unemployment rate

c)

NSC

$$V_E - V_U = V_E\left(1 - e^{-\frac{\rho}{a}}\right) \geq \frac{e}{q} \longrightarrow V_E \geq \frac{e}{q\left(1 - e^{-\frac{\rho}{a}}\right)}$$

$$\rho V_E = (w - e) - b(V_E - V_U) = (w - e) - bV_E\left(1 - e^{-\frac{\rho}{a}}\right)$$

$$V_E = \frac{(w-e)}{\rho + b\left(1 - e^{-\frac{\rho}{a}}\right)} \geq \frac{e}{q\left(1 - e^{-\frac{\rho}{a}}\right)}$$

$$w \geq e + \frac{\rho + b\left(1 - e^{-\frac{\rho}{a}}\right)}{\left(1 - e^{-\frac{\rho}{a}}\right)} \frac{e}{q} = e + \left(b + \frac{\rho}{1 - e^{-\frac{\rho}{a}}}\right)\frac{e}{q}$$

d)

Claim: $w < w^{SS} \longrightarrow u < u^{SS}$

$$\begin{aligned}
e + \left(b + \frac{\rho}{1 - e^{-\frac{\rho}{a}}}\right)\frac{e}{q} &< e + (a + b + \rho)\frac{e}{q} \\
\frac{\rho}{1 - e^{-\frac{\rho}{a}}} &< \rho + a \\
\rho &< \rho - \rho e^{-\frac{\rho}{a}} + a\left(1 - e^{-\frac{\rho}{a}}\right) \\
\frac{\rho}{a} e^{-\frac{\rho}{a}} &< 1 - e^{-\frac{\rho}{a}} \\
\left(1 + \frac{\rho}{a}\right) e^{-\frac{\rho}{a}} &< 1 \\
1 + \frac{\rho}{a} &< e^{\frac{\rho}{a}}
\end{aligned}$$

Indeed, since $e^x > 1 + x$ we have that $w < w^{SS}$, and so the unemployment rate in this version of the model will be lower than in the original Shapiro-Stiglitz model.

Problem 8

$$\begin{aligned}
\pi &= \frac{(eL)^{\alpha}}{\alpha} - wL \\
U &= w - x \\
e &= 1
\end{aligned}$$

a)

$$\begin{aligned}
\frac{\partial \pi}{\partial L} &= L^{\alpha - 1} - w = 0 \\
L(w) &= w^{\frac{1}{\alpha - 1}} \\
\pi(w) &= \frac{1}{\alpha} w^{\frac{\alpha}{\alpha - 1}} - w^{1 + \frac{1}{\alpha - 1}} = \left(\frac{1}{\alpha} - 1\right) w^{\frac{\alpha}{\alpha - 1}} = \frac{1 - \alpha}{\alpha} w^{\frac{\alpha}{\alpha - 1}}
\end{aligned}$$

b)

Nash maximand

$$\begin{aligned}
\Omega &= (w - x)^{\gamma} \left(\frac{1 - \alpha}{\alpha} w^{\frac{\alpha}{\alpha - 1}}\right)^{1 - \gamma} \\
\ln \Omega &= \gamma \ln(w - x) + (1 - \gamma)\left[\ln\left(\frac{1 - \alpha}{\alpha}\right) - \frac{\alpha}{1 - \alpha} \ln w\right] \\
\frac{\partial \ln \Omega}{\partial w} &= \gamma \frac{1}{w - x} - (1 - \gamma)\left(\frac{\alpha}{1 - \alpha}\right)\frac{1}{w} = 0 \\
\gamma \frac{1}{w - x} &= (1 - \gamma)\left(\frac{\alpha}{1 - \alpha}\right)\frac{1}{w} \\
\gamma w &= (1 - \gamma)\left(\frac{\alpha}{1 - \alpha}\right)(w - x)
\end{aligned}$$

$$w\left[\gamma-(1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)\right]=(1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)(-x)$$

$$0<\gamma<\alpha<1$$

$$(1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)=(1-\gamma)\left(\frac{1}{1-\alpha}-1\right)=\frac{1-\gamma}{1-\alpha}-1+\gamma=\frac{\alpha-\gamma}{1-\alpha}+\gamma>0$$

$$w\left[1-\frac{1-\gamma}{1-\alpha}\right]=-\left(\frac{\alpha-\gamma}{1-\alpha}+\gamma\right)x$$

$$w=\frac{\frac{\alpha-\gamma}{1-\alpha}+\gamma}{\frac{\alpha-\gamma}{1-\alpha}}x=\frac{\alpha-\gamma+\gamma-\alpha\gamma}{\alpha-\gamma}x=\frac{\alpha-\alpha\gamma}{\alpha-\gamma}x$$

$$w>x$$

c)

$$\begin{aligned}
\ln w &= \ln x+\ln(\alpha-\alpha\gamma)-\ln(\alpha-\gamma)\\
\frac{\partial(\ln w)}{\partial\gamma} &= \frac{-\alpha}{\alpha-\alpha\gamma}-\frac{-1}{\alpha-\gamma}=\frac{-1}{1-\gamma}+\frac{1}{\alpha-\gamma}>0\\
\frac{\partial(\ln w)}{\partial\gamma}|_{\gamma=0} &= -1+\frac{1}{\alpha}=\frac{1-\alpha}{\alpha}>0
\end{aligned}$$

d)

$$\begin{aligned}
\pi &= \frac{\left[((w-x)/x)^{\beta}L\right]^{\alpha}}{\alpha}-wL\\
&= \frac{(w-x)^{\alpha\beta}}{\alpha x^{\alpha\beta}}L^{\alpha}-wL\\
\frac{\partial\pi}{\partial L} &= \frac{(w-x)^{\alpha\beta}}{x^{\alpha\beta}}L^{\alpha-1}-w=0\\
L^{\alpha-1} &= \frac{w\cdot x^{\alpha\beta}}{(w-x)^{\alpha\beta}}\\
L(w) &= \left[\frac{w\cdot x^{\alpha\beta}}{(w-x)^{\alpha\beta}}\right]^{\frac{1}{\alpha-1}}=w^{\frac{1}{\alpha-1}}\left(\frac{x}{w-x}\right)^{\frac{\alpha\beta}{\alpha-1}}\\
\pi(w) &= \frac{1}{\alpha}\left(\frac{w-x}{x}\right)^{\alpha\beta}w^{\frac{\alpha}{\alpha-1}}\left(\frac{x}{w-x}\right)^{\frac{\alpha^2\beta}{\alpha-1}}-w^{1+\frac{1}{\alpha-1}}\left(\frac{x}{w-x}\right)^{\frac{\alpha\beta}{\alpha-1}}\\
&= \frac{1}{\alpha}w^{\frac{\alpha}{\alpha-1}}\left(\frac{w-x}{x}\right)^{\alpha\beta+\frac{\alpha^2\beta}{1-\alpha}}-w^{\frac{\alpha}{\alpha-1}}\left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}}\\
&\left\{\alpha\beta+\frac{\alpha^2\beta}{1-\alpha}=\frac{\alpha\beta}{1-\alpha}\right\}\\
&= w^{\frac{\alpha}{\alpha-1}}\left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)=\frac{1-\alpha}{\alpha}w^{\frac{\alpha}{\alpha-1}}\left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}}
\end{aligned}$$

e)

$$
\begin{aligned}
\Omega &= (w-x)^{\gamma}\left(\frac{1-\alpha}{\alpha}w^{\frac{\alpha}{\alpha-1}}\left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}}\right)^{1-\gamma} \\
\ln\Omega &= \gamma\ln(w-x)+(1-\gamma)\left[\ln\left(\frac{1-\alpha}{\alpha}\right)-\frac{\alpha}{1-\alpha}\ln w+\frac{\alpha\beta}{1-\alpha}\ln\left(\frac{1}{x}\right)+\frac{\alpha\beta}{1-\alpha}\ln(w-x)\right] \\
\frac{\partial\ln\Omega}{\partial w} &= \gamma\frac{1}{w-x}+(1-\gamma)\left[-\frac{\alpha}{1-\alpha}\frac{1}{w}+\frac{\alpha\beta}{1-\alpha}\frac{1}{w-x}\right]=0 \\
& (1-\gamma)\frac{\alpha}{1-\alpha}\frac{1}{w}=\frac{1}{w-x}\left[\gamma+(1-\gamma)\frac{\alpha\beta}{1-\alpha}\right] \\
& \left\{\gamma+(1-\gamma)\frac{\alpha\beta}{1-\alpha}=\frac{\gamma-\alpha\gamma+\alpha\beta-\alpha\beta\gamma}{1-\alpha}\right\} \\
& (\alpha-\alpha\gamma)\frac{1}{w}=\frac{1}{w-x}(\gamma-\alpha\gamma+\alpha\beta-\alpha\beta\gamma) \\
& (\alpha-\alpha\gamma)(w-x)=(\gamma-\alpha\gamma+\alpha\beta-\alpha\beta\gamma)w \\
& w(\alpha-\gamma-\alpha\beta+\alpha\beta\gamma)=\alpha(1-\gamma)x \\
& w=\frac{\alpha(1-\gamma)}{\alpha-\gamma-\alpha\beta+\alpha\beta\gamma}x=\{\beta=0\}=\frac{\alpha-\alpha\gamma}{\alpha-\gamma}x
\end{aligned}
$$

f)

$$
\begin{aligned}
\ln w &= \ln x+\ln(\alpha-\alpha\gamma)-\ln(\alpha-\gamma-\alpha\beta+\alpha\beta\gamma) \\
\frac{\partial(\ln w)}{\partial\gamma} &= \frac{-\alpha}{\alpha-\alpha\gamma}-\frac{-1+\alpha\beta}{\alpha-\gamma-\alpha\beta+\alpha\beta\gamma} \\
\frac{\partial(\ln w)}{\partial\gamma}|_{\gamma=0} &= -1+\frac{1-\alpha\beta}{\alpha-\alpha\beta}=\frac{1-\alpha\beta-\alpha+\alpha\beta}{\alpha-\alpha\beta}=\frac{1-\alpha}{\alpha(1-\beta)}>0
\end{aligned}
$$

$$
\frac{1-\alpha}{\alpha(1-\beta)}>\frac{1-\alpha}{\alpha}
$$

Problem 9

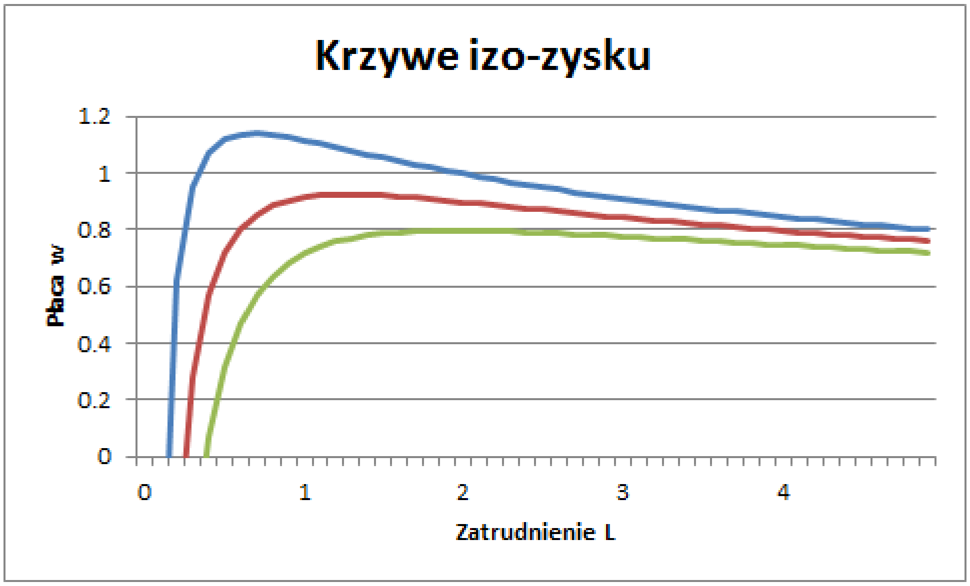
a)

Labor demand

$$
\begin{aligned}
\pi &= \frac{L^{\alpha}}{\alpha}-wL \\
\frac{\partial\pi}{\partial L} &= L^{\alpha-1}-w=0 \\
L^{D} &= w^{\frac{1}{\alpha-1}}
\end{aligned}
$$

Isoprofit

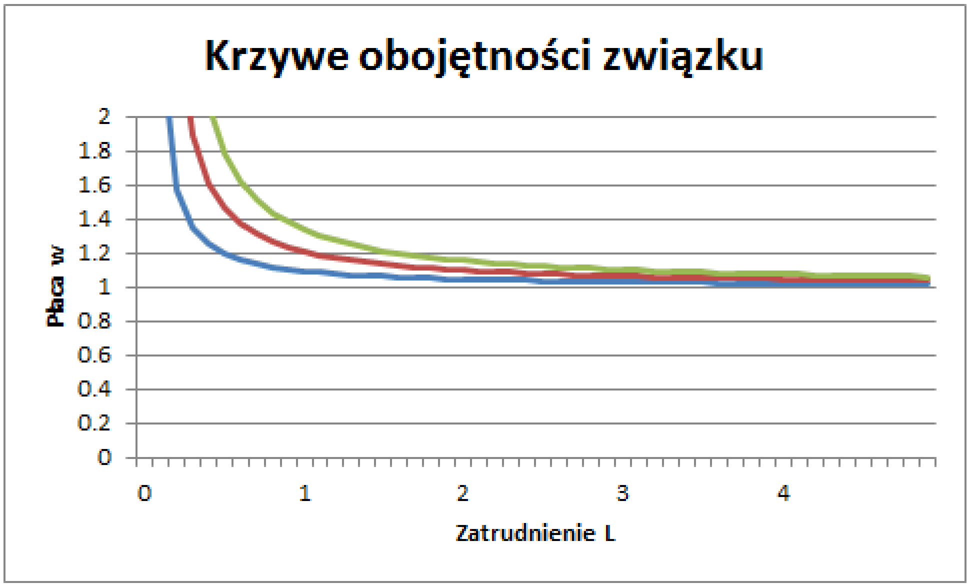
$$\bar{\pi} = \frac{L^{\alpha}}{\alpha} - wL$$
$$wL = \frac{L^{\alpha}}{\alpha} - \bar{\pi}$$
$$w = \frac{L^{\alpha-1}}{\alpha} - \frac{\bar{\pi}}{L}$$



b)

Labor union indifference curves

$$\bar{V} = \frac{L}{N}\ln(w) + \left(1 - \frac{L}{N}\right)\ln(B)$$
$$\exp\left(\bar{V}\right) = w^{\frac{L}{N}} B^{1-\frac{L}{N}}$$
$$w = \left(\frac{\exp\left(\bar{V}\right)}{B^{1-\frac{L}{N}}}\right)^{\frac{N}{L}}$$



c)

Profits

$$\begin{aligned}\pi &= \frac{L^{\alpha}}{\alpha} - wL \\ &= \frac{\left(w^{\frac{1}{\alpha-1}}\right)^{\alpha}}{\alpha} - w\left(w^{\frac{1}{\alpha-1}}\right) \\ &= \left(\frac{1-\alpha}{\alpha}\right) w^{\frac{\alpha}{\alpha-1}}\end{aligned}$$

Nash maximand

$$\begin{aligned}\Omega &= \left[V - \ln(B)\right]^{\beta}\left[\pi - 0\right]^{1-\beta} \\ &= \left[\frac{L}{N}\ln(w) + \left(1 - \frac{L}{N}\right)\ln(B) - \ln(B)\right]^{\beta}\left[\left(\frac{1-\alpha}{\alpha}\right) w^{\frac{\alpha}{\alpha-1}}\right]^{1-\beta} \\ &= \left[\frac{L}{N}\left(\ln(w) - \ln(B)\right)\right]^{\beta}\left[\left(\frac{1-\alpha}{\alpha}\right) w^{\frac{\alpha}{\alpha-1}}\right]^{1-\beta} \\ \ln(\Omega) &= \beta\ln[L] - \beta\ln[N] + \beta\ln[\ln(w) - \ln(B)] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right] + (1-\beta)\left(\frac{\alpha}{\alpha-1}\right)\ln[w] \\ &= \beta\ln\left[w^{\frac{1}{\alpha-1}}\right] - \beta\ln[N] + \beta\ln[\ln(w) - \ln(B)] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right] + (1-\beta)\left(\frac{\alpha}{\alpha-1}\right)\ln[w] \\ &= \left(\frac{\beta}{\alpha-1}\right)\ln[w] - \beta\ln[N] + \beta\ln[\ln(w) - \ln(B)] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right] + (1-\beta)\left(\frac{\alpha}{\alpha-1}\right)\ln[w] \\ &= \left(\frac{\alpha+\beta-\alpha\beta}{\alpha-1}\right)\ln[w] + \beta\ln[\ln(w) - \ln(B)] - \beta\ln[N] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right]\end{aligned}$$

d)

$$\begin{aligned}\frac{\partial \ln(\Omega)}{\partial \ln(w)} &= \left(\frac{\alpha+\beta-\alpha\beta}{\alpha-1}\right)+\frac{\beta}{\ln(w)-\ln(B)}=0\\ \frac{\alpha+\beta-\alpha\beta}{1-\alpha} &= \frac{\beta}{\ln(w)-\ln(B)}\\ \ln(w)-\ln(B) &= \frac{\beta(1-\alpha)}{\alpha+\beta-\alpha\beta}\\ \ln(w) &= \ln(B)+\frac{\beta(1-\alpha)}{\alpha+\beta-\alpha\beta}\end{aligned}$$

e)

$$\begin{aligned}\frac{\partial \ln(w)}{\partial \beta} &= \frac{(1-\alpha)(\alpha+\beta-\alpha\beta)-\beta(1-\alpha)(1-\alpha)}{(\alpha+\beta-\alpha\beta)^2}\\ &= \frac{(1-\alpha)(\alpha+\beta-\alpha\beta-\beta+\alpha\beta)}{(\alpha+\beta-\alpha\beta)^2}\\ &= \frac{(1-\alpha)\alpha}{(\alpha+\beta-\alpha\beta)^2}>0\\ \frac{\partial L}{\partial \beta} &= \frac{\partial L^D}{\partial w}\frac{\partial w}{\partial \ln(w)}\frac{\partial \ln(w)}{\partial \beta}\\ &= \left(\frac{1}{\alpha-1}\right)w^{\frac{1}{\alpha-1}-1}\cdot w\cdot\frac{(1-\alpha)\alpha}{(\alpha+\beta-\alpha\beta)^2}<0\end{aligned}$$

f)

$$\begin{aligned}|\varepsilon_D| &= -\frac{\partial L^D}{\partial w}\frac{w}{L^D}\\ &= -\left(\frac{1}{\alpha-1}\right)w^{\frac{1}{\alpha-1}-1}\cdot\frac{w}{w^{\frac{1}{\alpha-1}}}\\ &= \frac{1}{1-\alpha}\\ \frac{\partial|\varepsilon_D|}{\partial \alpha} &= \frac{1}{(1-\alpha)^2}>0\end{aligned}$$

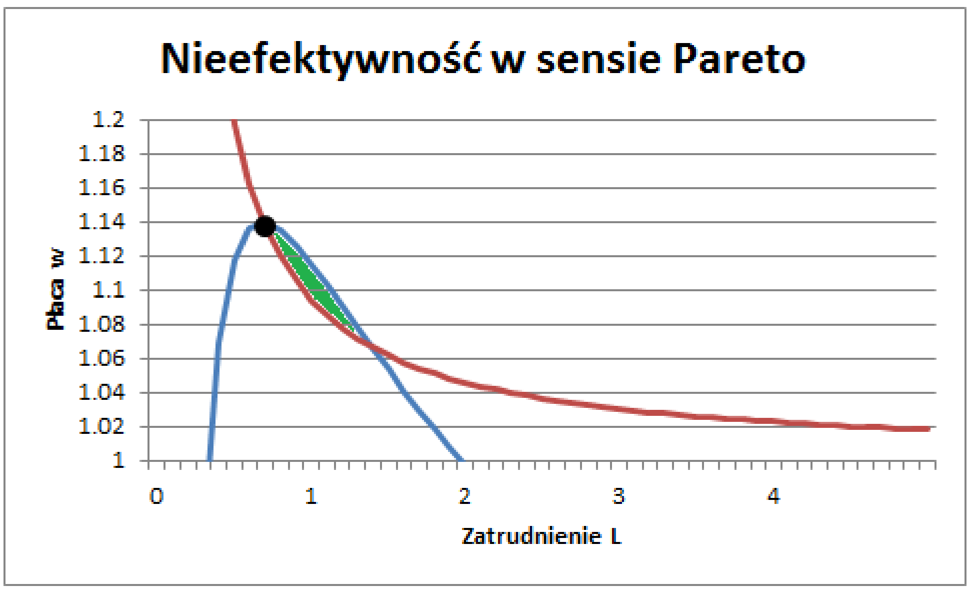
Useful to calculate $\partial \ln(w)/\partial\alpha$

$$
\begin{aligned}
\frac{\partial \ln(w)}{\partial \alpha} &= \frac{-(\alpha+\beta-\alpha\beta)-\beta(1-\alpha)(1-\beta)}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{-\alpha-\beta+\alpha\beta-\beta(1-\alpha-\beta+\alpha\beta)}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{-\alpha-\beta+\alpha\beta-\beta+\alpha\beta+\beta^2-\alpha\beta^2}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{\alpha\left(-1+\beta-\beta^2\right)+\beta(-2+\beta)}{(\alpha+\beta-\alpha\beta)^2} < 0 \\
\frac{\partial L}{\partial \alpha} &= \frac{\partial L^D}{\partial w}\frac{\partial w}{\partial \ln(w)}\frac{\partial \ln(w)}{\partial \alpha} \\
&= (-)\cdot(+)\cdot(-) > 0
\end{aligned}
$$

Then employment increases with α and wages will decrease (closer to competitive outcome)

g)

The equilibrium is not Pareto efficient since there exist allocations that would have been preferred by both the firm and the labor union (equilibrium marked with black dot, better allocations marked with green area).



Problem 10

Definitions

$$
\begin{aligned}
\theta &\equiv \frac{v}{u} \\
\frac{m(u,v)}{u} &\equiv p(\theta) \\
\frac{m(u,v)}{v} &= \frac{p(\theta)}{\theta} \equiv q(\theta)
\end{aligned}
$$

The dynamics of unemployment

$$\begin{aligned} \dot{u} &= s(1-u) - p(\theta)u \\ \dot{u} &= 0 \longrightarrow u = \frac{s}{s + p(\theta)} \end{aligned}$$

Job creation by firms

$$\begin{aligned} rJ &= (y-w) - s(J-V) + \dot{J} \\ rV &= -c + q(\theta)(J-V) + \dot{V} \end{aligned}$$

Steady state: $\dot{J} = \dot{V} = V = 0$

$$\begin{cases} J &= \frac{c}{q(\theta)} \\ J &= \frac{y-w}{r+s} \end{cases} \longrightarrow y - w = (r+s)\frac{c}{q(\theta)}$$

Job seeking by workers

$$\begin{aligned} rE &= w - s(E-U) + \dot{E} \\ rU &= z + p(\theta)(E-U) + \dot{U} \end{aligned}$$

Steady state: $\dot{E} = \dot{U} = 0$

$$E - U = \frac{w - z}{r + s + p(\theta)}$$

Match surplus

$$S = (J+E) - (V+U) = (J-V) + (E-U)$$

Nash bargaining

$$w = \arg\max_{w} \left\{ (E(w) - U)^{\beta} (J(w) - V)^{1-\beta} \right\}$$

FOC

$$\beta (E(w) - U)^{\beta-1} (J(w) - V)^{1-\beta} + (E(w) - U)^{\beta} (1-\beta)(J(w) - V)^{-\beta} (-1) = 0$$

$$(1-\beta)(E-U) = \beta(J-V)$$

$$E - U = \frac{\beta}{1-\beta}(J-V)$$

$$E - U = \beta[(J-V) + (E-U)] = \beta S$$

Negotiations outcome

$$\begin{aligned} (1-\beta)\left(\frac{w + sU}{r+s} - U\right) &= \beta\left(\frac{y-w}{r+s}\right) \\ (1-\beta)(w - rU) &= \beta(y-w) \end{aligned}$$

$$w = \beta y + (1-\beta) rU$$

Additionally

$$(E-U) = \frac{\beta}{1-\beta}(J-V) = \frac{\beta}{1-\beta}\frac{c}{q(\theta)}$$

$$\begin{aligned} rU &= z + p(\theta)(E-U) = z + \theta q(\theta)(E-U) \\ rU &= z + \theta q(\theta)\frac{\beta}{1-\beta}\frac{c}{q(\theta)} = z + \frac{\beta}{1-\beta}c\theta \end{aligned}$$

Join conditions

$$w = \beta y + (1-\beta) rU = \beta y + (1-\beta)\left(z + \frac{\beta}{1-\beta}c\theta\right) = z + \beta(y + c\theta - z)$$

Steady state equilibrium

$$\begin{aligned} &(BC) \qquad u = \frac{s}{s + p(\theta)} \\ &(JC) \qquad w = y - (r+s)\frac{c}{q(\theta)} \\ &(W) \qquad w = (1-\beta) z + \beta(y + c\theta) \end{aligned}$$

a)

$s\uparrow\longrightarrow BC\uparrow \quad JC\downarrow\longrightarrow w\downarrow \quad \theta\downarrow\longrightarrow u\uparrow \quad v?$

b)

$y\uparrow\longrightarrow JC\uparrow \quad W\uparrow\longrightarrow w\uparrow \quad \theta\uparrow\longrightarrow u\downarrow \quad v\uparrow$

c)

$r\uparrow\longrightarrow JC\downarrow\longrightarrow w\downarrow \quad \theta\downarrow\longrightarrow u\uparrow \quad v\downarrow$

d)

$\phi\uparrow\longrightarrow p(\theta)\uparrow \quad q(\theta)\uparrow\longrightarrow BC\downarrow \quad JC\uparrow\longrightarrow w\uparrow \quad \theta\uparrow\longrightarrow u\downarrow \quad v?$

Problem 11

a)

Job creation by firms

$$\begin{aligned} rV &= -c_0 w + q(\theta)(J-V) + \dot{V} \\ rJ &= (y-w) - s[J-V] + \dot{J} \end{aligned}$$

Steady state: $\dot{J} = \dot{V} = V = 0$

$$\begin{aligned} 0 &= -c_0 w + q(\theta) J \longrightarrow J = \frac{c_0 w}{q(\theta)} \\ rJ &= (y - w) - sJ \longrightarrow J = \frac{y - w}{r + s} \end{aligned}$$

Job creation condition

$$\frac{c_0 w}{q(\theta)} = \frac{y - w}{r + s} \longrightarrow y - w = (r + s) \frac{c_0 w}{q(\theta)} \longrightarrow y = w \left[1 + (r + s) \frac{c_0}{q(\theta)} \right]$$

Wage determination

$$\begin{aligned} w &= (1 - \beta) z_0 w + \beta (y + c_0 w \theta) \\ w [1 - (1 - \beta) z_0 - \beta c_0 \theta] &= \beta y \\ w &= \frac{\beta y}{1 - (1 - \beta) z_0 - \beta c_0 \theta} \end{aligned}$$

Equilibrium (BC, JC, W)

$$\begin{aligned} u &= \frac{s}{s + p(\theta)} \\ w &= y / \left[1 + (r + s) \frac{c_0}{q(\theta)} \right] \\ w &= \frac{\beta y}{1 - (1 - \beta) z_0 - \beta c_0 \theta} \end{aligned}$$

b)

$$\begin{aligned} \frac{\beta y}{1 - (1 - \beta) z_0 - \beta c_0 \theta} &= \frac{y}{1 + (r + s) \frac{c_0}{q(\theta)}} \\ \beta \left[1 + (r + s) \frac{c_0}{q(\theta)} \right] &= 1 - (1 - \beta) z_0 - \beta c_0 \theta \end{aligned}$$

The above equation is implicit in θ, however its solution does not depend on y. Wages increase proportionately to productivity.

c)

Productivity growth does not impact long-run unemployment rate.

Problem 12

a)

$$\begin{aligned} rV_E &= (w - e) - s (V_E - V_U) \\ rV_S &= w - (s + \mu) (V_S - V_U) \\ rV_U &= z + p(\theta) (V_N - V_U) \\ V_N &= \max \{V_E, V_S\} \end{aligned}$$

NSC

$$V_E \geq V_S \longrightarrow -V_S \geq -V_E$$

$$(w-e) - s(V_E - V_U) \geq w - (s+\mu)(V_E - V_U)$$

$$V_E - V_U \geq \frac{e}{\mu}$$

$$r(V_E - V_U) = (w-e) - s(V_E - V_U) - z - p(\theta)(V_E - V_U)$$

$$\begin{aligned} w &= e + z + (p(\theta) + s + r)(V_E - V_U) \\ w &\geq e + z + (p(\theta) + s + r)\frac{e}{\mu} \end{aligned}$$

b)

$$w = (1-\beta)z + \beta(y + c\theta)$$

c)

Steady state equilibrium

$$\begin{aligned} &(BC) \qquad u = \frac{s}{s + p(\theta)} \\ &(JC) \qquad w = y - (r+s)\frac{c}{q(\theta)} \\ &(W) \qquad w = e + z + (p(\theta) + s + r)\frac{e}{\mu} \end{aligned}$$

$y \uparrow \longrightarrow JC \uparrow \longrightarrow w \uparrow \quad \theta \uparrow \longrightarrow u \downarrow \quad v \uparrow$

d)

$s \uparrow \longrightarrow BC \uparrow \quad JC \downarrow \quad W \uparrow \longrightarrow w? \quad \theta \downarrow\downarrow \longrightarrow u \uparrow\uparrow \quad v?$

Yes, wages do not go down as much which results in lower tightness and higher unemployment

Problem 13

a)

$$\begin{aligned} m(u,v) &= \phi u^{\alpha} v^{1-\alpha} \\ p(\theta) &= \frac{m(u,v)}{u} = \phi u^{\alpha-1} v^{1-\alpha} = \phi \theta^{1-\alpha} \\ q(\theta) &= \frac{p(\theta)}{\theta} = \phi \theta^{-\alpha} \longrightarrow q'(\theta) = -\alpha \phi \theta^{-\alpha-1} \\ \dot{u} &= s(1-u) - p(\theta)u \\ \dot{u} = 0 &\Leftrightarrow u = \frac{s}{s + p(\theta)} = \frac{s}{s + \phi \theta^{1-\alpha}} \end{aligned}$$

b)

$$
\begin{aligned}
rV(t) &= -c + q(\theta(t))\,[J(t) - V(t)] + \dot{V}(t) \\
rJ(t) &= [y - w(t)] + s\,[V(t) - J(t)] + \dot{J}(t)
\end{aligned}
$$

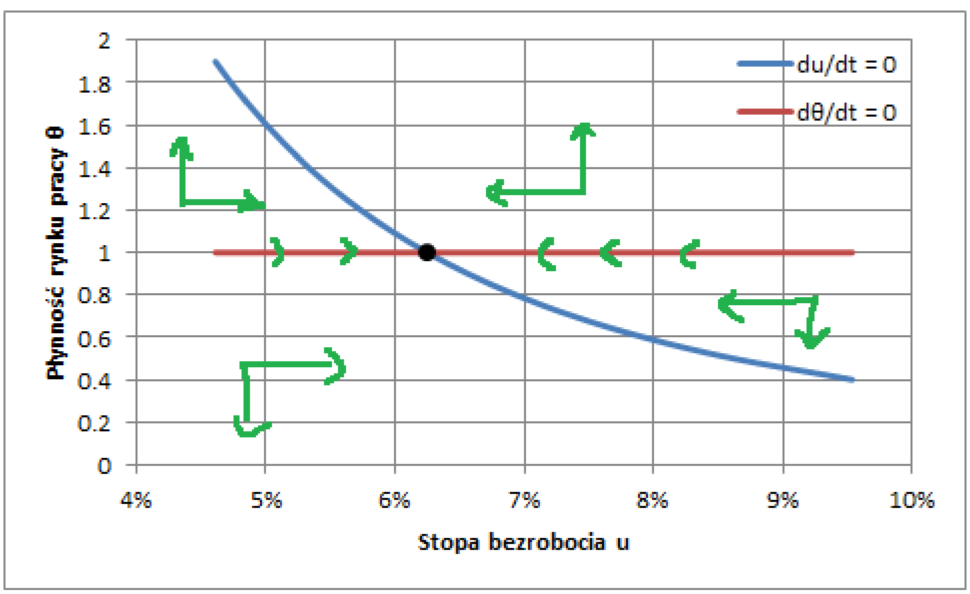
Assume $V = 0$ in equilibrium even outside the steady state

$$
\begin{aligned}
0 &= -c + q(\theta(t))\,J(t) \longrightarrow J(t) = \frac{c}{q(\theta(t))} \\
rJ(t) &= [y - w(t)] - sJ(t) + \dot{J}(t) \longrightarrow \dot{J}(t) = (r+s)\,J(t) - [y - w(t)] \\
\dot{J}(t) &= -\frac{c \cdot q'(\theta(t)) \cdot \dot{\theta}(t)}{[q(\theta(t))]^2} = \frac{c\alpha q(\theta)/\theta}{[q(\theta(t))]^2}\dot{\theta}(t) = \frac{c\alpha}{q(\theta(t))}\frac{\dot{\theta}(t)}{\theta(t)} \\
\frac{c\alpha}{q(\theta(t))}\frac{\dot{\theta}(t)}{\theta(t)} &= (r+s)\frac{c}{q(\theta(t))} - [y - w(t)] \\
\frac{\dot{\theta}(t)}{\theta(t)} &= \frac{r+s}{\alpha} - \frac{q(\theta(t))}{c\alpha}[y - w(t)] \\
w(t) &= (1-\beta)\,z + \beta\,(y + c\theta) \\
\frac{\dot{\theta}(t)}{\theta(t)} &= \frac{r+s}{\alpha} - \frac{q(\theta(t))}{c\alpha}[(1-\beta)(y-z) + \beta c\theta] \\
\dot{\theta}(t) &= \frac{\theta(t)}{\alpha}\left[(r+s) - (1-\beta)(y-z)\frac{q(\theta(t))}{c} + \beta p(\theta(t))\right]
\end{aligned}
$$

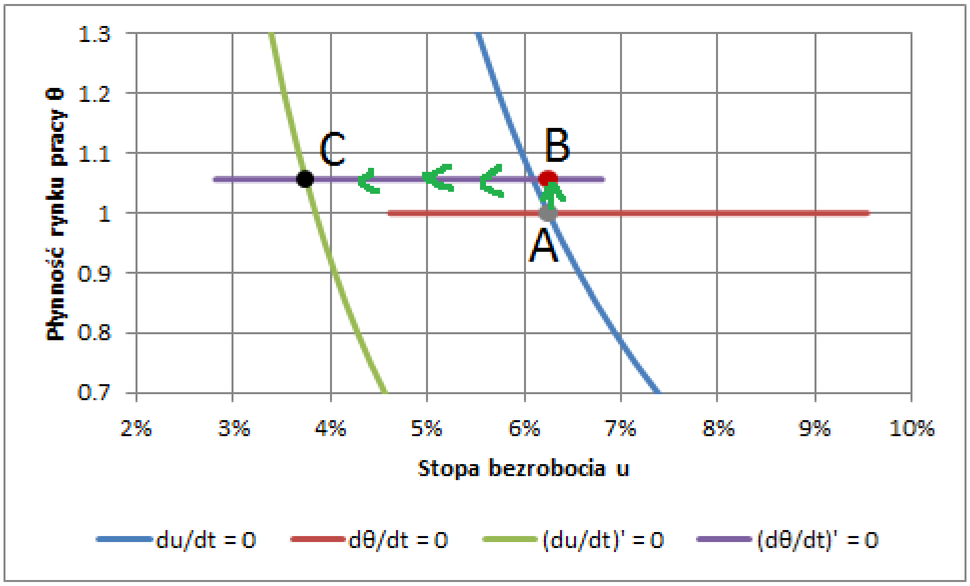
The above equation does not depend on u, so the separating curve $\dot{\theta} = 0$ in (u, θ) space is horizontal at level θ^*.

c)

The steady state of the system is given by $\theta = \theta^*$ and $u = s/\left(s + \phi(\theta^*)^{1-\alpha}\right)$. The system is saddle-path stable, and the saddle path lies at the Jest $\dot{\theta} = 0$ condition:



An increase of ϕ improves matching efficiency, increasing steady state tightness and lowering steady state unemployment rate. Labor market tightness θ increases momentarily ($A \to B$), and the unemployment rate u gradually decreases ($B \to C$):



Unemployment rate after an increase in ϕ: