

Macroeconomics of the Labour Market

Problem Set

dr Leszek Wincenciak

Problem 1

The utility of a consumer is given by $U(C, L) = \alpha \ln C + (1 - \alpha) \ln L$, where C is the aggregate consumption, and L is the leisure. Total time available in one week is denoted by T , market wage by w . Assume that the non-wage income is given by R .

- (a) Write the budget constraint for this consumer.
- (b) Set up the utility maximization problem and derive the function of labour supply. Find the expression for the reservation wage.
- (c) How does the labour supply respond to changes in the non-wage income?
- (d) How does the labour supply respond to changes in the wage? Is the labour supply curve „backward-bending”?
- (e) If the non-wage income is $R = 0$, what would be the answer to point (d)?

Assume that the non-wage income is $R = 100$ \$/week, while $\alpha = 0.125$. If the market wage is 10 \$/hour, what is the optimal labour supply (hours of work per week) of this consumer? If the working week is set to 40 hours and regulations make it impossible to work part-time, should the consumer take up such a job or remain inactive?

Problem 2

Consider the basic efficiency wages model. Suppose that fraction f of workers belong to unions that are able to obtain a wage that exceeds the nonunion wage by μ percent, so that $w_u = (1 + \mu)w_n$, where subscripts u and n denote union and nonunion wages, respectively. The average wage in the economy can be calculated as $w_a = fw_u + (1 - f)w_n$. Suppose the effort function is defined as:

$$e(w, x) = \begin{cases} \left(\frac{w-x}{x}\right)^\beta & \text{if } w > x \\ 0 & \text{otherwise,} \end{cases}$$

where $x = (1 - bu)w_a$ is a measure of labour market conditions, $0 < \beta < 1$ and $b > 0$.

- (a) Find the equilibrium unemployment rate in terms of the exogenous parameters of the model (β, b, f, μ) .
- (b) Suppose that $\mu = f = 0.15$. What is the equilibrium unemployment rate if $\beta = 0.06$ and $b = 1$? Is the effort exerted by unionized workers higher than nonunionized workers? By how much? How does it compare to the their wages ratio?
- (c) What is the cost of effective labor for unionized workers relative to nonunionized workers?

Problem 3

Describe how each of the following events affect the equilibrium employment and wage in the Shapiro-Stiglitz model. Use the graph and give an intuitive explanation.

- (a) An increase in workers' discount rate
- (b) An increase in the job breakup rate
- (c) An improvement in the shirking detection technology
- (d) An increase in the size of the labour force

Problem 4

Assume that in Shapiro-Stiglitz model unemployed workers are employed not at random but according to the length of unemployment period, specifically – those with longest unemployment spells are employed as first.

- (a) Analyse the steady-state without shirking. Find the expression for average time until finding new job by the unemployed as a function of b , L , N and $\bar{L}$.
- (b) Let V_U be the value of being just unemployed. Find the formula for V_U as a function of time needed to find another job, workers' discount rate (ρ) and the value of being employed (V_E).
- (c) Using answers from (a) and (b) find the non-shirking condition.
- (d) How does the unemployment rate in this version of the model compare to the standard Shapiro-Stiglitz model?

Problem 5

Analyse Shapiro-Stiglitz model of efficiency wages. In addition to version of the model analyzed in the lecture, let's introduce unemployment benefits z , which increase V_U – the utility of the Unemployed.

- (a) Reformulate the asset equation for the value of V_U .
- (b) Find the formula for the non-shirking condition. How does the parameter z influence incentives to provide effort and unemployment in equilibrium?
- (c) Is it possible that the model generates full employment equilibrium? Use your formula from point (a) to give one example of such situation and show it graphically.

Problem 6

Assume that in Shapiro-Stiglitz model of efficiency wages, dismissed workers are paid severance payments F . For the workers who are caught shirking however, the severance payment is paid with probability p only.

- (a) Write the Bellman equations for the values of V_E , V_S and V_U and find the 'No Shirking Condition'. For convenience of notation use r as the discount rate.
- (b) How does increase of F influence unemployment if $p = 0$? How does p affect the response of unemployment to F ? Explain the economic mechanism behind it.
- (c) Is unemployment in this modified version of the model higher than in the original Shapiro-Stiglitz model? Explain.

Problem 7

Assume that in Shapiro-Stiglitz model of efficiency wages, workers who are caught shirking are not fired but instead they have to pay a fine being fraction f of current wage.

- (a) Write the Bellman equations for the values of V_E , V_S and V_U and find the 'No Shirking Condition'.
- (b) Draw the 'No Shirking Condition' line on a graph. How does increase of f influence unemployment? Explain the relation between monitoring of workers' effort and the value of the fine.
- (c) Find the value of f for which the unemployment in this modified version of the model is lower than in the original Shapiro-Stiglitz model.

Problem 8

Assume that the representative firm's profits are given by $\pi = (eL)^\alpha/\alpha - wL$ with $0 < \alpha < 1$, where e is the effort level. The labour union objective function is $U = w - x$, where x is the workers' outside option. Assume that a firm and union bargain over wage, then firm chooses employment L for this negotiated wage (as in the 'right-to-manage' model).

Assume that $e = 1$, so we do not consider efficiency wages.

- (a) What is the level of L chosen by firm for given wage w ? What is the firm's profit?
- (b) Assume that the bargaining power of the union is γ , where $0 < \gamma < \alpha$. What is the negotiated wage?
- (c) What is the value of $\partial(\ln w)/\partial\gamma$ for $\gamma = 0$?

Assume now that $e = [(w - x)/x]^\beta$, where $0 < \beta < 1$.

- (d) What is now the level of L chosen by firm for given wage w ? What is the firm's profit?
- (e) What is the negotiated wage in this setting? (check if your solution for $\beta = 0$ is the same as in point (b)).
- (f) What is the value of $\partial(\ln w)/\partial\gamma$ for $\gamma = 0$? Is this elasticity higher with efficiency wages or not?

Problem 9

Assume that the firm's production function is $F(L) = L^\alpha/\alpha$, where $0 < \alpha < 1$ and the union objective function is $V = (L/N)\ln(w) + (1 - L/N)\ln(B)$, where B is the unemployment benefit.

- (a) Derive the firm's labour demand and iso-profit lines. Show them on appropriate graph.
- (b) Find the union indifference curves and show them on the same graph.
- (c) Assuming β as the union bargaining power and the 'right-to-manage' model of wage setting, state the wage negotiation problem. Assume that the firm's fallback is zero.
- (d) Find the negotiated wage (may be for convenience expressed in logarithmic terms).
- (e) How does the union bargaining power affect negotiated wage and employment?
- (f) How does α influence the elasticity of labour demand with respect to wage? How does it influence wage and employment in equilibrium?
- (g) Show that the equilibrium is not Pareto efficient.

Problem 10

Consider the search model of unemployment presented in the lecture. Using the graphs describing equilibrium, explain how each of the following events affect changes in wages, vacancies ratio and unemployment rate.

- (a) A decrease of the separation rate
- (b) An increase in the aggregate productivity
- (c) An increase in the real interest rate
- (d) An improvement in the matching efficiency

Problem 11

Consider the search model of unemployment presented in the lecture. Suppose that the flow cost of a vacancy c and the imputed value of free time z are functions of the wage rate w (instead being exogenous). In particular, assume that $c = c_0 w$ and $z = z_0 w$.

- (a) Determine the formula for job creation and wage setting.
- (b) How do θ and wages in steady-state equilibrium react to productivity changes?
- (c) Does a continuous growth of productivity lead to a decrease in the long run unemployment rate?

Problem 12

Consider the search model of unemployment presented in the lecture. In contrast to what was assumed that the wage is bargained over by the worker and the firm, assume that it is set according to Shapiro-Stiglitz model of efficiency wages (including unemployment benefits z , like in Problem 5, Problem set #2). Adopt notation from the search model and denote $p(\theta)$ – probability of finding new job per unit of time, s – exogenous job destruction rate, r – discount rate and μ – probability of detecting shirkers per unit of time.

- (a) Write the new formula for wage setting.
- (b) How does it differ from the original search model?
- (c) Assume there is a positive productivity shock. How does it affect labour market tightness (θ), wages and unemployment in equilibrium? Are these effects different than in the original search model?
- (d) Assume there is an increase of job destruction rate s . What happens to labour market tightness (θ), wages and unemployment in equilibrium? Are these effects different than in the original search model?

Problem 13

Consider the search model of unemployment presented in the lecture. Assume that the matching function is given by: $m(u, v) = \phi u^\alpha v^{1-\alpha}$, where $0 < \alpha < 1$. Let s be the separation rate, r – real interest rate, c – flow cost of vacancy, z – unemployment benefits and β – workers' bargaining power.

- (a) Write the equation of unemployment rate dynamics. Find the steady-state unemployment rate as a function of θ . In the space (u, θ) draw the line $\dot{u} = 0$ and a vector field showing how the unemployment rate changes in given regions.

- (b) Write the Bellman equations for job creation of firms. Using the condition of wage setting, i.e. $w = (1 - \beta)z + \beta(y + c\theta)$ find the equation of θ dynamics. In space (u, θ) draw the line $\dot{\theta} = 0$ and a vector field showing how θ changes in given regions.
- (c) Describe the stability of equilibrium in this model and show the effects of an increase of parameter ϕ (explain briefly the economic interpretation of this). Find the new steady-state equilibrium point and show the transitory dynamics.