Advanced Macroeconomics, Economic Growth: Problem set

Warm-up problems

1. Assume: $Y = AK^{\alpha}L^{1-\alpha}$, $A = 1$, $\alpha = 1/3$, $s = 0.2$, $\delta = 0.04$, $n = 0.01$. Find per worker levels of capital, GDP and consumption in the steady state.

2. Assume: $Y = AK^{\alpha}L^{1-\alpha}$, $A = 3/4$, $\alpha = 1/2$, $s = 0.32$, $\delta = 0.04$, $n = 0.02$. Find per worker levels of capital, GDP and consumption in the steady state.

3. Assume: $Y = AK^{\alpha}L^{1-\alpha}$, $A = 1$, $\alpha = 1/2$, $s = 0.3$, $\delta = 0.05$, $n = 0$. Find rates of growth of capital and GDP per worker assuming the initial level of capital per worker $k_0 = 9$. Find the level of capital per worker in the steady state.

4. Assume that a country has access to two production functions: $Y_1 = K^{\alpha}L^{1-\alpha}$ and $Y_2 = AK^{\alpha}L^{1-\alpha} - BL$, where $B > 0$ and $A > 1$. Imagine that the component linear in labor in the second production function captures costs of maintaining a "well" functioning state, etc. Rewrite both production functions in intensive forms per worker. Sketch them on a graph. Find the level of capital per worker for which it makes sense to switch to the second production function. Is it possible for a country to get "stuck" at the lower steady state?

5. Solow model well describes the dynamics of industrial economies. Prior to the industrial revolution all economies in the world where agrarian, with dynamics well described by the Malthus model, where in the production function instead of capital there is a fixed factor of production T (arable land) and the rate of growth of population is given by the difference in crude birth and death rates, depending respectively positively and negatively on GDP per person:

$$\begin{aligned} Y &= AT^{\alpha}N^{1-\alpha} \\ n &= b\left(Y/N\right) - d\left(Y/N\right), \quad b' > 0, \quad d' < 0 \end{aligned}$$

Sketch the graph of GDP per person as a function of population N. Sketch the graph of population dynamics as a function of GDP per person. Find graphically the model's steady state. Analyze graphically the dynamic effects of a one time improvement in technology

Exam-like problems

1. Solow-Swan model: technological progress and rate of convergence

In the traditional Solow-Swan model there are two factors of production: physical capital K and raw labor L, and the production function is of the Cobb-Douglas form:

$$Y_t = K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}$$

where A is the level of technology and we assume that L and A grow exogenously at rates n and g, respectively. Capital stock evolves over time:

$$K_{t+1} = sY_t + \left(1-\delta\right)K_t$$

The model predicts that the level of capital per effective labor $\hat{k}_t$ will converge to its long-run value $\hat{k}^*$, characteristic for the model's steady state (balanced growth path, BGP). Let $\hat{y}^*$ denote the level of output per effective labor along the BGP, and $\hat{y}_t$ denote its actual level at time t.

(a) Write down the equation for the rate of growth of capital per effective labor. Find the levels of capital per effective labor and output per effective labor along the BGP.

(b) Solow model assumes that consumption per worker c_t is a constant fraction $(1-s)$ of output per worker y_t. Does the result in (a) mean that a higher growth rate of technology lowers the level of consumption per worker? Why?

(c) Produce a linear approximation of the equation for the growth rate of capital per effective labor with respect to $\ln \hat{k}$ around the point $\ln \hat{k}^*$. Show that the equation for convergence of output per worker is of the following form:

$$g_y \approx g + (1-\alpha)(\delta + n + g)(\ln y_t^* - \ln y_t)$$

2. Solow-Swan model: alternative production function

Robert Solow in his 1956 article "A Contribution to the Theory of Economic Growth" considered the behavior of economy when output was produced according to other than Cobb-Douglas production functions. One of them was the following Constant Elasticity of Substitution (CES) function:

$$Y_t = \left[aK_t^\rho + (1-a) L_t^p\right]^{1/\rho}$$

where $a \in (0,1)$, $\rho \leq 1$ and for simplicity technology growth rate is set to 0. The CES function reduces to the Cobb-Douglas function in the limit of $\rho \to 0$.

(a) Transform the production function into per worker form, i.e. divide it by L_t.

(b) Find the steady state value for capital per worker k_t.

(c) Show graphically how an increase in the population growth rate affects the steady state level of capital per worker.

(d) Show graphically how an increase in the saving rate affects the steady state level of capital per worker.

3. Solow-Swan model: human capital

Consider the modified Mankiw-Romer-Weil model, where hyman capital H enters the production function (to simplify on notation, we assume that technology level is constant $A = 1$, $g = 0$):

$$Y_t = K_t^\alpha H_t^\beta L_t^{1-\alpha-\beta}$$

where $0 < \beta < 1$ and $\alpha + \beta < 1$. Let s_k denote the part of total income that is invested in physical capital, and s_h is invested in human capital. This lets us to write down the conditions describing the evolution of physical capital per worker k_t and human capital per worker h_t as follows:

$$\Delta k_{t+1} \simeq s_k y_t - (\delta + n) k_t$$
$$\Delta h_{t+1} \simeq s_h y_t - (\delta + n) h_t$$

Find the levels of physical capital per worker, human capital per worker and output per worker in the steady state.

4. Intertemporal choice: CRRA utility function

Consider the following two-period utility maximization problem. This utility function belongs to the Constant Relative Risk Aversion (CRRA) class of functions that will be often used throughout our course.

$$\begin{aligned} \max_{c_t,\, c_{t+1},\, a_t} \quad & U = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma} - 1}{1-\sigma} \\ \text{subject to} \quad & c_t + a_t = y_t \\ & c_{t+1} = y_{t+1} + (1+r) a_t \end{aligned}$$

where $\sigma \geq 0$,[1] $\beta \in [0,1]$, y_t, $y_{t+1} \geq 0$ and $r \geq -1$.

[1] For $\sigma = 1$ the CRRA function becomes logarithmic: $U = \ln c_t + \beta \ln c_{t+1}$. This can be easily proven by using the L'Hôpital's rule to compute the following limit: $\lim_{\sigma \to 1} \frac{c^{1-\sigma}-1}{1-\sigma}$.

(a) Rewrite the budget constraints into a single lifetime budget constraint and set up the Lagrangian.

(b) Obtain the first order conditions for c_t and c_{t+1}. Express c_{t+1} as a function of c_t.

(c) Using the lifetime budget constraint obtain the formulas for optimal c_t and c_{t+1}.

(d) Using your results from (c), set $\alpha = 1$ and verify that the formulas for optimal c_t and c_{t+1} are identical to the ones we obtained in class for the utility function $U = \ln c_t + \beta \ln c_{t+1}$.

(e) Return to expressions obtained in (c). Assume now that $y_{t+1} = 0$. How does c_t react when interest rate r increases? How does this reaction depend on σ?

5. Intertemporal choice: borrowing constraint

In this problem we will show how changes in taxation can change consumption of an agent if she is borrowing-constrained. The agent's income in the first period is 1/3 of her income in the second period: $y_{t+1} = y$, $y_t = y/3$. Assume that $\beta(1+r) = 1$ and consider the following utility maximization problem:

$$\begin{aligned} \max_{c_t,\, c_{t+1},\, a_t} \quad & U = \ln c_t + \beta \ln c_{t+1} \\ \text{subject to} \quad & c_t + a_t = y_t \\ & c_{t+1} = y_{t+1} + (1+r)\, a_t \end{aligned}$$

(a) Using the Lagrangian method find optimal c_t, c_{t+1} and a_t. Are assets positive or negative?

(b) Assume now that the agent cannot borrow and faces an additional non-borrowing constraint: $a_t \geq 0$. What is now the optimal choice?

(c) Suppose that the government arranges a transfer v to this agent by issuing bonds. In the future, the government will tax the agent to be able to buy back the bonds. The new constraints of the agent are:

$$\begin{aligned} & c_t + a_t = y_t + v \\ & c_{t+1} = y_{t+1} + (1+r)\, a_t - (1+r)\, v \\ & a_t \geq 0 \end{aligned}$$

What is the impact of the government transfer on the agent's first period consumption?

(d) Show graphically he effect of the transfer scheme from (c).

(e) Suppose that the transfer is large enough so that the agent's assets become positive. What would be the impact of even bigger transfers on first period consumption?

6. Intertemporal choice: life expectancy and decisions

Suppose that we have an agent that lives for T periods, from period 1 to period T. Her lifetime utility is

$$U = \sum_{t=1}^{T} \beta^{t-1} \ln c_t$$

The agent starts with no wealth, $a_0 = 0$, and plans to die with no wealth as well, $a_T = 0$. Over her lifetime, the agent faces a sequence of budget constraints

$$c_t + a_t = y_t + (1+r)\, a_{t-1} \qquad \text{for all } t = 1, 2, \ldots, T$$

(a) Set up the lifetime budget constaint.

(b) Using the Lagrangian method find the optimal initial level of consumption c_1.

(c) Assume that $r = 0$, $\beta = 1$, and that the agent's labor income $y_t = y$ for $t = 1, 2, \ldots, R$ and 0 afterwards. Determine the level of consumption in all periods.

(d) Find the level of agent's assets at the end of period R, a_R.

(e) Imagine now that the agent expects to live longer, $T' > T$. How does it affect c_1 and a_R?

(f) Imagine now that the agent will retire later, $R' > R$. How does it affect c_1 and assets at the end of period R'?

7. Overlapping generations: Ricardian equivalence?

Suppose you have a two-period overlapping generations (OLG) model. Each generation of agents has the same number of agents, N. There is no production; each agent receives an endowment in each period. However, agents are able to buy government bonds when they are young and redeem them when they are old. Let superscript y denote the value of a variable when an agent in young and o denote the value of a variable when an agent is old. The interest rate is constant over time. Each yount agent born at time t maximizes:

$$U_t = \ln c_t^y + \beta \ln c_{t+1}^o$$

subject to the following constraints:

$$\begin{aligned} c_t^y + b_t &= y^y - \tau_t \\ c_{t+1}^o &= y^o - \tau_{t+1} + (1+r)\, b_t \end{aligned}$$

where $\beta = 1/(1+r)$ and $y^y - \tau_t > y^o - \tau_{t+1}$, whereas the old of period t (those born in $t-1$) consume all resources they have at hand:

$$c_t^o = y^o - \tau_t + (1+r)\, b_{t-1}$$

Suppose the government reduces the taxes in period t by issuing bonds in period t (bought by the young) and retires the bonds in $t+1$ by raising the taxes in period $t+1$. In short,

$$\begin{aligned} \Delta\tau_t &= -T \\ \Delta\tau_{t+1} &= -(1+r)\,\Delta\tau_t = (1+r)\,T \end{aligned}$$

where T is a positive value denoting the size of the tax change.

(a) What would be the value of aggregate period t consumption $C_t = Nc_t^y + Nc_t^o$, if the tax change was not implemented?

(b) What happens to aggregate period t consumption if taxes are changed at the start of period t, i.e., what is $\Delta C_t / \Delta\tau_t$?

(c) Given (a) and (b), does the Ricardian equivalence hold? Why or why not?

8. Ramsey model: social planner's solution

The social planner solves the following discrete time utility maximization problem:

$$\begin{aligned} \max_{\{c_t,\, k_{t+1}\}_{t=0}^{\infty}} \quad & U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ \text{subject to} \quad & c_t + k_{t+1} = A_t k_t^{\alpha} + (1-\delta)\, k_t \\ & k_0 > 0 \end{aligned}$$

where β is the household's discount factor.

(a) Find the first order conditions for the choice of c_t and k_{t+1}. Obtain the Euler equation.

(b) Compute the steady state values of k and c.

(c) How do they change in response to changes in A, δ and β?

(d) Use the following assumptions $\sigma = 1$ and $\delta = 1$. Assuming that household behavior can be expressed as $c_t = (1-s)\, y_t$, where s is a constant, find the value of s. *Hint: use the Euler equation first.*

(e) Find the expression for k_{t+1} as a function of k_t and model parameters.

9. Ramsey model: effects of taxing overall income

Consider a Ramsey economy where for simplicity we assume $n = g = 0$ and $N = A = 1$. The representative households solve the following utility maximization problem:

$$\max_{\{c_t,\, k_{t+1}\}_{t=0}^{\infty}} \quad U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$
$$\text{subject to} \quad c_t + a_t = w_t + (1 + r_t)\, a_{t-1} + v_t$$

where v is the lump-sum transfer from the government to households.

The representative firm solves the following profit maximization problem:

$$\max \quad D_t = (1 - \tau^y)\, Y_t - (r_t + \delta)\, K_t - w_t L_t$$
$$\text{subject to} \quad Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

where τ^y is the firm revenue tax (equivalent to taxing all households' income at the same rate regardless of its source).

(a) Derive the first order conditions of the household.

(b) Recast the problem of the firm in per worker terms. Derive the first order conditions of the firm.

(c) Write down the government budget constraint. Using the assumptions of closed economy and balanced government budget, find the conditions for general equilibrium in this economy.

(d) Find the steady state level of capital per worker k^* and consumption per worker c^* in this economy. Discuss how they depend on the tax rate τ_y.

10. Expanding variety model: effects of taxing monopolists

Consider an economy where final goods are produced according to the following production function:

$$Y_t = L^{1-\alpha} \sum_{i=1}^{M_t} x_{it}^{\alpha}$$

where L is the constant labor force, M_t is the number of invented types of intermediate goods and x_{it} denotes usage of intermediate good type i in the final goods production. The inventor of type i holds a perpetual patent that gives exclusive, monopolistic rights to produce this type of an intermediate good, with marginal cost of production equal to 1. Assume that the government taxes the monopolists and each of them pays lump-sum tax T per period: $D_{it} = (p_{it} - 1)\, x_{it} - T$.

(a) Solve the profit maximization problem of the final goods producer to obtain the (inverse) demand function for intermediate goods.

(b) Solve the profit maximization problem of the intermediate goods producers (monopolists). Find the optimal price, quantity produced and maximal after-tax profit per period.

(c) Assume that inventing a new type of an intermediate good costs $1/\eta$ units of the final good. Equalize the cost of invention with the discounted after-tax value of profit flows of a new monopolist.

(d) Transform the expression from (c) to obtain the real interest rate. Use the Euler equation $g = (r - \rho)/\sigma$ to obtain the equilibrium growth rate of the economy.

(e) Discuss how the growth rate of the economy depends on the level of taxation T. Should the government aim to reduce the after-tax profits of the monopolists to 0?

11. Expanding variety model: alternative production function

Suppose that the production function of final goods is

$$Y = L^{1-\alpha}\left(\sum_{i=1}^{M_t} x_{it}^{\sigma}\right)^{\alpha/\sigma}$$

where $\sigma \in (0,1)$. The parameter σ, rather than α, will now determine the elasticity of demand for each type of intermediate

(a) How are the monopolized intermediates priced, and what is the quantity of each intermediate, x_{it}?

(b) What is the free-entry condition for R&D, and how is the rate of return determined?

(c) What are the growth rates of M and Y in the steady state?

Advanced Macroeconomics, Business cycles: Problem set

1. Real business cycles model: special case

Consider the following model. Households maximize expected utility subject to the standard budget constraint:

$$\max_{\{c_{t+j},\,h_{t+j},\,k_{t+j+1}\}_{j=0}^{\infty}} \quad U = \mathrm{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\ln c_{t+j} - \psi\frac{h_{t+j}^{1+\varphi}}{1+\varphi}\right)\right]$$
$$\text{subject to} \quad c_t + k_{t+1} = w_t h_t + (1+r_t)\,k_t + d_t$$

Firms maximize profits subject to the Cobb-Douglas production function:

$$\max \quad d_t = y_t - w_t h_t - (r_t+\delta)\,k_t$$
$$\text{subject to} \quad y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$$

where $\delta = 1$ (capital deprecates fully). The technology constant z evolves according to the process:

$$z_t = \rho_z z_{t-1} + (1-\rho_z) + \epsilon_{z,t}$$

(a) Derive the first order conditions of the households and their optimality conditions.

(b) Derive the first order conditions of the firm and expressions for prices in equilibrium.

(c) Find the steady state of the system, assuming that $h^* = 1$. Find corresponding value of ψ.

(d) Assuming that household behavior can be expressed as $c_t = (1-s)\,y_t$ where s is a constant, find the value of s as a function of model parameters.

(e) Show that $h_t = h^*$. Find the expression for k_{t+1} as a function of variables at time t.

2. Real business cycles model: government expenditure

Consider the effects of increasing government expenditure in the RBC model. Households maximize expected utility subject to the standard budget constraint:

$$\max_{\{c_{t+j},\,h_{t+j},\,k_{t+j+1}\}_{j=0}^{\infty}} \quad U = \mathrm{E}_t\left[\sum_{j=0}^{\infty}\beta^i\left(\ln c_{t+j} + \chi\ln g_{t+j} - \psi\frac{h_{t+j}^{1+\varphi}}{1+\varphi}\right)\right]$$
$$\text{subject to} \quad c_t + k_{t+1} = w_t h_t + (1+r_t)\,k_t + d_t - \tau_t$$

The problem of the firms is the same as in the previous problem. The government maintains a balanced budget at all times:

$$g_t = \tau_t$$

And the government spending is a constant fraction of GDP:

$$g_t = \gamma \cdot y_t$$

(a) Derive the first order conditions of the households and their optimality conditions.

(b) Find the steady state of the system.

(c) Suppose now that the government increases its expenditure to $\gamma' > \gamma$. What is the effect on this higher government spending on GDP?

(d) What is the effect of increased government expenditure from (c) on capital, consumption and hours worked? If $\chi = 0$, what would be the impact of higher government expenditure on welfare (utility)?

(e) The RBC model postulates that households dislike working. Why is then unemployment a problem? Can you argue for a setup where more hours worked is welfare improving?

3. Baumol-Tobin model

In the lecture we have been treating the number of tranfers as a continuous variable. Consider now the case where the number of transfers has to be a natural number. We still aim to minimize overall costs:

$$\min_{n} \quad Pkn + iM = Pkn + i\frac{PY}{2n}$$

Assume the following: monthly spending needs $PY = 2000\,\text{EUR}$, transfer costs $Pk = 2.5\,\text{EUR}$, nominal interest rate 3% per annum (assume that the monthly interest rate is 1/12 of the annual).

(a) Calculate the optimal number of transfers per month and the average amount on the checking account.

(b) Find the level of nominal interest rate for which it would pay off to executre one more transfer than in (a).

(c) What does the result from (b) mean for the elasticity of money demand with respect to the nominal interest rate?

4. Cash-in-Advance model: constraint applied to investment

In this Cash-in-Advance variant money is needed for both consumption and investment purchases. For simplicity consider the model without uncertainty (where $m_t \equiv M_{t-1}/P_t$)

$$\begin{aligned}\max_{c_t,\, k_{t+1},\, m_{t+1}} \quad & U = \sum_{t=0}^{\infty} \beta^t \ln c_t \\ \text{s. t.} \quad & c_t + (1+\pi_{t+1})\, m_{t+1} + k_{t+1} = f(k_t) + (1-\delta)\, k_t + m_t \\ & c_t + k_{t+1} - (1-\delta)\, k_t \le m_t\end{aligned}$$

(a) Derive the first order conditions and obtain the Euler equation.

(b) How does a positive rate of inflation in the steady state affect the level of capital per worker?

(c) Which firms in reality can be subject to the CIA constraint? What are the consequences?

5. Price expectations and the labor market

Assume that the labor market functions at the time of wage negotiations to be:

$$N^s = b_1 \frac{W}{P^e}$$
$$N^d = a_0 - a_1 \frac{W}{P^e}$$

where W is the nominal wage; P^e is the expected price level for the period ahead, and all parameters are positive.

(a) What will be the contractual nominal wage in equilibrium?

(b) Derive the expected level of employment. Does an increase in expectations of the general price level affect the expected level of employment?

(c) Suppose that at the time of production the nominal wage is set by the wage contract at the value calculated in (a). What will be the level of employment if the actual price level is equal to P?

6. New classical model

Consider the following new classical model:

- The AD (aggregate demand) curve:

$$y_t = m_t - p_t + u_t \qquad \mathrm{E}_{t-1}[u_t] = 0$$

  where y is real output (all variables are in logarithms), m is nominal stock of money balances, p is the price level, and u is an unforecastable demand shock.

- The AS (aggregate supply) curve:

$$y_t = y^* + a(p_t - p_t^e)$$

  where y^* is natural level of output, and p_t^e is the expected price level.

Expectations are rational and the central bank supplies money on the basis of a gradual adjustment rule:

$$m_t = \bar{m} + b(m_{t-1} - \bar{m}) + \varepsilon_t$$

where $0 < b < 1$ and ε is a monetary policy shock with expected value equal to 0.

(a) Write down the AS and AD curves in their expectational form.

(b) Find the expected price level.

(c) Calculate the level of output in equilibrium.

7. Central bank independence

The society's loss function is:

$$V_S = \lambda(y - y^* - k)^2 + \pi^2$$

To mitigate the resulting inflation bias, the society can make itself better off by appointing a "hawkish" central banker who does not share the social objective function, but instead places a larger weight ($\delta > 0$) on inflation rate stabilization relative to output stabilization:

$$V_{CB} = \lambda(y - y^* - k)^2 + (1 + \delta)\pi^2$$

Output is given by the Lucas supply function:

$$y = y^* + a(\pi - \pi^e) + e$$

The independence of a central bank can be seen as the extent to which it determines monetary policy without interference of the government. It can be incorporated in the loss function that determines monetary policy as a weighted average of the central bank's loss function V_{CB} and society's loss function V_S:

$$L = \omega V_{CB} + (1 - \omega) V_S$$

where the weight $0 < \omega < 1$ is the degree of central bank independence.. Derive the rate of inflation in rational expectations equilibrium (assume that inflation is fully controlled by the central bank).

8. New Keynesian model

Consider the Rotemberg scheme where costs of price changes resulted in the NKPC. Assume that customers are angry at the firm if it changes its price unexpectedly and consider an anticipated price change to be fair. Hence the costs of price changes are given by:

$$\phi\left(p_t - p_t^e\right)^2$$

Further, assume that customers have the following price expectations:

$$p_t^e = p_{t-1} + \pi_{t-1}$$

The loss function of the firm is given by:

$$L = \sum_{i=0}^{\infty} \beta^i \cdot \mathrm{E}_t \left[\left(p_{t+i} - p_{t+i}^*\right)^2 + \phi\left(p_{t+i} - p_{t+i}^e\right)^2\right]$$

Derive the modified equation for the NKPC. What are the similarities and differences between the derived equation, the standard NKPC curve, and the hybrid NKPC?

9. Monetary policy in the New Keynesian model

Consider the following New Keynesian model:

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \left(i_t - \mathrm{E}_t \pi_{t+1}\right) \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t \end{aligned}$$

where x is output gap, i is the nominal interest rate, π is the inflation rate, β is the households' discount rate and $\kappa > 0$ is a constant that depends on model parameters. The central bank obeys a strict inflation targeting rule. In particular, let π_t^* be an exogenous inflation target. The central bank will adjust i_t so that $\pi_t = \pi_t^*$ is consistent with these equations holding. Assume that the inflation target follows an exogenous AR(1) process:

$$\pi_t^* = \rho \pi_{t-1}^* + e_t, \quad \rho \in (0,1)$$

(a) Derive an analytic expression for i_t as a function of π_t^*.

(b) Suppose that ρ is 0 (approximating standard policymaking). In which direction must the central bank adjust i_t in order to achieve a decrease in π_t?

(c) Suppose that ρ is 1 (approximating disinflation in the long run). In which direction must the central bank adjust i_t in order to achieve a decrease in π_t?

(d) Provide intuition behind the difference in results in (b) and (c).

10. Monetary policy at the Effective Lower Bound (forward guidance)

Consider a simplified New Keynesian model:

$$x_t = \mathrm{E}_t x_{t+1} - \left(i_t - \mathrm{E}_t \pi_{t+1} - r^f\right) + u_t$$
$$\pi_t = \mathrm{E}_t \pi_{t+1} + x_t$$

where x is the output gap, i is the nominal interest rate, π is the inflation rate, $r^f > 0$ is the natural real rate of interest, and u is a demand shock.
In time period t the economy is affected by a strong negative demand shock: $u_t = -r^f - v$, which lasts for one period only. The central bank, subject to the zero lower bound constraint, sets $i_t = 0$. After the shock recedes, in time period $t+1$ it will be possible to set $x_{t+1} = \pi_{t+1} = 0$. Additionally, the central bank credibly commits to maintain $x_{t+k} = \pi_{t+k} = 0$ for all $k \geq 2$.

(a) What level of nominal interest rate in $t+1$ will be set by the central bank aiming to minimize $\left(\pi_{t+1}^2 + x_{t+1}^2\right)$?

(b) What will be the levels of output gap and inflation in period t if the agents expect the central bank to act according to (a)?

(c) Assume the central bank credibly commits to set $i_{t+1} = r^f - e$. What will be the levels of output gap and inflation in periods $t+1$ and t?

(d) What is the optimal level of e for a central bank aiming to minimize $\frac{1}{2}\left[\left(\pi_t^2 + x_t^2\right) + \left(\pi_{t+1}^2 + x_{t+1}^2\right)\right]$? Why does the optimal value of e differ from zero?