University of Warsaw
Faculty of Economic Sciences

New Keynesian Model

Advanced Macroeconomics: Lecture 9

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Nominal rigidities and sticky prices

Monopolistic competition

Under perfect competition firms are selling homogeneous goods and are price-takers

In reality many firms sell differentiated products and are able to set their own prices

Convenient framework: **monopolistic competition**

- Households enjoy consuming many different goods (“love for variety”)
- Firms’ market power depends on elasticity of substitution $\varepsilon > 1$
- Perfect competition: infinitely high elasticity of substitution
- One sector: all firms “compete” with each other
- Can easily extend to a multisector setup: higher elasticity of substitution within industries, smaller across industries

Monopolistic competition: consumers

Consumer derives utility from overall consumption C,
generated by consuming differentiated goods C_j

Expenditure minimization problem with $\rho \equiv \dfrac{\varepsilon - 1}{\varepsilon} \in (0, 1)$

$$\min \quad \sum P_{jt} C_{jt} \qquad \text{s. t.} \quad \left(\sum C_{jt}^{\rho} \right)^{1/\rho} \geq C_t$$

Lagrangian (Lagrange multiplier is the ideal consumer price index)

$$\mathcal{L} = -\sum P_{jt} C_{jt} + P_t \left[\left(\sum C_{jt}^{\rho} \right)^{1/\rho} - C_t \right]$$

First order condition with respect to C_{jt}

$$-P_{jt} + P_t \left[\frac{1}{\rho} \left(\sum C_{jt}^{\rho} \right)^{(1/\rho)-1} \rho C_{jt}^{\rho-1} \right] = 0 \quad \rightarrow \quad P_{jt} = P_t \left(\sum C_{jt}^{\rho} \right)^{(1-\rho)/\rho} C_{jt}^{\rho-1}$$

$$P_{jt} = P_t C_t^{1-\rho} C_{jt}^{\rho-1} \quad \rightarrow \quad C_{jt} = \left(P_{jt} / P_t \right)^{1/(\rho-1)} C_t \quad \rightarrow \quad C_{jt} = P_{jt}^{-\varepsilon} P_t^{\varepsilon} C_t$$

Monopolistic competition: producers

Under constant returns to scale marginal cost MC_j is constant

$$\max_{P_{jt},\, Y_{jt}} \quad D_{jt} = P_{jt}Y_{jt} - MC_{jt}Y_{jt}$$
$$\text{s.t.} \quad P_{jt} = P_t Y_t^{1-\rho} Y_{jt}^{\rho-1}$$

Plug in the inverse demand function into the profit function

$$\max_{Y_{jt}} \quad D_{jt} = P_t Y_t^{1-\rho} Y_{jt}^{\rho} - MC_{jt}Y_{jt}$$

First order condition with respect to Y_{jt} ($MR = MC$)

$$\rho P_t Y_t^{1-\rho} Y_{jt}^{\rho-1} - MC_{jt} = 0 \quad \rightarrow \quad \rho P_{jt} = MC_{jt}$$

"Markup pricing" is the profit-maximizing strategy

$$P_{jt}^* = \frac{1}{\rho} MC_{jt} = \frac{\varepsilon}{\varepsilon - 1} MC_{jt} \equiv (1+\mu)\, MC_{jt}$$

If we allow to differentiate ε across sectors we get $\quad P_{jt}^* = (1+\mu_j)\, MC_{jt}$

Empirical evidence on markups in US and EA

Perfect competition can be rejected for almost all sectors in all countries
Markups are generally higher in services than manufacturing

Table 1. Weighted average markup, 1981-2004

Country	Manufacturing & Construction		Market Services		All (Manufacturing, Construction & Market Services)	
Germany	1.16	(0.01)*	1.54	(0.03)*	1.33	(0.01)*
France	1.15	(0.01)*	1.26	(0.02)*	1.21	(0.01)*
Italy	1.23	(0.01)*	1.87	(0.02)*	1.61	(0.01)*
Spain	1.18	(0.00)*	1.37	(0.01)*	1.26	(0.01)*
Netherlands	1.13	(0.01)*	1.31	(0.02)*	1.22	(0.01)*
Belgium	1.14	(0.00)*	1.29	(0.01)*	1.22	(0.01)*
Austria	1.20	(0.02)*	1.45	(0.03)*	1.31	(0.02)*
Finland	1.22	(0.01)*	1.39	(0.02)*	1.28	(0.01)*
Euro Area	1.18	(0.01)*	1.56	(0.01)*	1.37	(0.01)*
USA	1.28	(0.02)*	1.36	(0.03)*	1.32	(0.02)*

Christopoulou and Vermeulen (2008)

Prices do not change every period

Survey about price setting practices carried out by the Banco de Portugal

Firms in the sample are generally quicker to react to cost shocks,
in particular when they are positive, than to demand shocks

TABLE 1

Distribution of the price responses to demand and cost shocks

	Cost shocks		*Demand shocks*	
Price adjustment lag	*Positive*	*Negative*	*Positive*	*Negative*
1 – less than one week	4.7	3.5	2.8	4.8
2 – from one week to one month	16.8	15.2	12.2	16.8
3 – from one month to three months	25.0	25.7	19.3	23.4
4 – from three to six months	17.6	14.9	13.4	13.6
5 – from six months to one year	26.3	21.2	17.7	14.0
6 – more than one year	9.6	19.5	34.6	27.4
Total	100.0	100.0	100.0	100.0

Dias et al. (2014)

Stylized facts on price stickiness

Benefits of price stickiness: no need to survey all prices everytime we go to store, easy to plan expenditures ahead

Average price duration

- US: average time between price changes is 2-4 quarters
  Blinder et al. (1998), Klenow and Kryvstov (2008), Nakamura and Steinsson (2008)
- EA: average time between price changes is 4-5 quarters
  Dhyne et al. (2005), Altissimo et al. (2006)
- PL: average time between price changes is 4 quarters
  Macias and Makarski (2013)

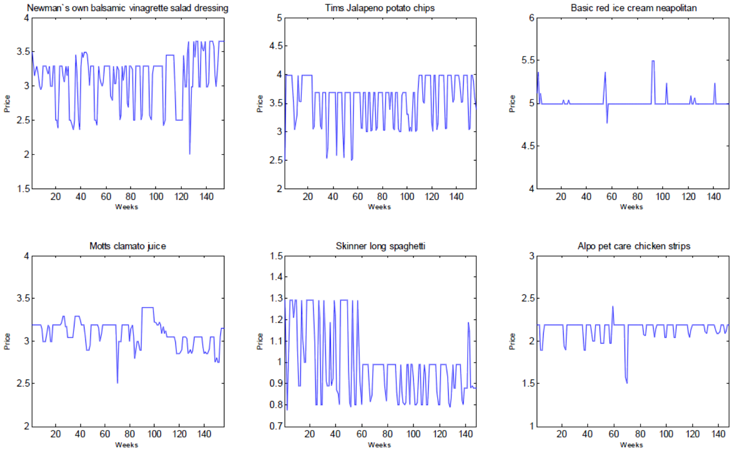
Cross-industry heterogeneity

- Prices of tradables less sticky than those of nontradables
- Retail prices usually more sticky than producer prices

Gagnon (2009): for inflation above 10-15% prices change more frequently with higher inflation

Example retail prices behavior

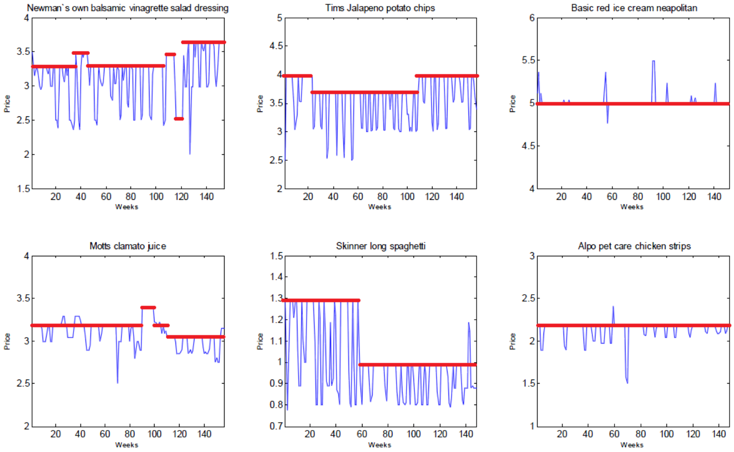
Raw retail scanner data



Koning (2015)

Example retail prices behavior

After “controlling” for short-lived sales prices: reference prices



Price stickiness depends on sector

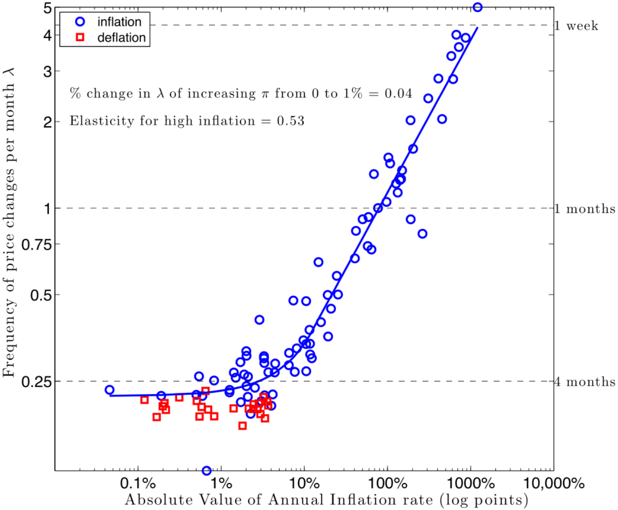
Table 4.1 Frequency of consumer price changes by product type, in %

Country	Unprocessed food	Processed food	Energy (oil products)	Non-energy industrial goods	Services	Total, country weights	Total, Euro area weights
Belgium	31.5	19.1	81.6	5.9	3.0	17.6	15.6
Germany	25.2	8.9	91.4	5.4	4.3	13.5	15.0
Spain	50.9	17.7	n.a.	6.1	4.6	13.3	11.5
France	24.7	20.3	76.9	18.0	7.4	20.9	20.4
Italy	19.3	9.4	61.6	5.8	4.6	10.0	12.0
Luxembourg	54.6	10.5	73.9	14.5	4.8	23.0	19.2
The Netherlands	30.8	17.3	72.6	14.2	7.9	16.2	19.0
Austria	37.5	15.5	72.3	8.4	7.1	15.4	17.1
Portugal	55.3	24.5	15.9	14.3	13.6	21.1	18.7
Finland	52.7	12.8	89.3	18.1	11.6	20.3	-
Euro Area	28.3	13.7	78.0	9.2	5.6	15.1	15.8

Source: Dhyne et al. (2005). Figures presented in this table are computed on the basis of the 50 product sample, with the only exception of Finland for which figures based on the entire CPI are presented. The total with country weights is calculated using country-specific weights for each item, the total with euro area weights using common euro area weights for each sub-index. No figures are provided for Finland because of a lack of comparability of the sample of products used in this country.

Altissimo, Ehrmann and Smets (2006)

Frequency of price changes when inflation is low or high



Alvarez i in. (2018)

New Keynesian Phillips Curve

Convex costs of price changes

Based on Rotemberg (1982)

Assumes that bigger price changes are more costly, e.g. due to losses in consumer loyalty

Zbaracki et al. (2004): menu costs are constant, managerial and customer costs are convex

The dynamic profit maximizing problem can be recast as a simpler problem of minimizing a loss function (where p is log of firm's price and $\phi > 0$)

$$L = \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \left[\left(p_{t+i} - p^*_{t+i} \right)^2 + \phi \left(p_{t+i} - p^e_{t+i} \right)^2 \right]$$

- $\mathrm{E}_t[(p_{t+i} - p^*_{t+i})^2]$ is the loss of profit by setting price other than $(1+\mu) MC$
- $\mathrm{E}_t[\phi(p_{t+i} - p^e_{t+i})^2]$ is the convex cost of price changes
- p^e_t is the price expected by consumers, assume $p^e_t = p_{t-1}$

For simplicity assume firm symmetry

Rotemberg model: solution

Expand the loss function for convenience

$$L = (p_t - p_t^*)^2 + \phi (p_t - p_{t-1})^2 + \beta \mathrm{E}_t \left[(p_{t+1} - p_{t+1}^*)^2 + \phi (p_{t+1} - p_t)^2 \right] + \ldots$$

First order condition with respect to p_t

$$2 (p_t - p_t^*) + 2\phi (p_t - p_{t-1}) + \beta \mathrm{E}_t [2\phi (p_{t+1} - p_t) (-1)] = 0$$

$$(p_t - p_{t-1}) = \beta \mathrm{E}_t [p_{t+1} - p_t] - \frac{1}{\phi} (p_t - p_t^*)$$

Since all firms set the same price (symmetry) the inflation rate is $\pi_t \equiv p_t - p_{t-1}$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{\phi} (p_t^* - p_t)$$

Current inflation depends on inflation expecations!

Staggered price adjustment

Based on Calvo (1983)

In Rotemberg firms make many small price changes

In Calvo firms are not "allowed" to do so

- Firms can change their price only if they receive a "signal"
- Price remains unchanged with probability θ
- If a firm set the price in period t, then the price remains unchanged in period $t+i$ with probability θ^i
- Average price duration is $1/\left(1-\theta\right)$ periods
- Denote price set in period t with $\tilde{p}_t$ ("reset" price)

The dynamic profit maximizing problem can be recast as a simpler problem of minimizing a loss function

$$L = \sum_{i=0}^{\infty} \left(\beta\theta\right)^i \mathrm{E}_t \left[\left(\tilde{p}_t - p^*_{t+i}\right)^2\right]$$

Calvo model: solution

First order condition

$$\sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t \left[2 \left(\tilde{p}_t - p^*_{t+i} \right) \right] = 0$$

$$\tilde{p}_t \sum_{i=0}^{\infty} (\beta\theta)^i = \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{t+i}$$

$$\tilde{p}_t \frac{1}{1-\beta\theta} = \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{t+i} = p^*_t + \beta\theta \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{t+i+1}$$

$$\tilde{p}_t = (1-\beta\theta) \sum_{i=0}^{\infty} (\beta\theta)^i \, \mathrm{E}_t p^*_{t+i}$$

Reset price $\tilde{p}_t$ is the weighted average of today's and future prices that would be optimal in a frictionless setting, which can be expressed as

$$\tilde{p}_t = (1-\beta\theta)\, p^*_t + \beta\theta \mathrm{E}_t \tilde{p}_{t+1}$$

Dynamics of inflation in the Calvo scheme

Within each period a fraction θ of firms keeps prices unchanged, the remaining $1-\theta$ fraction resets prices to $\tilde{p}_t$

$$p_t = \theta p_{t-1} + (1-\theta)\,\tilde{p}_t$$

After a series of algebraic manipulations we get that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(p_t^* - p_t\right)$$

Compare to Rotemberg's outcome

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{\phi}\left(p_t^* - p_t\right)$$

In both settings expectations on future inflation affect inflation today!

Comparing Rotemberg and Calvo

Identical (up to the first-order approximation) functional form, different economic conclusions: see Lombardo and Vestin (2007) and Ascari and Rossi (2012)

- Rotemberg: inflation is “costly” due to costs of changing prices, relative prices across firms are unaffected
- Calvo: inflation is “costly” since not every firm adjusts prices within each period, price dispersion arises
- Price dispersion introduces inefficiencies into the economy ($P_i/P_j \neq MC_i/MC_j$)
- Welfare costs of inflation are higher in the Calvo scheme
- Calvo scheme fits data well under single-digit inflation (constant price change frequency), for higher inflation rate price adjustment models perform better

Dynamics of inflation in the Calvo scheme: algebra

$$p_t = (1-\theta)\,\tilde{p}_t + \theta p_{t-1}$$

$$\tilde{p}_t = \frac{p_t - \theta p_{t-1}}{1-\theta} = \frac{p_t - p_{t-1} + (1-\theta)\,p_{t-1}}{1-\theta} = \frac{\pi_t}{1-\theta} + p_{t-1}$$

$$\tilde{p}_t = (1-\beta\theta)\,p_t^* + \beta\theta \mathrm{E}_t \tilde{p}_{t+1} = (1-\beta\theta)\,p_t^* + \frac{\beta\theta}{1-\theta}\mathrm{E}_t\pi_{t+1} + \beta\theta p_t$$

$$(1-\theta)\,\tilde{p}_t = p_t - \theta p_{t-1} = \theta\,(p_t - p_{t-1}) + (1-\theta)\,p_t$$

$$\theta\,(p_t - p_{t-1}) = (1-\theta)\,(\tilde{p}_t - p_t)$$

$$\theta\pi_t = (1-\theta)\left[\frac{\beta\theta}{1-\theta}\mathrm{E}_t\pi_{t+1} + (1-\beta\theta)\,p_t^* + \beta\theta p_t - p_t\right]$$

$$\pi_t = \beta \mathrm{E}_t\pi_{t+1} + \frac{(1-\theta)\,(1-\beta\theta)}{\theta}\,(p_t^* - p_t)$$

New Keynesian Phillips Curve

Under both schemes we have that (where $\chi > 0$)

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \left(p_t^* - p_t \right)$$

Relate it to the expression for "optimal" price

$$P_t^* = (1 + \mu) \, MC_t \quad \rightarrow \quad \ln P_t^* = \ln (1 + \mu) + \ln MC_t \quad \rightarrow \quad p_t^* = \mu + mc_t$$

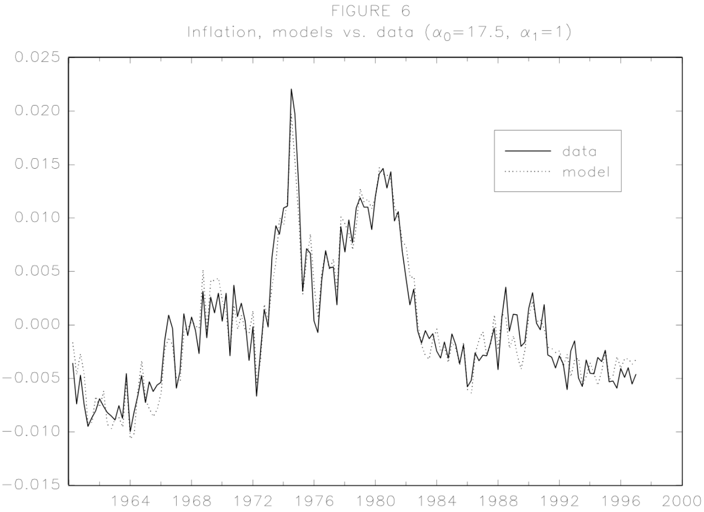
$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \left(\mu + mc_t - p_t \right) \equiv \beta \mathrm{E}_t \pi_{t+1} + \chi \tilde{mc}_t^r$$

where $mc - p$ is the log of **real marginal cost**, in the steady state equal to $-\mu$ and $\tilde{mc}^r$ is the percentage deviation of the real MC from the steady state, dependent on the ratio of real wages to labor productivity

Using the forward-looking character of inflation we can express it as

$$\pi_t = \chi \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \tilde{mc}_{t+i}^r$$

Inflation vs its one period ahead forecast from NKPC



Sbordone (2002)

Hybrid NKPC

Gali and Gertler (1999) construct a proxy for the real marginal cost gap

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \quad \rightarrow \quad \tilde{mc}_t^r \equiv \frac{W_t/P_t}{\partial Y_t/\partial N_t} = \frac{W_t/P_t}{(1-\alpha)\, Y_t/N_t} = \frac{1}{1-\alpha}\frac{W_t N_t}{P_t Y_t}$$

And estimate the following NKPC

$$\pi_t = 0.942\, \mathrm{E}_t \pi_{t+1} + 0.023\, \tilde{mc}_t^r$$

What if not all firms reset their prices optimally?

- Fraction $1-\omega$ is **forward looking** and set $p_t^f = \tilde{p}_t$
- Fraction ω is **backward looking** and reset prices basing on what their competitors did in $t-1$, "error" not very costly in low inflation environment

$$p_t^b = \tilde{p}_{t-1} + \pi_{t-1}, \quad p_t^b - \tilde{p}_t = \frac{\theta}{1-\theta}\pi_t$$

After aggregating we get the hybrid NKPC (where $\gamma_b + \gamma_f \leq 1$)

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f \mathrm{E}_t \pi_{t+1} + \chi \tilde{mc}_t^r$$

Empirical tests of Hybrid NKPC: US

Table 2
Estimates of the New Hybrid Phillips Curve

	ω	θ	β	γ_b	γ_f	λ
GDP Deflator						
(1)	0.265 (0.031)	0.808 (0.015)	0.885 (0.030)	0.252 (0.023)	0.682 (0.020)	0.037 (0.007)
(2)	0.486 (0.040)	0.834 (0.020)	0.909 (0.031)	0.378 (0.020)	0.591 (0.016)	0.015 (0.004)
Restricted β						
(1)	0.244 (0.030)	0.803 (0.017)	1.000	0.233 (0.023)	0.766 (0.015)	0.027 (0.005)
(2)	0.522 (0.043)	0.838 (0.027)	1.000	0.383 (0.020)	0.616 (0.016)	0.009 (0.003)
NFB Deflator						
(1)	0.077 (0.030)	0.830 (0.016)	0.949 (0.019)	0.085 (0.031)	0.871 (0.018)	0.036 (0.008)
(2)	0.239 (0.043)	0.866 (0.025)	0.957 (0.021)	0.218 (0.031)	0.755 (0.016)	0.015 (0.006)

Note: Table 2 reports GMM estimates of parameters of equation (26). Rows (1) and (2) correspond to the two specifications of the orthogonality conditions found in equations (27) and (28) in the text, respectively. Estimates are based on quarterly data and cover the sample period 1960:1-1997:4. Instruments used include four lags of inflation, labor income share, long-short interest rate spread, output gap, wage inflation, and commodity price inflation. A 12 lag Newey-West estimate of the covariance matrix was used. Standard errors are shown in brackets.

Gali and Gertler (1999), λ is "ours" χ

Empirical tests of Hybrid NKPC: EA

Phillips curves - Estimation output (endogenous variable: core inflation)				
	I Baseline	II Flattened	III Exponential	IV With oil price
Sample period	*1991q1 - 2010q4*	*2000q1 - 2010q4*	*1991q1 - 2010q4*	*1991q1 - 2010q4*
Core inflation lagged one quarter	0.830***	0.881***	0.799***	0.776***
Inflation expectations	0.164*	0.143**	0.179***	0.199***
Output gap	0.071***	0.038***		0.033*
EXP (output gap)			0.057***	
Constant			-0.072	
Oil price change lagged four quarters				0.002***
R^2	0.97	0.88	0.97	0.95
J statistic	0.046	0.09	0.049	0.030

Estimation by GMM. Dependent variable: core inflation. Instruments: First lag of output gap; first and second lags of inflation expectations; short-term interest rates and their first lag; change in the rate of capacity utilisation; oil price change; fifth lag of oil price change (the latter two for panel IV only).

*, **, *** denote significance at 5, 2 and 1% confidence level

Source: Commission services

European Commission (2011)

New Keynesian IS curve

Households' problem

For simplicity consider a model with no physical capital (can save via bonds)

$$\max \quad U = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$\text{s.t.} \quad P_t C_t + B_t = W_t N_t + (1 + i_{t-1}) B_{t-1} + D_t$$

Langrangian (already expanded "from the point of view" of period t)

$$\mathcal{L} = \frac{\boldsymbol{C_t}^{1-\sigma}}{1-\sigma} - \psi \frac{\boldsymbol{N_t}^{1+\varphi}}{1+\varphi} + \lambda_t \left[W_t \boldsymbol{N_t} + (1 + i_{t-1}) B_{t-1} + D_t - P_t \boldsymbol{C_t} - \boldsymbol{B_t} \right]$$

$$+ \beta \mathrm{E}_t \left[\frac{C_{t+1}^{1-\sigma}}{1-\sigma} - \psi \frac{N_{t+1}^{1+\varphi}}{1+\varphi} + \lambda_{t+1} \left[W_{t+1} N_{t+1} + (1 + i_t) \boldsymbol{B_t} + D_{t+1} - P_{t+1} C_{t+1} - B_{t+1} \right] \right] + \dots$$

First order conditions

$$\boldsymbol{C_t} : \quad C_t^{-\sigma} - \lambda_t P_t = 0 \quad \rightarrow \quad \lambda_t = C_t^{-\sigma} / P_t$$

$$\boldsymbol{N_t} : \quad -\psi N_t^{\varphi} + \lambda_t W_t = 0 \quad \rightarrow \quad \lambda_t = \psi N_t^{\varphi} / W_t$$

$$\boldsymbol{B_t} : \quad -\lambda_t + \beta \mathrm{E}_t \left[\lambda_{t+1} (1 + i_t) \right] = 0 \quad \rightarrow \quad \lambda_t = \beta \mathrm{E}_t \left[\lambda_{t+1} (1 + i_t) \right]$$

Households' problem

Euler equation

$$\frac{C_t^{-\sigma}}{P_t} = \beta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} (1 + i_t) \right] \quad \rightarrow \quad C_t^{-\sigma} = \beta \mathrm{E}_t \left[C_{t+1}^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \right]$$

Labor supply

$$\frac{C_t^{-\sigma}}{P_t} = \frac{\psi N_t^{\varphi}}{W_t} \quad \rightarrow \quad N_t^{\varphi} = \frac{1}{\psi} \frac{W_t}{P_t} C_t^{-\sigma} \quad \rightarrow \quad N_t = \left(\frac{1}{\psi} \frac{W_t}{P_t} C_t^{-\sigma} \right)^{1/\varphi}$$

Can always add a money demand equation like

$$M_t^d = \nu \mathrm{E}_t \left[P_{t+1} C_{t+1} \right] \cdot i_t^{-1/\sigma}$$

Add a few assumptions

Assume no investment and government spending, so that $C_t = Y_t$

$$Y_t^{-\sigma} = \frac{1}{1+\rho} \mathrm{E}_t \left[Y_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \right]$$

Apply logs and the first order approximation ("forget" about covariance)

$$-\sigma \ln Y_t \approx -\ln(1+\rho) - \sigma \mathrm{E}_t \ln Y_{t+1} + \ln(1+i_t) - \ln(1+\mathrm{E}_t \pi_{t+1})$$

$$y_t \approx \mathrm{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - \rho)$$

In the flexible price world (where r^f is the natural real interest rate)

$$y_t^f = \mathrm{E}_t y_{t+1}^f - \frac{1}{\sigma}(r_t^f - \rho)$$

Subtract the "natural" from the "actual" equation (where $x_t \equiv y_t - y_t^f$ is the **output gap**)

$$y_t - y_t^f = \mathrm{E}_t[y_{t+1} - y_{t+1}^f] - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f)$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f)$$

New Keynesian IS curve

Natural real interest rate

$$r_t^f = \rho + \mathrm{E}_t[\Delta y_{t+1}^f]$$

In general case where $C_t \neq Y_t$, one can capture the influence of other expenditures as a demand shock $\tilde{u}$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + \frac{1}{\sigma}\mathrm{E}_t[\Delta y_{t+1}^f] + \tilde{u}_t$$
$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + u_t$$

Cet. par. higher nominal interest rate leads to more negative output gap

Caution: positive output gap ($x > 0$) means the level of output is higher than in the counterfactual flex-price world ($y > y^f$)

Output gap in NKPC

Assume $C_t = Y_t$ and $Y_t = Z_t N_t$ to easily relate x_t with $\tilde{mc}_t^r$

$$Y_t = Z_t N_t \quad \rightarrow \quad MC_t = W_t / Z_t \quad \rightarrow \quad W_t = Z_t MC_t$$

Labor supply

$$\frac{\psi N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t} = Z_t \frac{MC_t}{P_t} \qquad \rightarrow \quad \ln \psi + \varphi n_t - \sigma c_t = z_t + mc_t - p_t$$

$$\ln \psi + \varphi (y_t - z_t) - \sigma y_t = z_t + mc_t - p_t \qquad \rightarrow \quad y_t = \frac{1+\varphi}{\varphi+\sigma} z_t + \frac{1}{\varphi+\sigma}(mc_t - p_t) - \frac{\ln \psi}{\varphi+\sigma}$$

In the flexible prices world

$$P_t = (1+\mu) MC_t \quad \rightarrow \quad p_t = \mu + mc_t \quad \rightarrow \quad y_t^f = \frac{1+\varphi}{\varphi+\sigma} z_t + \frac{1}{\varphi+\sigma}(-\mu) - \frac{\ln \psi}{\varphi+\sigma}$$

Output gap

$$x_t = y_t - y_t^f = \frac{1}{\varphi+\sigma}(mc_t - p_t - (-\mu)) = \frac{1}{\varphi+\sigma}(\mu + mc_t - p_t) = \frac{1}{\varphi+\sigma} \tilde{mc}_t^r$$

Output gap in NKPC

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \chi \tilde{mc}_t^r$$

Output gap

$$x_t = \frac{1}{\varphi + \sigma} \tilde{mc}_t^r$$

Final form of NKPC

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

where $\kappa \equiv \chi (\varphi + \sigma) > 0$, and e_t is a cost-push shock: influence of non-wage costs of production when $Y_t \neq Z_t N_t$

Key equations of the New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + u_t$$

A forward-looking system of three variables:
output gap x, inflation π and nominal interest rate i

Need an additional equation to close the system
$\hookrightarrow$ need to specify monetary policy rule

Monetary policy in the New Keynesian model

Optimal policy: long run

Two distortions in the basic model

1. Monopolistic competition: $P > MC$
2. Price dispersion: $P_i/P_j \neq MC_i/MC_j$

The first distortion cannot be eliminated by monetary policy

But the second can, by keeping inflation rate at 0%

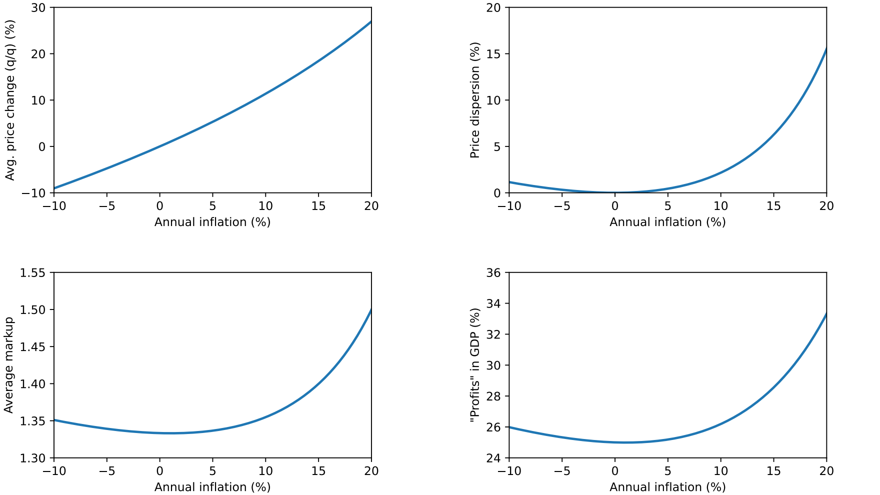
Welfare losses of annual inflation in 2-3% range
are very small, and other considerations matter

A positive inflation target (in advanced economies usually 2%)

- Decreases price dispersion under declining MC due to technological progress
- Increases labor market fluidity (nominal wage stickiness)
- Increases monetary policy space to reduce interest rate in recessions
- Lowers probability of hitting the effective lower bound on interest rates

Costs of inflation under Calvo scheme ($\varepsilon = 4$ and $\theta = 0.75$)

Inflation other than 0% $\rightarrow$ price dispersion and lower effective output



Optimal policy: short run

If there are no cost-push shocks and sticky prices are the only distortion then optimal monetary policy in the short run is to stabilize inflation perfectly

This would also perfectly stabilize output gap: divine coincidence, see Blanchard and Gali (2007)

If cost-push shocks occur and there are other distortions, e.g. sticky wages, then optimal policy becomes more complicated, e.g. has to also stabilize wage inflation

Divine coincidence no longer holds (impossible to stabilize all variables at the same time)

Funkcja straty optymalnej polityki

Due to many distortions optimal policy involves trade-offs

Rotemberg and Woodford (1998): when real imperfections are present, the second order approximation to social welfare is

$$W = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\lambda x_t^2 + \pi_t^2 \right) \right]$$

Consistent with behavior of central banks, who aim to stabilize both inflation and output gaps

Question arises whether policy should be conducted discretionary or under commitment

Optimal policy under discretion

Under optimal discretionary policy (ODP) the central bank is not able to influence expectations about future policy

Optimization boils down to solving a series of static problems

$$\begin{aligned} \min \quad & \lambda x_t^2 + \pi_t^2 \\ \text{subject to} \quad & \pi_t = \beta \pi_t^e + \kappa x_t + e_t \end{aligned}$$

Note that expectations π_t^e are taken as given, since the central bank is assumed not to influence them

Solution:

$$\pi_t = -\frac{\lambda}{\kappa} x_t$$

Caution: This is a targeting rule, without specifying instruments

After an inflationary cost-push shock the central bank allows the output gap to become negative

Optimal policy under commitment

Under **credible** commitment the central bank is able to influence expectations about future policy

The problem is now dynamic

$$\min \quad \frac{1}{2}\sum_{t=0}^{\infty}\beta^t \, \mathrm{E}_0\left[\lambda x_t^2 + \pi_t^2\right]$$

$$\text{subject to} \quad \pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty}\beta^t \, \mathrm{E}_0\left[\frac{1}{2}\left(\lambda x_t^2 + \pi_t^2\right) + \mu_t\left(\beta\pi_{t+1} + \kappa x_t + e_t - \pi_t\right)\right]$$

First order conditions

$$x_t \quad : \quad \beta^t \, \mathrm{E}_0\left[\lambda x_t + \mu_t \kappa\right] = 0 \qquad \rightarrow \qquad \mu_t = -\frac{\lambda}{\kappa} x_t$$

$$\pi_t \quad : \quad \beta^{t-1} \, \mathrm{E}_0\left[\mu_{t-1}\beta\right] + \beta^t \, \mathrm{E}_0\left[\pi_t - \mu_t\right] = 0 \qquad \rightarrow \qquad \pi_t = \mu_t - \mu_{t-1}$$

Optimal policy under commitment

For the current period the past is not a constraint ($\mu_{-1} = 0$)

$$\pi_0 = \mu_0 = -\frac{\lambda}{\kappa} x_0$$

Same as under discretion

For the future periods ($t \geq 1$) we get

$$\pi_t = \mu_t - \mu_{t-1} = -\frac{\lambda}{\kappa} \left(x_t - x_{t-1} \right)$$

Different than for today: will take the past into account

Optimal commitment policy (OCP) means pursuing a discretionary policy today, but promising a non-discretionary policy from tomorrow on!

But when we'll arrive in the next period, we will be tempted to act as in the current period: time inconsistency

Optimal policy under commitment

OCP is time inconsistent – solutions?

1. Appoint very credible central bankers
2. To build credibility, adopt a timeless perspective: pretend that OCP has been applied long ago and apply the formula for $t \geq 1$ from the beginning

Which is better: OCP or ODP?

- Neither invokes inflation bias
- ODP generates **stabilization bias**, making economy more volatile

The superiority of commitment calls for a credible, long-term arrangement for the central bank that will sometimes act **against** short-term welfare

Stabilization bias: discretion vs commitment

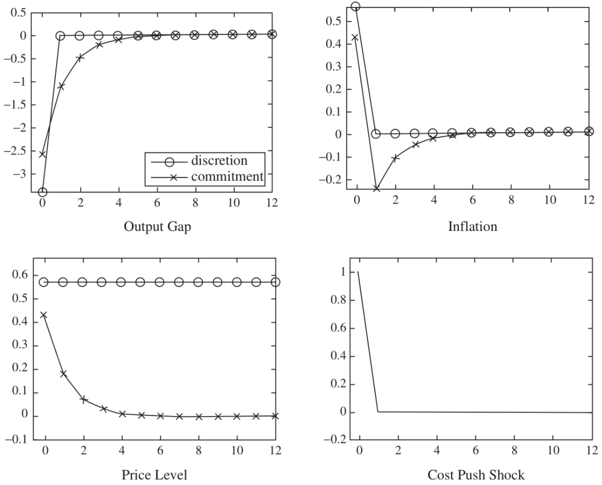


Figure 5.1 Optimal Responses to a Transitory Cost Push Shock

Gali (2008)

Stabilization bias: discretion vs commitment

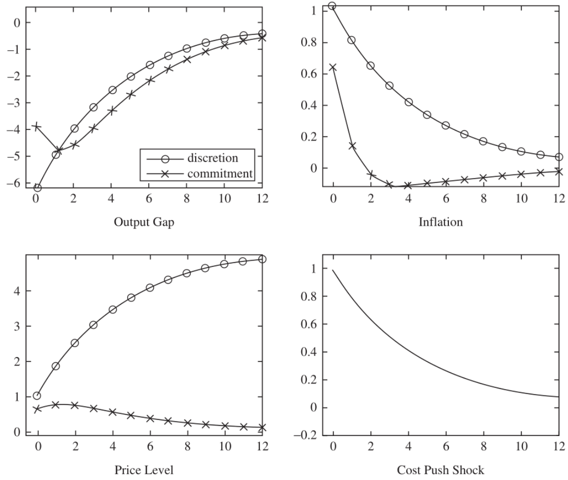


Figure 5.2 Optimal Responses to a Persistent Cost Push Shock

Gali (2008)

Digression: problems with identifying the NKPC

Recently, many papers have claimed that either

1. NKPC has become more "flat", or
2. NKPC relationship has disappeared at all

McLeay and Tenreyro (2019): Under ODP, observed inflation will be unrelated to the measure of slack in the economy!

Assume the cost-push shock follows an AR(1) process

$$e_t = \rho_e \, e_{t-1} + \epsilon_t$$

Combining above with NKPC and ODP rule, we get

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda\,(1 - \beta\rho_e)} e_t \equiv \lambda\alpha \, e_t = \lambda\alpha\,(\rho_e \, e_{t-1} + \epsilon_t)$$

Digression: problems with identifying the NKPC

Under ODP inflation is a function of cost-push shock

$$\pi_t = \lambda\alpha\, e_t = \lambda\alpha\left(\rho_e\, e_{t-1} + \epsilon_t\right)$$

But this means that current inflation will be very well forecastable using past inflation

$$\pi_t = \rho_e\, \pi_{t-1} + \lambda\alpha\epsilon_t$$

Realized inflation and output gap will be **negatively** correlated

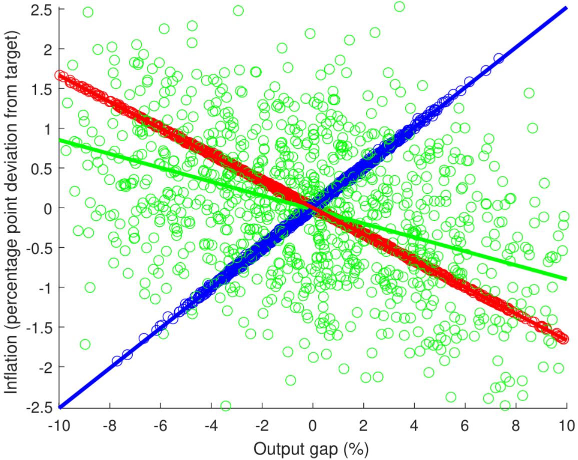
$$\pi_t = -\frac{\lambda}{\kappa} x_t \quad \rightarrow \quad \pi_t = b x_t + \varepsilon_t$$

$$\hat{b} < 0 \quad \text{and} \quad \mathrm{Corr}\left(x_t,\, \varepsilon_t\right) \neq 0$$

Additionally, if there are shocks to monetary policy rule, any estimate of $\hat{b}$ is possible, including $\hat{b} > 0$ and $\hat{b} \approx 0$!

Estimate for $\hat{b}$ depends on relative magnitude of shocks

Figure 5: *Inflation/output gap correlation in model-simulated data: optimal discretion with shocks to the targeting rule*

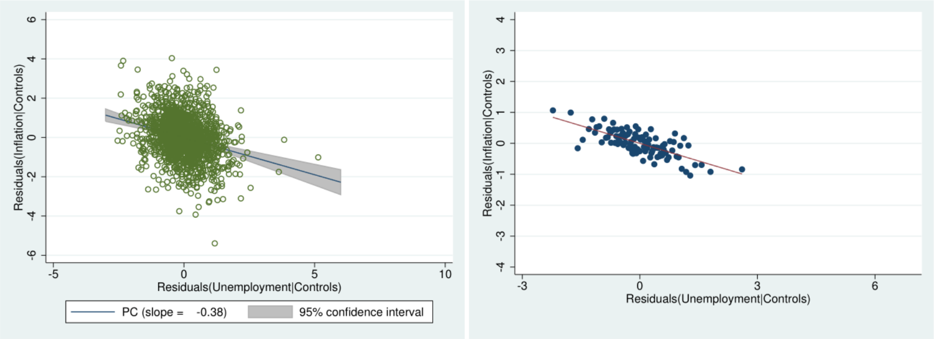


McLeay and Tenreyro (2019)

One solution: Identification using regional data

Figure 12: *Year and metro area fixed effects: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*

(a) *Raw residuals* **(b)** *Residuals grouped into bins*



Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (4) in table 3. See the notes to Figures 10a and 10b for details.

McLeay and Tenreyro (2019)

Instrument rules

We have been assuming that the CB can "choose" x_t and π_t

In reality, the CB can influence these variables indirectly, by e. g. changing the nominal interest rate

Recall the NKIS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - \rho\right) + u_t$$

where the CB can affect output gap by varying i

Note that monetary policy by changing i affects output gap "first" and inflation rate "second"

Determinacy concerns

Under ODP inflation and output gap react to cost-push shock

$$\pi_t = \lambda\alpha\, e_t \quad \text{and} \quad x_t = -\kappa\alpha\, e_t$$

The CB could vary the level of i to try implementing the ODP

$$-\kappa\alpha\, e_t = -\kappa\alpha\rho_e\, e_t - \frac{1}{\sigma}\left(i_t - \lambda\alpha\rho_e\, e_t - r_t^*\right)$$
$$i_t = r_t^* + \alpha\left(\kappa\sigma\left(1-\rho_e\right) + \lambda\rho_e\right) e_t$$

But then our forward-looking system would have multiple solutions, only one of which is consistent with ODP

Such instrument rule would be “too weak”

And would require observing e_t **perfectly** in real-time!

Taylor rules

Instead of constructing the instrument rule as function of shocks, construct the rule as function of endogenous variables

$$i_t = \bar{i} + \gamma_\pi \, \pi_t$$

where $\gamma_\pi = (1 - \rho_e)\, \kappa\sigma/\alpha + \rho_e$

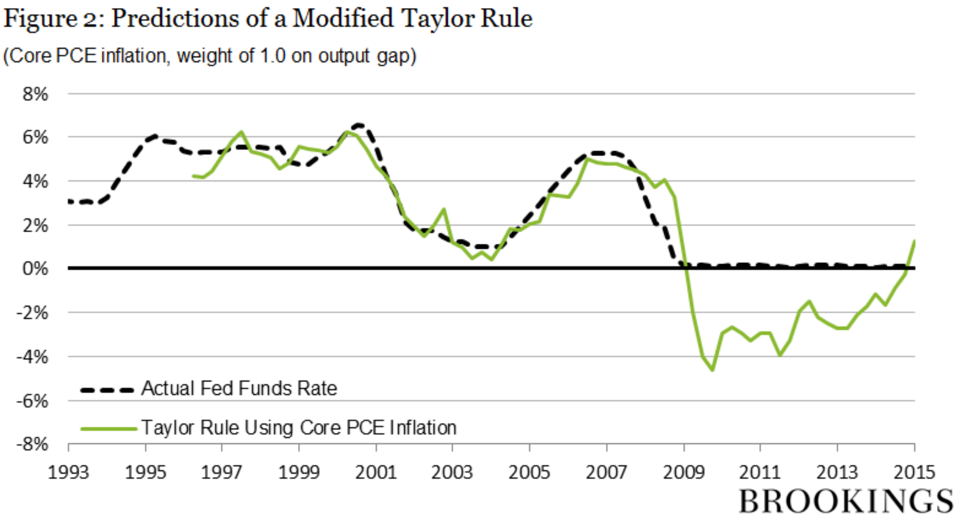
It can be shown that if only $\gamma_\pi > 1$, the system has a unique solution, and the central bank can "select" the ODP equilibrium

A more general Taylor rule allows for reactions to output gap and smoothing of policy rate changes

$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i) \left(\bar{i} + \gamma_\pi \, \pi_t + \gamma_x \, x_t \right)$$

A CB should raise the interest rate when inflation is above target and / or output gap is positive

A Taylor rule can capture Fed's actual policy prior to 2009



Bernanke (2015)

Taylor principle and stability of inflation

Taylor principle: when inflation increases by 1 p.p., the central bank should raise the interest rate by $\gamma_\pi > 1$ p.p. (in a dynamic sense)

Failure to do so results in inflation instability

The estimate for the pre-Volcker rule is significantly less than unity. Monetary policy over this period was accommodating increases in expected inflation, in clear violation of the [Taylor principle – MB].

TABLE 1
ESTIMATES OF POLICY REACTION FUNCTION

	γ_π	γ_x	ρ
Pre-Volcker	0.83	0.27	0.68
	(0.07)	(0.08)	(0.05)
Volcker–Greenspan	2.15	0.93	0.79
	(0.40)	(0.42)	(0.04)

Clarida, Gali and Gertler (1999)

Basic three-equation New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t \pi_{t+1} - r_t^f) + \tilde{u}_t$$

Taylor rule

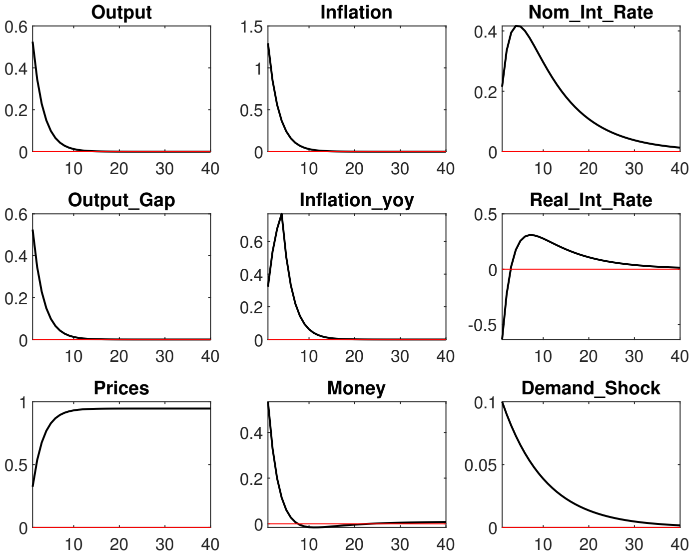
$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i) \left(\bar{i} + \gamma_\pi \, \pi_t + \gamma_x \, x_t\right) + v_t$$

where v is an **exogenous** monetary policy shock

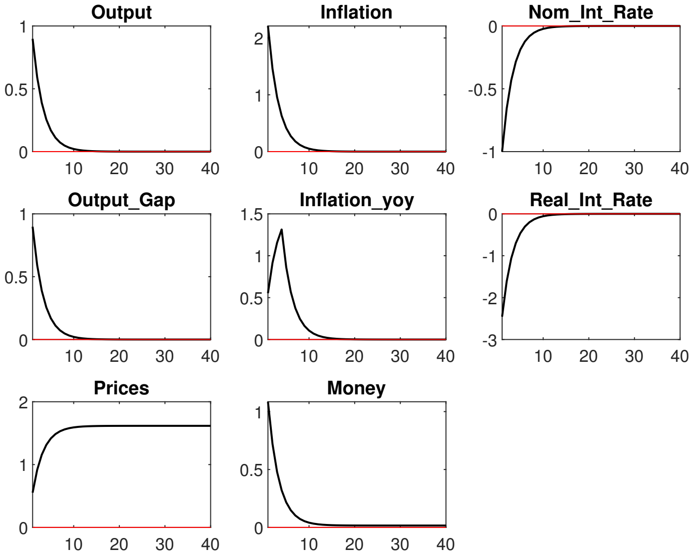
Plus (optinally) the natural real interest rate equation

$$r_t^f = \rho + \sigma \mathrm{E}_t[\Delta y_{t+1}^f]$$

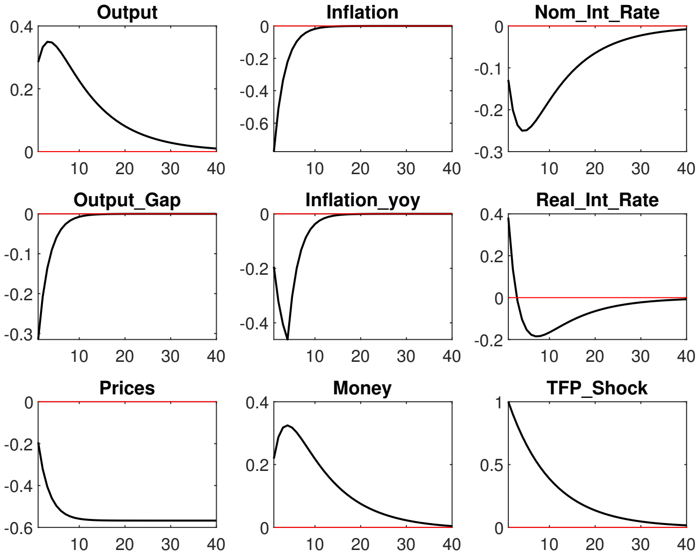
Positive demand shock ($\tilde{u} > 0$)



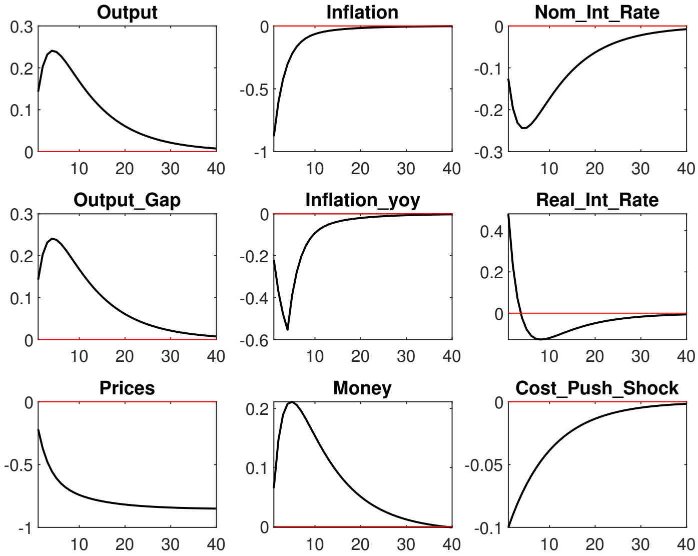
Positive monetary shock ($v < 0$)



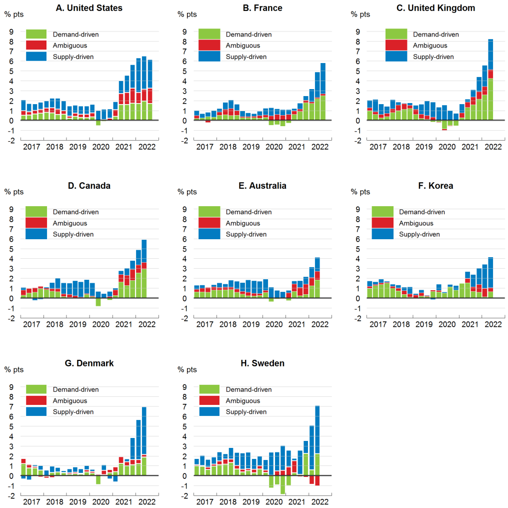
Positive TFP shock ($z > 0$)



Positive cost-push shock ($e < 0$)



Model-based inflation shock decomposition

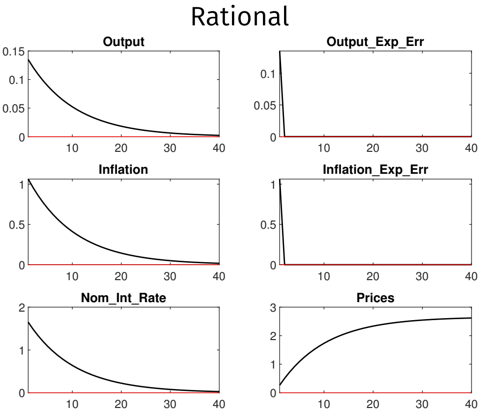


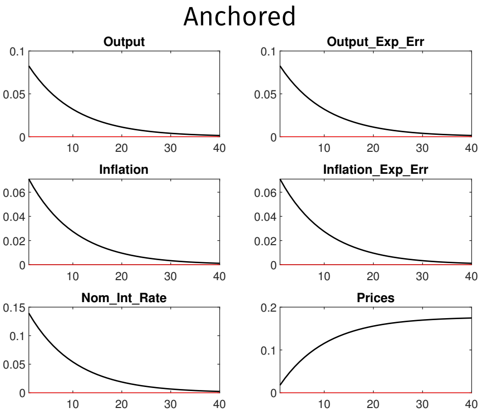
Role of expectations

Modern monetary policy: management of expectations

Woodford (2005, p. 3): For not only do expectations about policy matter, but, at least under current conditions, very little *else* matters

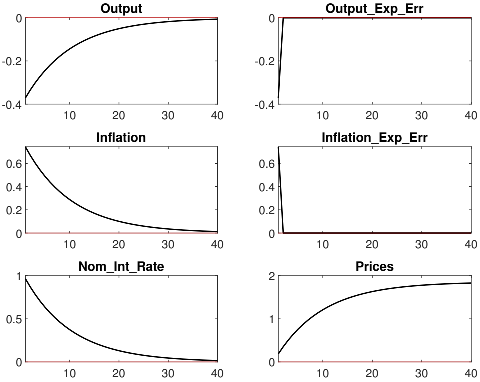
Response to demand shock under alternative expectations



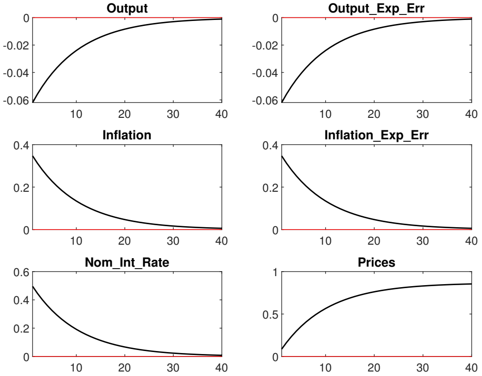


Response to cost-push shock under alternative expectations

Rational

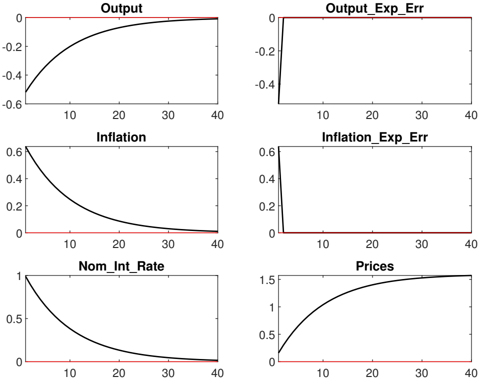


Anchored



Response to TFP shock under alternative expectations

Rational



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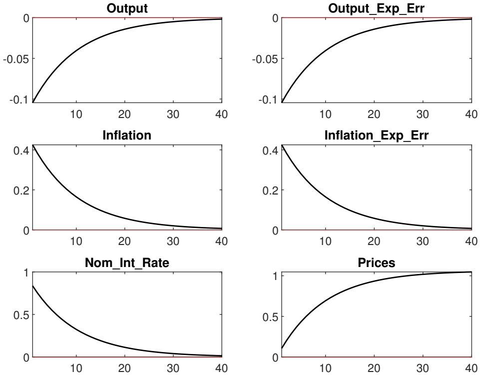


Empirical performance of NK

Shocks affect the economy in the directions indicated by empirical evidence

Basic model is too stylized to take it directly to data

Some standard extensions introduced to applied NK models:

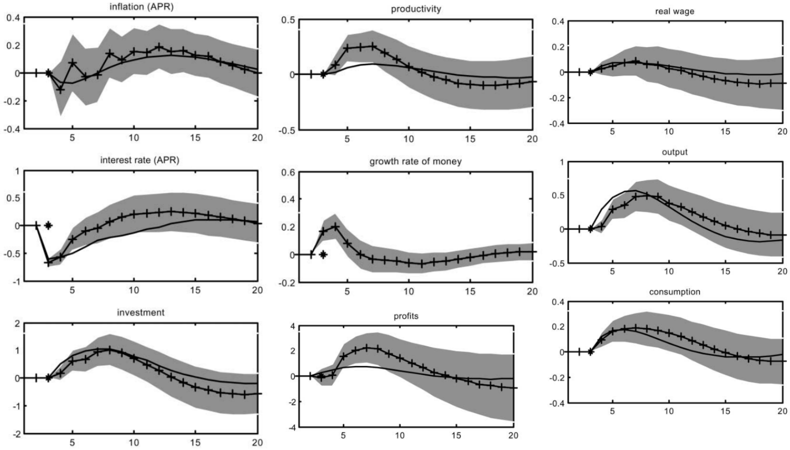
- Nominal wage stickiness
- Mechanisms that generate hybrid NKPC
- Habits in the utility function add backward-looking terms to the NKIS
- Investment adjustment costs delay the response of investment to shocks

More complicated extensions:

- Financial frictions (Bernanke, Gertler and Gilchrist 1999, Kiyotaki and Moore 1997, Iacoviello 2005)
- Unemployment (Gertler, Sala and Trigari 2008, Gali 2010)

Empiryczne dopasowanie modelu NK o średniej skali

Model- and VAR-based impulse responses. Solid lines are benchmark model impulse responses; solid lines with plus signs are VAR-based impulse responses. Grey areas are 95 percent confidence intervals about VAR-based estimates.



Christiano, Eichenbaum and Evans (2005)