University of Warsaw
Faculty of Economic Sciences

Money, price expectations and inflation bias

Advanced Macroeconomics: Lecture 8

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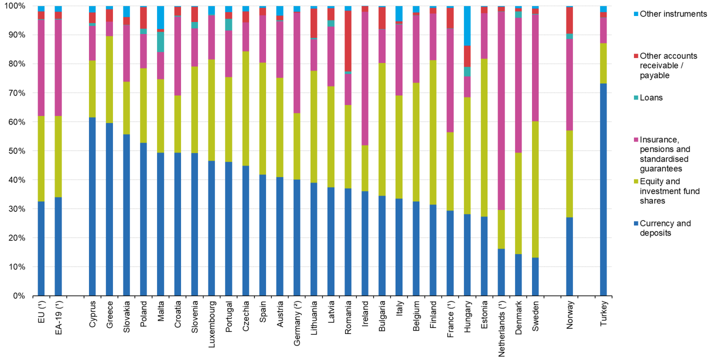
Demand for money

Money demand

Share of type of assets of households, 2020
(% share of total financial assets of households)



Note:
The Other category refers to the sum of four groups:
[F1] Monetary gold and special drawing rights (SDRs)
[F3] Debt securities
[F7] Financial derivatives and employee stock options

(¹) Provisional.
(²) No data for Germany on Loans
Source: online data code (nasa_10_f_bs)

eurostat

Money demand

Money buys goods, and goods buy money, but rarely goods buy goods (barter)

Literature on "deep" theory of money: Kiyotaki and Wright (1991), Lagos and Wright (2005)

Not interested here in **why** people want to hold money

Interested in what affects the observed **demand** for money

Motives for holding money:

- **transaction demand**
- precautionary demand
- asset / speculatory demand

Transaction demand for money

Modeled in spirit of Baumol (1952) and Tobin (1956)

Consider the following real-life example:

- Ignore cash, focus on electronic money
- Two accounts in bank: checking and savings
- Costless transfers / purchases via checking account
- Savings account pays interest i
- Transfer from savings account costs K
- Monthly purchasing needs are PY, at uniform rate
- How much money M should I hold on average?
- Equivalently: how many transfers n should I make?

Baumol-Tobin model

Number of transfers n, amount transferred m, $PY = mn$

Average money holding

$$M = \frac{m}{2} = \frac{PY}{2n}$$

Optimization problem

$$\min \quad Kn + iM = Kn + \frac{iPY}{2n}$$

- Kn are nominal transferring costs
- iM is opportunity cost of holding M outside the savings account

First order condition

$$K - \frac{iPY}{2n^2} = 0$$

Baumol-Tobin model

Optimal number of transfers, where $k \equiv K/P$

$$n^2 = \frac{iY}{2K/P} \quad \rightarrow \quad n = \sqrt{\frac{iY}{2k}}$$

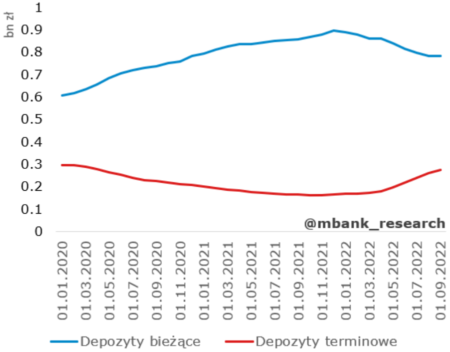
Average money holding

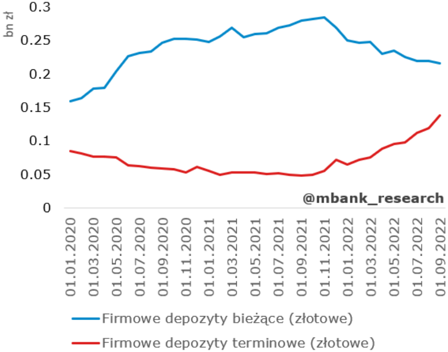
$$M = \frac{PY}{2n} = P \cdot \sqrt{\frac{Yk}{2i}}$$

Taking logarithms

$$\ln M = \ln P + 0.5 \ln Y - 0.5 \ln i + 0.5 \ln k$$

Reallocation of deposit portfolio in Poland following changes in i





mBank

Empirical studies on money demand

The sample used in this paper that consists of 381 empirical estimations relating to 16 different OECD countries yields an average income elasticity estimate **almost equal to 1.0** with a sizeable standard deviation of 0.37.

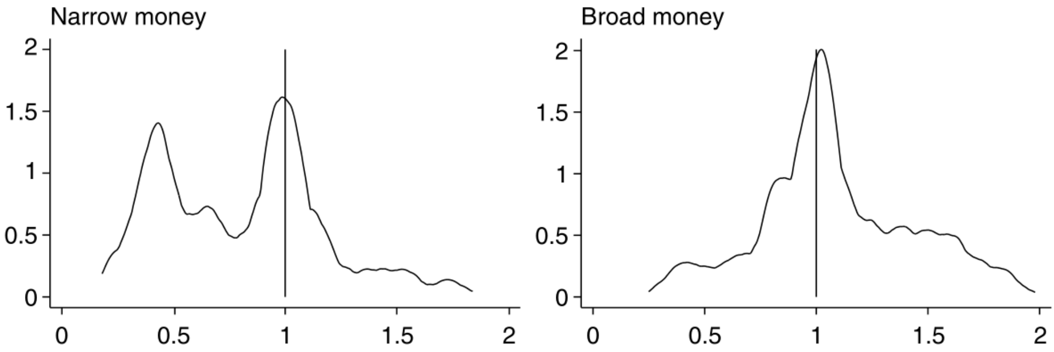


Figure 1. Smoothed histogram of point estimates.

Knell and Stix (2005)

Digression: monetary aggregates

Money = Central bank money + Commercial bank money (created when issuing loans)

M0: cash in circulation

MB: M0 + cash in commercial and central banks' vaults

M1: M0 + demand deposits + traveller's checques

M2: M1 + savings deposits + some time deposits and money market funds

M3: M2 + remaining time deposits

MZM: M2 + remaining money market funds

In previous graph:

- Narrow money: M0 or M1
- Broad money: M2 or MZM

“Fixing” the Baumol-Tobin model

Checking vs savings accounts: narrow money

Think now on choosing “broad” money vs bonds

Buying / selling bonds: broker fees b

Assume m is exogenous (due to fee structure it is not optimal to convert $m < \bar{m}$)

$$\min \quad bmn + \frac{iPY}{2n}$$

First order condition

$$bm - \frac{iPY}{2n^2} = 0 \quad \rightarrow \quad \frac{bPY}{n} = \frac{iPY}{2n^2} \quad \rightarrow \quad b = \frac{i}{2n}$$

Optimal number of transfers

$$n = \frac{i}{2b}$$

“Fixing” the Baumol-Tobin model

Average money holding

$$M = \frac{PY}{2n} = PY \cdot \frac{b}{i}$$

Taking logarithms

$$\ln M = \ln P + \ln Y - \ln i + \ln b$$

Only a fraction ω of people participates in asset markets

$$M = \omega \cdot PY \cdot \frac{b}{i} + (1-\omega) \cdot PY = PY\left[1 + \omega\left(\frac{b}{i} - 1\right)\right]$$

$$\ln M = \ln P + \ln Y + \ln\left[1 + \omega\left(\frac{b}{i} - 1\right)\right]$$

Interest (semi-)elasticity of money demand depends (in absolute value) positively on ω (see e.g. Reynard 2004)

Empirical studies on money demand

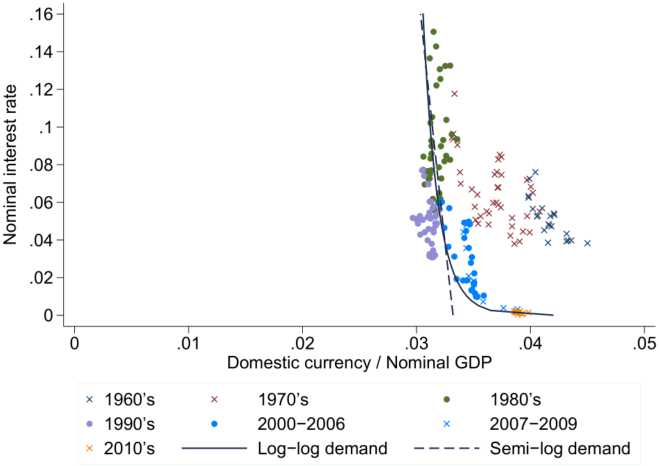


Figure 5: U.S. domestic currency demand 1964-2012

Briglevics and Schuh (2014)

General Equilibrium approaches to modeling money demand

Cash In Advance: cash is required to buy certain goods (similar to Baumol-Tobin)

Shopping Time: money (vs barter) facilitates trade, less time spent making transactions

Money In Utility: “reduced form” of Shopping Time

For simplicity, we’ll focus on models without uncertainty and physical capital

General equilibrium models all result in a similar real money demand function

$$\frac{M}{P} = L\left(Y, i\right) \quad \text{with} \quad L_Y \equiv \frac{\partial L}{\partial Y} \approx 1 \quad \text{and} \quad L_i \equiv \frac{\partial L}{\partial i} < 0$$

Attention: from now on we change our notation convention

- Levels of variables will be written in big letters
- Their logarithms will be written in small letters
- Small letters will also be used for rates: interest rates, inflation rates, etc.

Cash In Advance (CIA)

A fraction χ of goods (C_1) are bought with cash, others (C_2) can be bought with credit

$$\max_{\{C_{1t}, C_{2t}, B_t, M_t\}_{t=0}^{\infty}} \quad U = \sum_{t=0}^{\infty} \beta^t \left(\chi \ln C_{1t} + (1-\chi) \ln C_{2t}\right)$$

$$\text{subject to} \quad P_t \left(C_{1t} + C_{2t}\right) + B_t + M_t = W_t + \left(1 + i_{t-1}\right) B_{t-1} + M_{t-1}$$

$$P_t C_{1t} \leq M_{t-1}$$

where P is the consumer price index (assume „cash" and „credit" goods have the same prices), W is nominal wage and B are nominal bonds yielding nominal interest rate i

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \chi \ln C_{1t} + (1-\chi) \ln C_{2t} + \mu_t \left[M_{t-1} - P_t C_{1t}\right] \\ +\lambda_t \left[W_t + \left(1 + i_{t-1}\right) B_{t-1} + M_{t-1} - P_t \left(C_{1t} + C_{2t}\right) - B_t - M_t\right] \end{array} \right\}$$

Cash In Advance (CIA)

Expanded Lagrangian

$$\mathcal{L} = \ldots + \beta^t \left\{ \begin{array}{c} \chi \ln \boldsymbol{C_{1t}} + (1-\chi) \ln \boldsymbol{C_{2t}} + \mu_t \left[M_{t-1} - P_t \boldsymbol{C_{1t}} \right] \\ + \lambda_t \left[W_t + (1+i_{t-1}) B_{t-1} + M_{t-1} - P_t \boldsymbol{C_{1t}} - P_t \boldsymbol{C_{2t}} - \boldsymbol{B_t} - \boldsymbol{M_t} \right] \end{array} \right\}$$

$$+ \beta^{t+1} \left\{ \begin{array}{c} \chi \ln C_{1,t+1} + (1-\chi) \ln C_{2,t+1} + \mu_{t+1} \left[\boldsymbol{M_t} - P_{t+1} C_{1,t+1} \right] \\ + \lambda_{t+1} \left[W_{t+1} + (1+i_t) \boldsymbol{B_t} + \boldsymbol{M_t} - P_{t+1} C_{1,t+1} - P_{t+1} C_{2,t+1} - B_{t+1} - M_{t+1} \right] \end{array} \right\} + \ldots$$

First Order Conditions

$$\boldsymbol{C_{1t}} : \quad \beta^t \left\{ \chi \frac{1}{C_{1t}} - \mu_t P_t - \lambda_t P_t \right\} = 0 \quad \rightarrow \quad \lambda_t = \frac{\chi}{P_t C_{1t}} - \mu_t$$

$$\boldsymbol{C_{2t}} : \quad \beta^t \left\{ (1-\chi) \frac{1}{C_{2t}} - \lambda_t P_t \right\} = 0 \quad \rightarrow \quad \lambda_t = \frac{1-\chi}{P_t C_{2t}}$$

$$\boldsymbol{B_t} : \quad \beta^t \{-\lambda_t\} + \beta^{t+1} \{\lambda_{t+1} (1+i_t)\} = 0 \quad \rightarrow \quad \lambda_t = \beta \lambda_{t+1} (1+i_t)$$

$$\boldsymbol{M_t} : \quad \beta^t \{-\lambda_t\} + \beta^{t+1} \{\lambda_{t+1} + \mu_{t+1}\} = 0 \quad \rightarrow \quad \lambda_t = \beta (\lambda_{t+1} + \mu_{t+1})$$

Cash In Advance (CIA)

If CIA constraint does not bind ($\mu = 0$), consumption of goods reflects their shares in U

$$\frac{\chi}{P_t C_{1t}} = \frac{1-\chi}{P_t C_{2t}} \quad \rightarrow \quad C_{1t} = \frac{\chi}{1-\chi} C_{2t} \quad \rightarrow \quad s = \frac{C_{1t}}{C_{1t} + C_{2t}} = \frac{\frac{\chi}{1-\chi} C_{2t}}{\frac{\chi}{1-\chi} C_{2t} + C_{2t}} = \frac{\frac{\chi}{1-\chi}}{\frac{1}{1-\chi}} = \chi$$

The FOC for bonds B results in the Euler equation

$$\frac{\chi}{P_t C_{1t}} = \beta \frac{\chi}{P_{t+1} C_{1,t+1}} (1 + i_t) \quad \rightarrow \quad \frac{1}{C_t} = \beta (1 + i_t) \frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}} \quad \rightarrow \quad C_{t+1} = \beta \frac{1 + i_t}{1 + \pi_{t+1}} C_t$$

where $\pi_{t+1} \equiv P_{t+1}/P_t - 1$ is inflation (relative change in CPI) from period t to $t+1$

Real interest rate is equal to nominal corrected for inflation

$$1 + r_{t+1} \equiv \frac{1 + i_t}{1 + \pi_{t+1}} \quad \rightarrow \quad C_{t+1} = \beta (1 + r_{t+1}) C_t$$

We get the standard Euler equation for logarithmic utility function

Cash In Advance (CIA)

If the CIA constraint binds, Euler equation applies only for „credit" goods

$$\frac{1-\chi}{P_t C_{2t}} = \beta \frac{1-\chi}{P_{t+1} C_{2,t+1}} (1+i_t) \quad \rightarrow \quad C_{2,t+1} = \beta \frac{1+i_t}{1+\pi_{t+1}} C_{2t}$$

For „cash" goods we use the FOC for money M and then use FOCs for C_1 and C_2, as well as the „credit" Euler equation

$$\beta\left(\lambda_{t+1} + \mu_{t+1}\right) = \lambda_t \quad \rightarrow \quad \beta \frac{\chi}{P_{t+1} C_{1,t+1}} = \frac{1-\chi}{P_t C_{2t}} = \beta \frac{1-\chi}{P_{t+1} C_{2,t+1}} (1+i_t)$$

$$C_{1,t+1} = \frac{\chi}{1-\chi} \frac{C_{2,t+1}}{1+i_t} \quad \rightarrow \quad C_{1t} = \frac{\chi}{1-\chi} \frac{C_{2t}}{1+i_{t-1}}$$

Positive nominal interest rate distorts the shares of consumed goods

$$s\left(i_{t-1}\right) = \frac{C_{1t}}{C_{1t} + C_{2t}} = \frac{\frac{\chi}{1-\chi} \frac{C_{2t}}{1+i_{t-1}}}{\frac{\chi}{1-\chi} \frac{C_{2t}}{1+i_{t-1}} + C_{2t}} = \frac{\chi}{\chi + (1-\chi)\left(1+i_{t-1}\right)} \quad \rightarrow \quad \frac{\partial s}{\partial i_{t-1}} < 0$$

Cash In Advance (CIA)

In an equilibrium with no capital, investment and public spending, consumption is equal to income

$$C_{1t} + C_{2t} = C_t = Y_t$$

Money demand

$$M_{t-1} = P_t C_{1t} = P_t \cdot s\left(i_{t-1}\right) Y_t \quad \rightarrow \quad \ln M = \ln P + \ln Y + \ln s\left(i\right)$$

Household welfare would be maximized if only

$$s\left(i\right) = \chi \quad \Longleftrightarrow \quad i = 0$$

In the steady state

$$C_2 = \beta \frac{1+i}{1+\pi} C_2 \quad \rightarrow \quad 1 + \pi = \beta\left(1+i\right) \quad \rightarrow \quad \pi_{i=0} = \beta - 1 < 0$$

Since a positive nominal interest rate distorts households' decisions, it would be optimal to set it at 0, requiring deflation (Friedman rule)

Shopping Time

If we have money, we can easily purchase goods from any vendor

Without money we need to seek for a vendor willing to barter with us

Real money (M/P) and time spent shopping S generate „trading services"

$$C = S^{\kappa} \left(M/P\right)^{1-\kappa}$$

An inverse function maps from real money and consumption to required shopping time

$$S = C^{1/\kappa} \left(M/P\right)^{(\kappa-1)/\kappa}$$

Utility within a period becomes a Money In Utility function

$$u\left(C, S\right) = \ln C - \psi \ln S = \ln C - \frac{\psi}{\kappa} \ln C - \psi \frac{\kappa - 1}{\kappa} \ln \left(\frac{M}{P}\right) \equiv a \ln C + b \ln \left(\frac{M}{P}\right)$$

where $a = 1 - \psi/\kappa > 0$ and $b = -\psi \left(\kappa - 1\right)/\kappa > 0$ (which requires $0 < \psi < \kappa < 1$)

Money In Utility (MIU)

Real money holdings M/P enter households' utility function

$$\max_{\{C_t, B_t, M_t\}_{t=0}^{\infty}} \quad U = \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \nu \ln\left(\frac{M_{t-1}}{P_t}\right)\right]$$

$$\text{subject to} \quad P_t C_t + B_t + M_t = W_t + (1 + i_{t-1}) B_{t-1} + M_{t-1}$$

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{\ln C_t + \nu \ln\left(\frac{M_{t-1}}{P_t}\right) + \lambda_t \left[W_t + (1 + i_{t-1}) B_{t-1} + M_{t-1} - P_t C_t - B_t - M_t\right]\right\}$$

$$= \ldots + \beta^t \left\{\ln \boldsymbol{C_t} + \nu \ln\left(\frac{M_{t-1}}{P_t}\right) + \lambda_t \left[W_t + (1 + i_{t-1}) B_{t-1} + M_{t-1} - P_t \boldsymbol{C_t} - \boldsymbol{B_t} - \boldsymbol{M_t}\right]\right\}$$

$$+ \beta^{t+1} \left\{\ln C_{t+1} + \nu \ln\left(\frac{\boldsymbol{M_t}}{P_{t+1}}\right) + \lambda_{t+1} \left[W_{t+1} + (1 + i_t) \boldsymbol{B_t} + \boldsymbol{M_t} - P_{t+1} C_{t+1} - B_{t+1} - M_{t+1}\right]\right\}$$

$$+ \ldots$$

Money In Utility (MIU)

Expanded Lagrangian

$$\mathcal{L} = \ldots + \beta^t \left\{ \ln \boldsymbol{C_t} + \nu \ln \left(\frac{M_{t-1}}{P_t} \right) + \lambda_t \left[W_t + (1 + i_{t-1}) B_{t-1} + M_{t-1} - P_t \boldsymbol{C_t} - \boldsymbol{B_t} - \boldsymbol{M_t} \right] \right\}$$

$$+ \beta^{t+1} \left\{ \ln C_{t+1} + \nu \ln \left(\frac{\boldsymbol{M_t}}{P_{t+1}} \right) + \lambda_{t+1} \left[W_{t+1} + (1 + i_t) \boldsymbol{B_t} + \boldsymbol{M_t} - P_{t+1} C_{t+1} - B_{t+1} - M_{t+1} \right] \right\}$$

$$+ \ldots$$

First Order Conditions

$$\boldsymbol{C_t} : \quad \beta^t \left\{ \frac{1}{C_t} - \lambda_t P_t \right\} = 0 \quad \rightarrow \quad \lambda_t = \frac{1}{P_t C_t}$$

$$\boldsymbol{B_t} : \quad \beta^t \{ -\lambda_t \} + \beta^{t+1} \{ \lambda_{t+1} (1 + i_t) \} = 0 \quad \rightarrow \quad \lambda_t = \beta \lambda_{t+1} (1 + i_t)$$

$$\boldsymbol{M_t} : \quad \beta^t \{ -\lambda_t \} + \beta^{t+1} \left\{ \nu \frac{1}{M_t / P_{t+1}} \cdot \frac{1}{P_{t+1}} + \lambda_{t+1} \right\} = 0 \quad \rightarrow \quad \lambda_t = \beta \left[\frac{\nu}{M_t} + \lambda_{t+1} \right]$$

Money In Utility (MIU)

Euler equation (bonds)

$$\frac{1}{P_t C_t} = \beta \left(1 + i_t\right) \frac{1}{P_{t+1} C_{t+1}} \quad \rightarrow \quad C_{t+1} = \beta \left(1 + i_t\right) \frac{P_t}{P_{t+1}} C_t$$

$$C_{t+1} = \beta \left[\left(1 + i_t\right) / \left(1 + \pi_{t+1}\right)\right] C_t \quad \rightarrow \quad C_{t+1} = \beta \left(1 + r_{t+1}\right) C_t$$

Money demand

$$\beta \lambda_{t+1} \left(1 + i_t\right) = \beta \left[\frac{\nu}{M_t} + \lambda_{t+1}\right] \quad \rightarrow \quad \lambda_{t+1} \cdot i_t = \frac{\nu}{M_t}$$

$$\frac{1}{P_{t+1} C_{t+1}} \cdot i_t = \frac{\nu}{M_t} \quad \rightarrow \quad M_t = \nu \cdot P_{t+1} C_{t+1} \cdot i_t^{-1}$$

In an equilibrium with no capital, investment and public spending, consumption is equal to income

$$M_{t-1} = \nu \cdot P_t Y_t \cdot i_{t-1}^{-1} \quad \rightarrow \quad \ln M = \ln P + \ln Y - \ln i + \ln \nu$$

In the MIU just as in CIA household welfare is maximized for $i = 0$

RBC model with MIU

$$
\begin{aligned}
\text{Euler equation :} \quad & C_t^{-\sigma} = \beta \mathrm{E}_t \left[C_{t+1}^{-\sigma} \left(1 + r_{t+1} \right) \right] && (1) \\
\text{Labor supply :} \quad & \psi N_t^{\varphi} = \left(W_t / P_t \right) C_t^{-\sigma} && (2) \\
\text{Production function :} \quad & Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} && (3) \\
\text{Real return on capital :} \quad & r_t = \alpha Y_t / K_t - \delta && (4) \\
\text{Labor demand :} \quad & W_t / P_t = \left(1 - \alpha \right) Y_t / N_t && (5) \\
\text{Investment :} \quad & I_t = K_{t+1} - \left(1 - \delta \right) K_t && (6) \\
\text{Output accounting :} \quad & Y_t = C_t + I_t + G_t && (7) \\
\text{Government spending :} \quad & G_t / Y_t = \rho_G \left(G_{t-1} / Y_{t-1} \right) + \left(1 - \rho_G \right) \left(G / Y \right) + \epsilon_{G,t} && (8) \\
\text{TFP AR(1) process :} \quad & \ln Z_t = \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} && (9) \\
\text{Money demand (CRRA) :} \quad & M_t = \nu \cdot \mathrm{E}_t \left[P_{t+1} C_{t+1} \right] \cdot i_t^{-1/\sigma} && (10) \\
\text{Fisher equation :} \quad & \mathrm{E}_t \left[1 + r_{t+1} \right] = \mathrm{E}_t \left[\left(1 + i_t \right) / \left(1 + \pi_{t+1} \right) \right] && (11) \\
\text{Money supply :} \quad & \ln M_t = \rho_M \ln M_{t-1} + \left(1 - \rho_M \right) \ln M + \epsilon_{M,t} && (12) \\
\text{Money demand :} \quad & \ln \nu_t = \rho_\nu \ln \nu_{t-1} + \left(1 - \rho_\nu \right) \ln \nu + \epsilon_{\nu,t} && (13) \\
\text{Inflation rate :} \quad & \pi_t = P_t / P_{t-1} - 1 && (14)
\end{aligned}
$$

RBC model with MIU

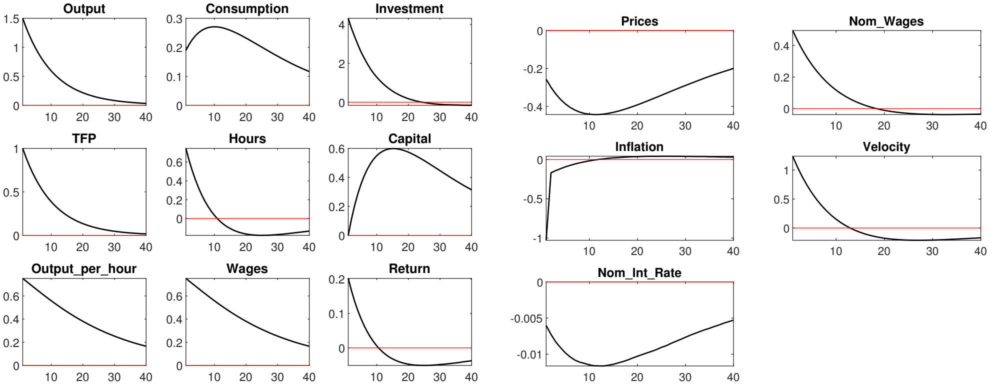
The model exhibits **classical dichotomy**: nominal variables don't influence real ones

Equations (1)-(9) can be solved separately (RBC model)

The nominal block (10)-(14) describes dynamics of prices etc.

RBC model with MIU: TFP shock

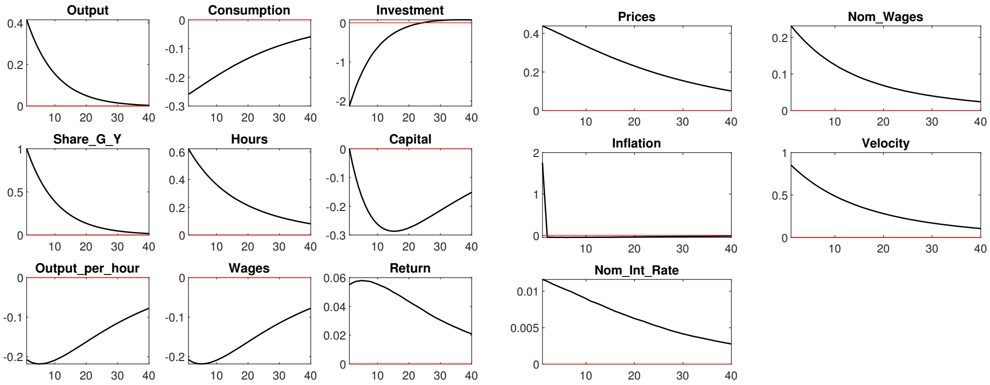
TFP shock (constant money supply)



Temporarily higher productivity leads to temporary increase of output and fall of prices

RBC model with MIU: G shock

Government spending shock (constant money supply)



Temporarily higher G leads to temporary increase of output and prices

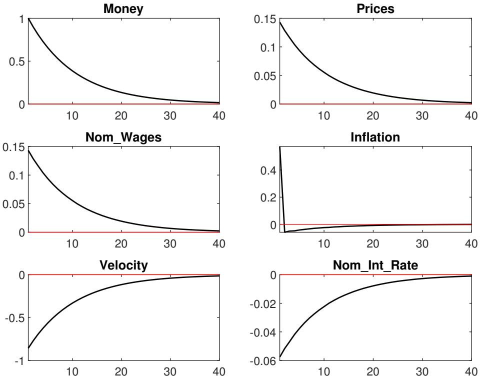
RBC model with MIU: ν shock

Increase in money demand via ν (constant money supply)

Temporarily increased demand for money leads to temporary fall of prices

RBC model with MIU: M shocks

Transitory increase in money supply M



Price level increases by less than M and goes back to the steady state with M

RBC model with MIU: M shocks

Permanent increase in money supply M

Immediate, permanent increase in price level exactly proportional to increase in M

Quantity theory of money

Equation of exchange

From money demand models we get the equation of the form

$$M = PY \cdot f(i) \quad \to \quad M/f(i) = PY$$

Define money velocity

$$V = 1/f(i) \quad \to \quad \frac{\partial V}{\partial i} > 0$$

Equation of exchange (here M is money supply)

$$MV = PY$$

In logarithms

$$m + v = p + y$$

In rates of change

$$\Delta m + \Delta v = \Delta p + \Delta y$$

Quantity theory of money

If V (relatively) stable, then $\Delta v \approx 0$ and

$$\pi \equiv \Delta p \approx \Delta m - \Delta y$$

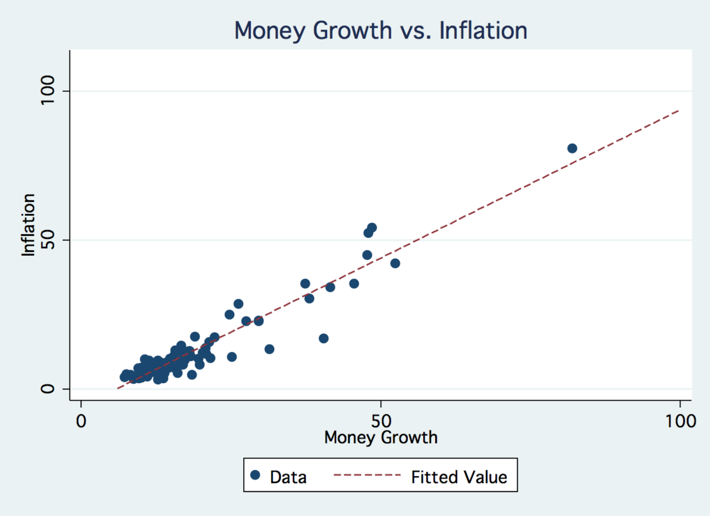
For countries where $\Delta m \gg 0$, we will have $\pi \approx \Delta m$

For countries with low money growth rate,
we expect $\pi \neq \Delta m$, as Δy and Δv become important

McCandless and Weber (1995):

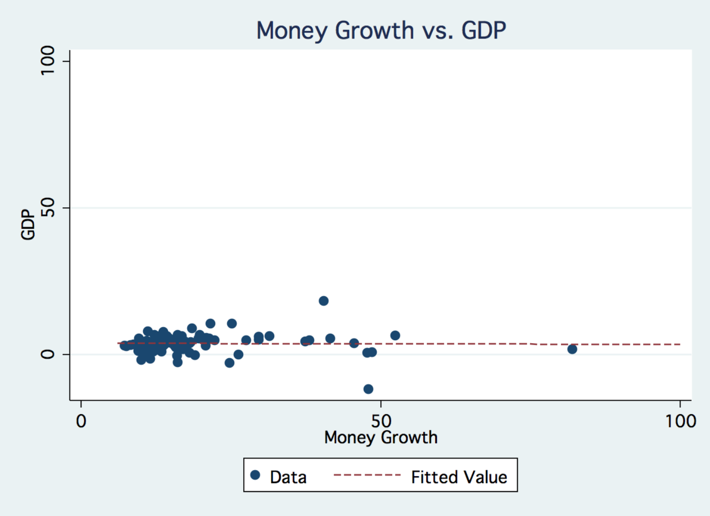
- High, almost unity correlation between Δm and π.
  Holds for all money aggregates
- No correlation between Δm and Δy. Exception: OECD
- No correlation between π and Δy

Money, inflation and real GDP growth



McCandless and Weber (1995), graph by Corrections: Page One

Money, inflation and real GDP growth



McCandless and Weber (1995), graph by Corrections: Page One

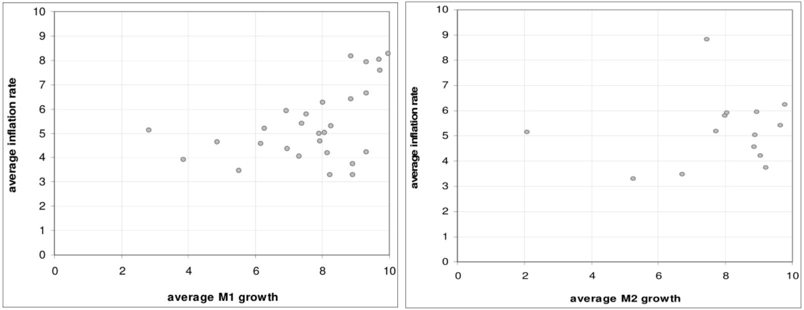
Money, inflation and real GDP growth

Strong link between inflation and money growth is almost wholly due to the presence of high (or hyper-) inflation countries in the sample.

Relationship between inflation and money growth for low inflation countries is weak.

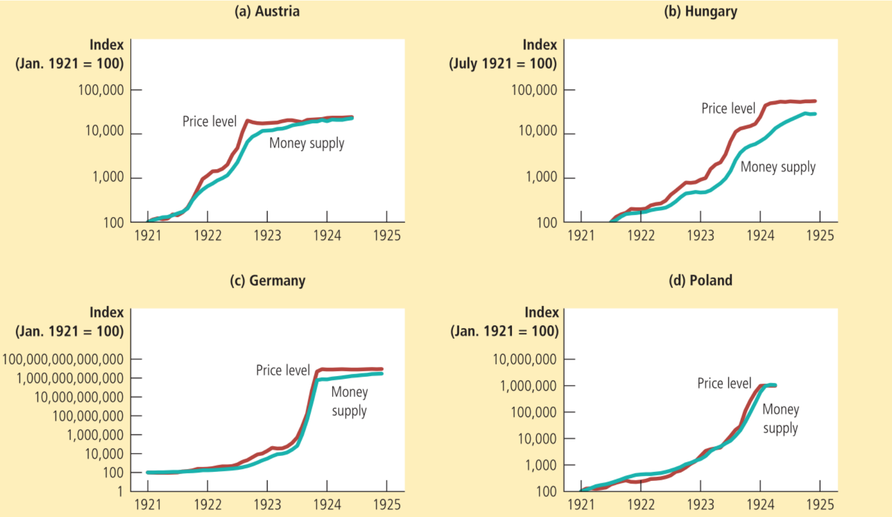
Higher growth rates of money do not lead to higher growth rates of output.

Figure 2: Inflation and money supply growth lower than 10%.



De Grauwe and Polan (2001)

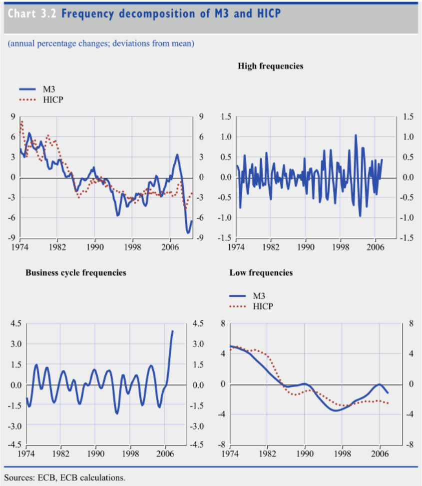
Four big hyperinflations in 1920s



Sargent (1982), graph from Mankiw (2014)

Money and inflation in euro area

The link between money growth and inflation is most robust in the long run.



ECB (2011)

Non-neutrality of money

Money neutrality: long vs short run

Empirical studies tend to support long run money neutrality

- No effect of money growth on output growth
- Money growth correlates highly with inflation
  (except for low inflation countries)

But what about the short run?

- **Exogenous** monetary policy tightening
  $\rightarrow$ fall in output (temporary) and price level (permanent)
- Empirical studies often encounter the price puzzle: price level / inflation temporarily rising after a monetary policy action

Identifying effects of monetary policy is not easy

Standard econometric tests for whether money causes output will be meaningless
if monetary policy is chosen optimally to smooth fluctuations in output.

This paper shows that U.S. monetary policy does not cause U.S. output, but does cause Hong Kong output.

Table 4
Variance decomposition

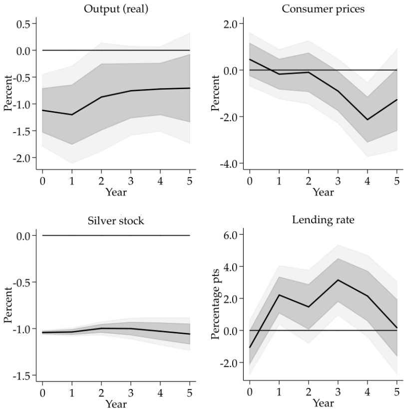
Error variance of	Contribution from	Horizon (in quarters)			
		$h=1$	$h=6$	$h=12$	$h=24$
y^{hk}	y^{hk}	100.0	62.7	38.1	38.7
	y^{us}	0.0	12.4	35.5	43.6
	m^{us}	0.0	24.8	26.4	17.6
y^{us}	y^{hk}	2.8	1.5	3.8	3.5
	y^{us}	97.2	94.2	91.9	94.2
	m^{us}	0.0	4.5	4.3	2.3
m^{us}	y^{hk}	9.7	14.8	14.6	21.0
	y^{us}	1.8	8.7	17.4	58.1
	m^{us}	88.5	76.5	67.9	20.8

Rodriguez and Rowe (2007)

Maritime disasters in the Spanish Empire (1531-1810)

Permanent 1% reduction in the money supply led to a 1% drop in real output that persists for around four years. The price level fell permanently, but only with a lag. Tighter credit markets temporarily increased lending rates by 200 basis points.

Figure 2: Impulse responses for -1% money shock

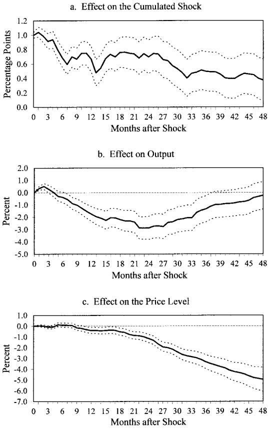


Brzezinski et al. (2019)

“Exogenous” monetary policy shocks in US (1969-1996)

This paper develops a measure of U.S. monetary policy shocks for the period 1969-1996 that is relatively free of endogenous and anticipatory movements.

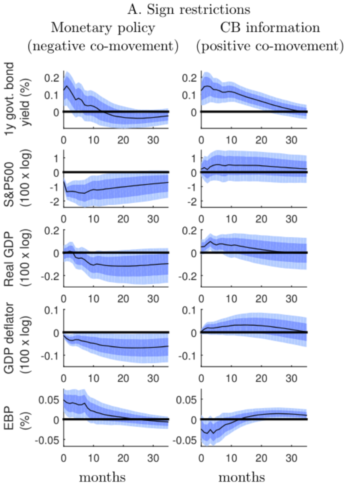
Estimates indicate that policy has large, relatively rapid, and statistically significant effects on both output and inflation.



Romer and Romer (2004)

High frequency identification: US (1990-2016)

We separate monetary policy shocks from central bank information shocks in a structural VAR and track the dynamic response of key macroeconomic variables.



Jarocinski and Karadi (2018)

Price expectations

Nominal wage contracts

Households and firms negotiate over the nominal wage in advance

The negotiated nominal wage is binding within a period

Actual price level might differ from expected

Employment is determined by the resulting **real wage** and firms' demand for labor

Gives rise to positively sloped short-run Phillips curve

In the long run, Phillips curve is vertical

Nominal wage contracts: negotiations

Negotiated wage W^c equates labor demand and supply under price expectations P^e

Labor demand

$$W_t^c / P_t^e = (1-\alpha)\, Y_t / N_t = (1-\alpha)\, Z_t K_t^{\alpha} N_t^{-\alpha}$$

Labor supply

$$\psi N_t^{\varphi} = (W_t^c / P_t^e)\, C_t^{-\sigma}$$

For simplicty assume constant $\bar{K}$ and $\delta = 0$, therefore $Y = C$, as well as $\sigma = 1$, $Z = 1$

$$\psi N_t^{\varphi} = \frac{(1-\alpha)\, Y_t / N_t}{Y_t} \quad \rightarrow \quad N_t^e = \left(\frac{1-\alpha}{\psi}\right)^{1/(1+\varphi)} \quad \rightarrow \quad Y_t^e = \bar{K}^{\alpha} \left(N_t^e\right)^{1-\alpha}$$

Negotiated nominal wage

$$W_t^c = P_t^e\, (1-\alpha)\, \bar{K}_t^{\alpha} \left(N_t^e\right)^{-\alpha}$$

Nominal wage contracts: equilibrium within period

Within a period firms observe prices P and adjust employment to match marginal product of labor with resulting real wage

$$W_t^c/P_t = (1-\alpha)\,\bar{K}^{\alpha} N_t^{-\alpha} \quad \rightarrow \quad N_t = \left(\frac{W_t^c}{P_t}\frac{1}{(1-\alpha)\,\bar{K}^{\alpha}}\right)^{-1/\alpha}$$

Plug in the nominal wage

$$N_t = \left(\frac{P_t^e\,(1-\alpha)\,\bar{K}_t^{\alpha}\,(N_t^e)^{-\alpha}}{P_t\,(1-\alpha)\,\bar{K}_t^{\alpha}}\right)^{-1/\alpha} = \left(\frac{P_t}{P_t^e}\right)^{1/\alpha} N_t^e$$

Employment depends positively on price level (negatively on real wage)

Output in equilibrium

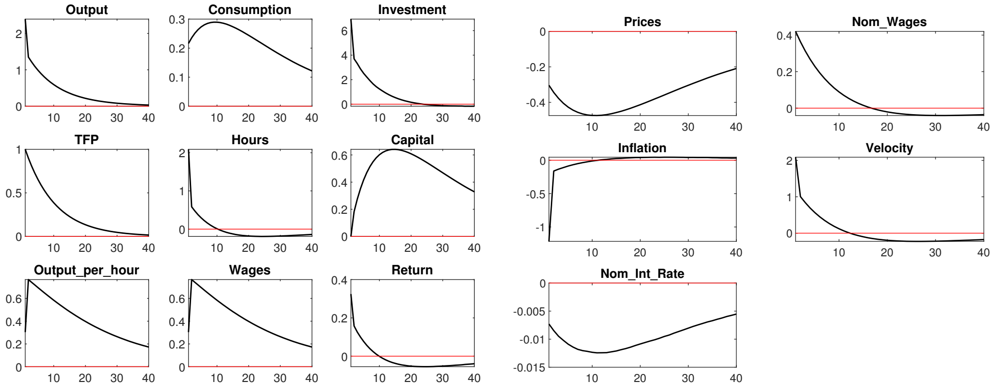
$$Y_t = \bar{K}^{\alpha} N_t^{1-\alpha} = \bar{K}^{\alpha}\,(N_t^e)^{1-\alpha} \cdot (N_t/N_t^e)^{1-\alpha} = Y_t^e \cdot (P_t/P_t^e)^{(1-\alpha)/\alpha}$$

After taking logarithms we obtain Friedman's aggregate supply curve

$$y = y^e + \frac{1-\alpha}{\alpha}\,(p-p^e) \equiv y^e + a\,(p-p^e)$$

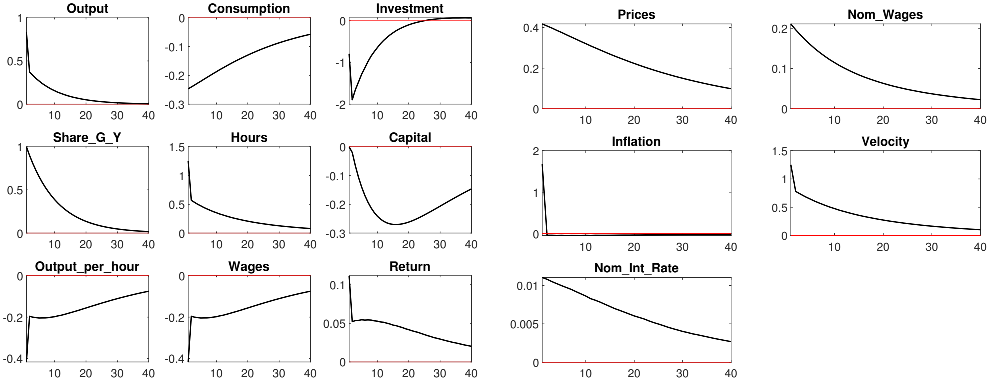
RBC model with MIU and one-period wage rigidity: TFP shock

TFP shock (constant money supply)



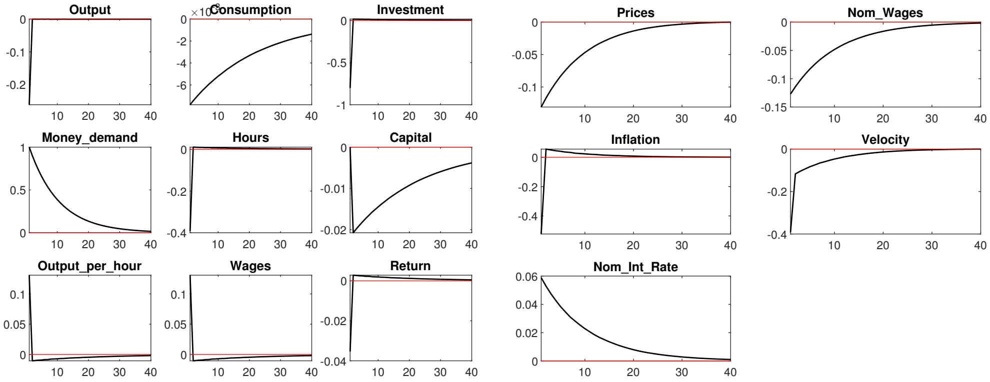
RBC model with MIU and one-period wage rigidity: TFP shock

Government expenditure shock (constant money supply)



RBC model with MIU and one-period wage rigidity: ν shock

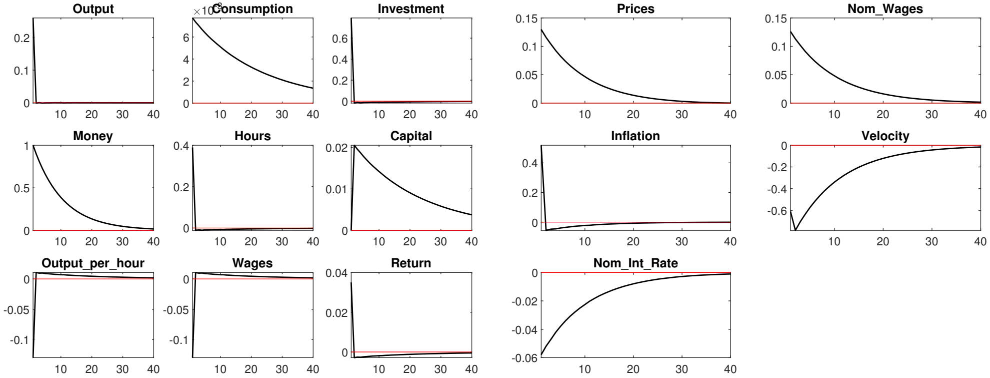
Increase in money demand via ν (constant money supply)



Results in a shortlived recession

RBC model with MIU and one-period wage rigidity: M shocks

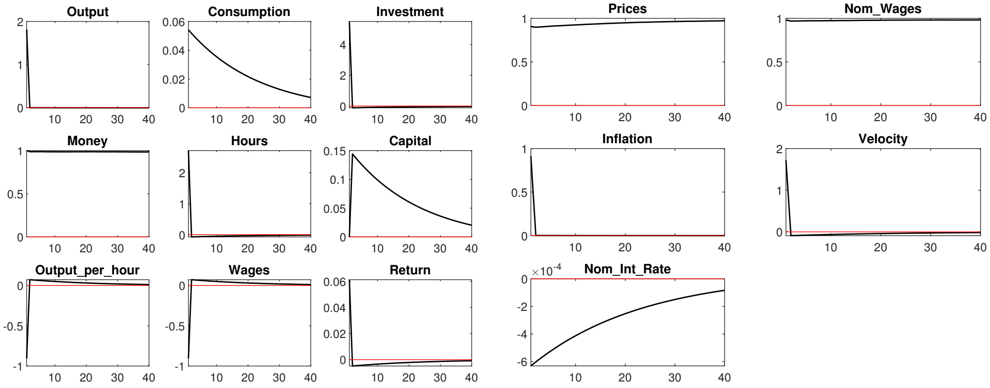
Transitory increase in money supply M



This time we get a shortlived effect on real variables

RBC model with MIU and one-period wage rigidity: M shocks

Permanent increase in money supply M



Prices do not adjust fully on impact, in the long run increase 1:1 with M

Imperfect information: Lucas "islands" model

Based on Lucas (1973)

Differentiated markets, each with local demand conditions

Firms have no pricing power, choose how much to produce

Timing assumptions:

1. Firm learns of local price $P_i = P \cdot Z_i$ (Z_i is local demand)
2. Firm commits to produce Y_i which requires inputs M_i
3. Price of inputs is proportional to the aggregate price level P, unknown in advance

Firm does not know whether high P_i is due to higher relative demand Z_i, or due to rising price level P

- If due to higher Z_i, optimal to expand production
- If only due to higher P, optimal to produce as usual

Local supply function

Profit maximization problem (if P was known)

$$\max_{Y_i,\, M_i} \quad P_i Y_i - P M_i \quad \rightarrow \quad \max_{M_i} \quad P_i M_i^{1-\alpha} - P M_i$$
$$\text{subject to} \quad Y_i = M_i^{1-\alpha}$$

First order condition

$$(1-\alpha)\, P_i M_i^{-\alpha} - P = 0$$

Optimal inputs and production

$$M_i = \left[\frac{(1-\alpha)\, P_i}{P}\right]^{1/\alpha} \quad \text{and} \quad Y_i = \left[\frac{(1-\alpha)\, P_i}{P}\right]^{(1-\alpha)/\alpha}$$

In logarithms

$$y_i = \frac{1-\alpha}{\alpha} \ln(1-\alpha) + \frac{1-\alpha}{\alpha}(p_i - p) = y^* + \gamma\,(p_i - p)$$

Signal extraction problem

Firm observes only local price level P_i

$$P_i = P \cdot Z_i \quad \rightarrow \quad p_i = p + z_i$$

Firms possess a prior distribution of the price level

$$p \sim \mathcal{N}(p^e,\, \sigma_p^2)$$

Local demand conditions z_i are independent of p

$$z_i \sim \mathcal{N}(0,\, \sigma_z^2)$$

Optimally extract information on p given the signal p_i

$$\mathrm{E}_i\left[p|p_i\right] = \kappa p^e + (1 - \kappa)\, p_i \quad \text{where} \quad \kappa = \sigma_z^2/(\sigma_p^2 + \sigma_z^2)$$

If $\sigma_z^2 > \sigma_p^2$, expected posterior p closer to prior p^e

If $\sigma_z^2 < \sigma_p^2$, expected posterior p closer to local price p_i

Lucas aggregate supply function

Local supply function

$$y_i = y^* + \gamma \left(p_i - \mathrm{E}_i \left[p|p_i\right]\right) = y^* + \kappa\gamma \left(p_i - p^e\right)$$

Aggregate over markets

$$y = \int_0^1 y_i \, \mathrm{d}i \quad \text{and} \quad p = \int_0^1 p_i \, \mathrm{d}i$$

Lucas aggregate supply function

$$y = y^* + \kappa\gamma \left(p - p^e\right) \equiv y^* + a \left(p - p^e\right)$$

Supply functions: Friedman vs Lucas

Friedman's expectations augmented Phillips curve is based on errors in expectations in the labor market and contractual rigidities

Lucas supply function is based on errors in the expectations of relative prices in commodity markets

Similar functional form, giving rise to positively sloped aggregate supply function

$$y = y^* + a\,(p - p^e)$$

where the level of production under fulfilled price expectations ($p = p^e$) is called the **natural output** and (its logarithm) is denoted with y^*

Aggregate demand and equilibrium

We can obtain some key results in a reduced form model with only the aggregate supply curve and the aggregate demand curve based on the equation of exchange

$$MV = PY$$

Take logs and normalize $V = 1$

$$m = p + y \quad \rightarrow \quad y = m - p$$

Equilibrium

$$\begin{aligned} \text{[AS]} \quad & y = y^* + a\left(p - p^e\right) \\ \text{[AD]} \quad & y = m - p \end{aligned}$$

Rational expectations equilibrium

Prior on price level p^e is given by the expected value operator

$$p^e \equiv \mathrm{E}\left[p\right]$$

Equilibrium in expectations

$$\begin{aligned} \text{[EAS]} \quad & \mathrm{E}\left[y\right] = \mathrm{E}\left[y^*\right] + a\left(\mathrm{E}\left[p\right] - \mathrm{E}\left[p\right]\right) = y^* \\ \text{[EAD]} \quad & \mathrm{E}\left[y\right] = \mathrm{E}\left[m\right] - \mathrm{E}\left[p\right] \\ & \mathrm{E}\left[p\right] = \mathrm{E}\left[m\right] - y^* \end{aligned}$$

Actual equilibrium

$$y = y^* + a\left(m - y - \left(\mathrm{E}\left[m\right] - y^*\right)\right)$$

Effects of monetary policy

After a few steps of algebra

$$y = y^* + \frac{a}{1+a}\left(m - \mathrm{E}\left[m\right]\right)$$
$$p = \mathrm{E}\left[m\right] - y^* + \frac{1}{1+a}\left(m - \mathrm{E}\left[m\right]\right)$$

Expected money supply level $\mathrm{E}\left[m\right]$ affects only prices

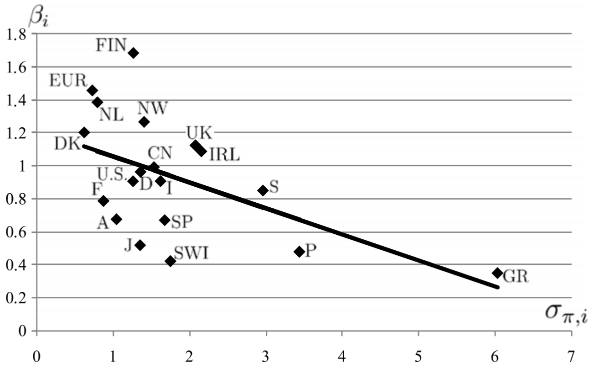
Only the surprise component $(m - \mathrm{E}\left[m\right])$ has real effects

Random shifts in aggregate demand cause changes in output **only if** firms believe that some part of the resulting increase in their prices is a relative price increase

Monetary surprise has bigger effects if there were fewer surprises in the past!

Evidence on the Lucas Supply Function

Evidence based on actual inflation surprises $\pi_t - \mathrm{E}_{t-4}\pi_t$. Inflation surprises are positively correlated ($\beta_i > 0$) with the output gap. This relationship is negatively related to inflation variability $\sigma_{\pi,i}$



Fendel and Rülke (2012)

Inflation bias

Central bank's objective function

Desire to stabilize both output / employment and inflation

$$\min \quad L = \lambda \left(y_t - y^*\right)^2 + \left(\pi_t - \pi^T\right)^2$$

Ad-hoc loss function postulated by Tinbergen (1952) and Theil (1958)

Given microfoundations by Rotemberg and Woodford (1998)
as a quadratic approximation to the welfare of the representative agent

The value of parameter λ can be derived within a theoretical model, but it cannot be measured in reality: hence the split of policymakers into “hawks” or “doves”

Inflation bias

Based on Barro and Gordon (1983)

Natural level of output $\equiv$ level of actual output whenever expected inflation equals actual inflation

For given expectations, policymakers have an incentive to generate positive unexpected inflation to increase the level of output and employment

But expectations are rational!

People know this and expect higher inflation

In equilibrium output is unaffected and inflation is higher

Inflation bias: modified objective function

Suppose policymakers want output to be higher than natural

$$\min \quad L = \lambda \left(y_t - (y^* + k)\right)^2 + \left(\pi_t - \pi^T\right)^2$$

- inflation around the target π^T
- output k percent higher than natural (unemployment below natural)

US Senators write to Fed Chairman Powell on the eve of elections…

United States Senate
COMMITTEE ON BANKING, HOUSING, AND URBAN AFFAIRS
WASHINGTON, DC 20510–6075

October 25, 2022

The Honorable Jerome Powell
Chair
Board of Governors of the Federal Reserve System
20th Street and Constitution Avenue NW
Washington, DC 20551

Dear Chair Powell:

As you know, the Federal Reserve is charged with the dual mandate of promoting maximum employment, stable prices, and moderate long-term interest rates in the U.S. economy. It is your job to combat inflation, but at the same time, you must not lose sight of your responsibility to ensure that we have full employment.

For the first time in decades, we have seen historic job growth, and workers have begun to see wage gains, gains that your prior actions to stabilize the economy helped achieve. Yet, many workers and their families are struggling under the weight of inflation. As you explained in your September 21, 2022, FOMC remarks, "If your family is one where you spend most of your paycheck, every paycheck cycle, on gas, food, transportation, clothing, basics of life, and prices go up the way they've been going up, you're in trouble right away." High inflation affecting household needs such as food, healthcare, and transportation strains middle- and lower-income budgets.[1] The Federal Reserve's tools work to lower inflation by reducing demand for economic activities sensitive to interest rates. However, a family's "pocketbook" needs have little to do with interest rates, and potential job losses brought about by monetary over-tightening will only worsen these matters for the working class.

Source

Aggregate Supply

Aggregate output is given by an aggregate supply function under some expectations of the inflation rate π_t^e

$$y_t = y^* + a\left(\pi_t - \pi_t^e\right) + e_t$$

where e_t is an unforecastable supply shock, $\mathrm{E}_{t-1}e_t = 0$

Sequence of events:

1. Formation of expectations
2. Private sector sets nominal contracts
3. The supply shock is realized
4. The policy instrument is set
5. The rate of inflation and output in equilibrium is realized

Central bank's choice

Assume $\pi^T = 0$ for simplicity and that the central bank can directly choose inflation

Optimization problem taking expectations as given

$$\begin{aligned} \min \quad & L = \lambda \left(y_t - (y^* + k)\right)^2 + \pi_t^2 \\ \text{subject to} \quad & y_t = y^* + a\left(\pi_t - \pi_t^e\right) + e_t \end{aligned}$$

Plug constraint into objective

$$\min \quad L = \lambda \left(a\left(\pi_t - \pi_t^e\right) + e_t - k\right)^2 + \pi_t^2$$

First order condition

$$\frac{\partial L}{\partial \pi_t} = \lambda \cdot 2\left(a\left(\pi_t - \pi_t^e\right) + e_t - k\right) \cdot a + 2\pi_t = 0$$

Inflation in rational expectations equilibrium

Rearranging the FOC gives the desired inflation rate

$$\pi_t = \frac{a^2\lambda\pi_t^e + a\lambda\left(k - e_t\right)}{1 + a^2\lambda}$$

Assuming rational expectations

$$\mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-1}\left[\frac{a^2\lambda\mathrm{E}_{t-1}\pi_t + a\lambda\left(k - e_t\right)}{1 + a^2\lambda}\right] \quad \rightarrow \quad (1 + a^2\lambda)\mathrm{E}_{t-1}\pi_t = a^2\lambda\mathrm{E}_{t-1}\pi_t + a\lambda k$$

Agents expect inflation exceeding target

$$\pi_t^e = \mathrm{E}_{t-1}\pi_t = a\lambda k > 0$$

Resulting in actual inflation (given supply shock e_t)

$$\pi_t = a\lambda k - \frac{a\lambda}{1 + a^2\lambda}e_t$$

Failure to raise output above natural

Because private agents understand the incentives facing the central bank, average inflation is fully anticipated

Equilibrium produces average rate of inflation above target (inflation bias)

This has no systematic effect on output

$$y_t = y^* + \frac{1}{1 + a^2\lambda} e_t$$

Counteracting inflation bias

Appointment of a "hawkish" central bank governor

"Hawks" place an additional weight ($\delta > 0$) on inflation stabilization compared with other members of the society

$$L = \lambda \left(y_t - (y^* + k)\right)^2 + (1 + \delta) \left(\pi_t - \pi^T\right)^2$$

The rate of inflation under discretion will equal

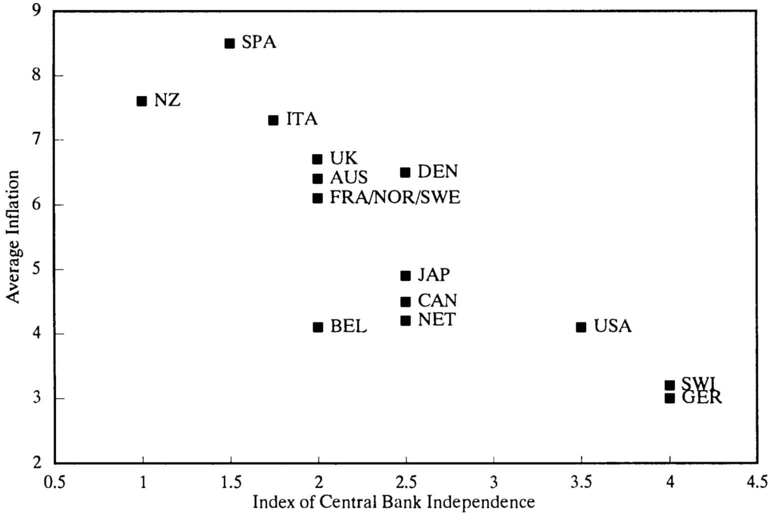
$$\pi_t = \pi^T + \frac{a\lambda}{1+\delta} k - \frac{a\lambda}{1 + \delta + a^2\lambda} e_t$$

Setting output target to potential output $(k = 0)$

The rate of inflation under discretion will equal

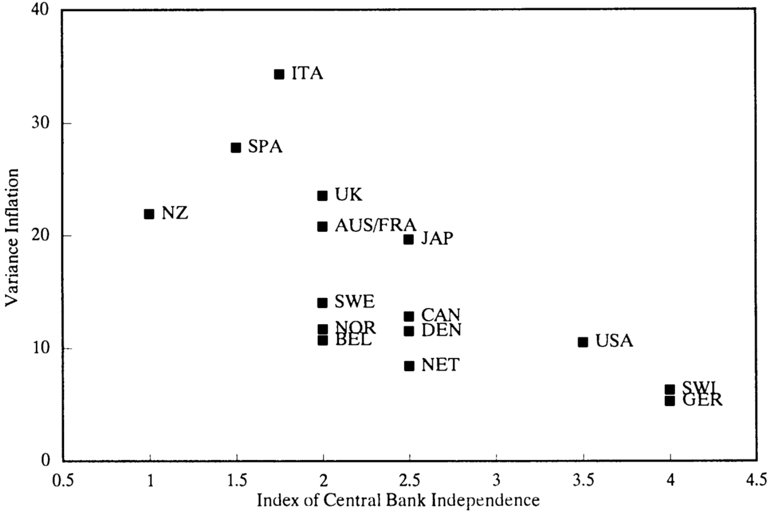
$$\pi_t = \pi^T - \frac{a\lambda}{1 + a^2\lambda} e_t$$

Central bank independence and average inflation (1955-1988)



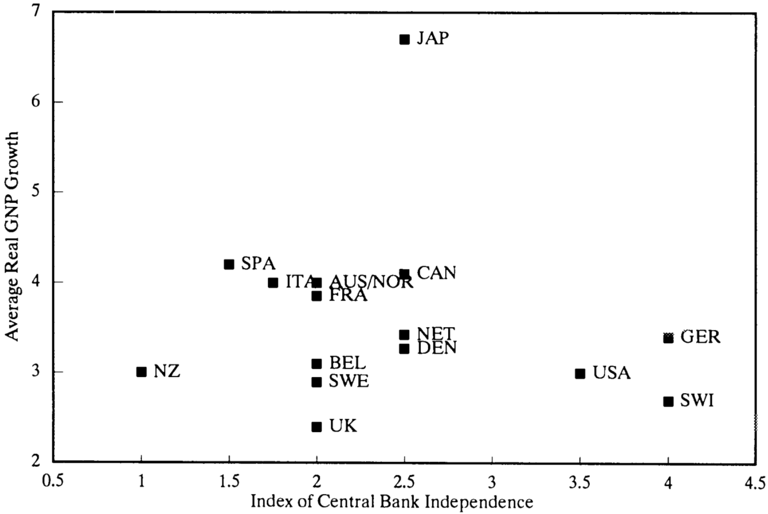
Alesina and Summers (1993)

Central bank independence and inflation volatility (1955-1988)



Alesina and Summers (1993)

Central bank independence and average real GNP growth (1955-1988)



Alesina and Summers (1993)

Counteracting inflation bias

Rules rather than discretion

Kydland and Prescott (1977): discretionary policy fails to maximize social welfare, relying on policy rules improves performance

Inflation targeting strategy

The central bank is mandated, and commits to, a unique numerical target in the form of a level or a range for annual inflation

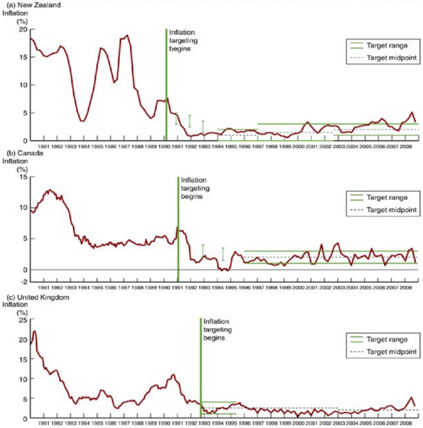
Formal adoption of inflation targeting

Country	Year	Country	Year	Country	Year
New Zealand	1989	South Africa	2000	Ghana	2007
Canada	1991	Thailand	2000	Serbia	2009
United Kingdom	1992	Mexico	2001	Georgia	2009
Australia	1993	Iceland	2001	United States	2012
Sweden	1995	Norway	2001	Japan	2013
Israel	1997	Hungary	2001	Russia	2014
Czechia	1997	Peru	2002	Kazakhstan	2015
South Korea	1998	Philippines	2002	Armenia	2016
Poland	1998	Guatemala	2005	India	2016
Brazil	1999	Indonesia	2005	Argentina	2016
Chile	1999	Romania	2005		
Colombia	1999	Turkey	2006	*ECB*	*1999*

Hammond (2012) & Wikipedia

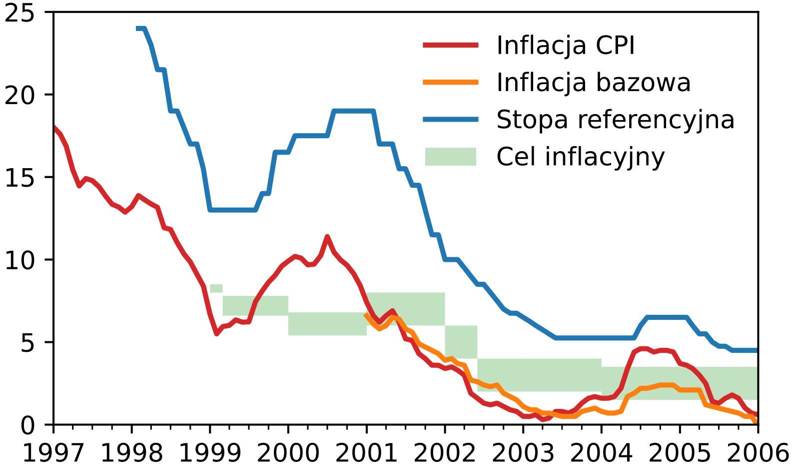
Success of inflation targeting for early adopters

Inflation Rates and Inflation Targets for New Zealand, Canada, and the United Kingdom, 1980–2008

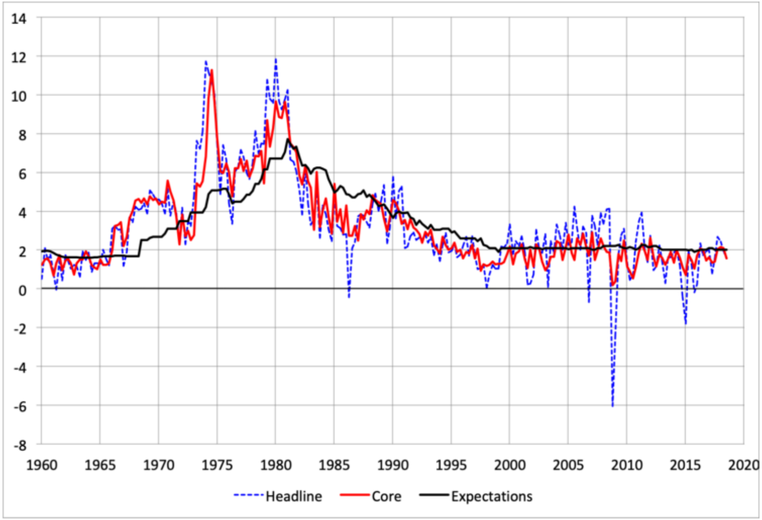


Source

Success of inflation targeting in Poland



Inflation and inflation expectations in the US (1960-2018)



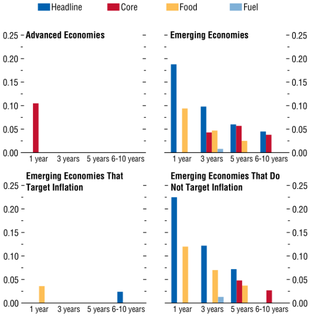
Source

Anchoring of inflation expectations in IT countries

Figure 3.12. Changes in Expected Inflation in Response to Changes in Actual Inflation[1]

(Expected inflation 1, 3, 5, and 6–10 years ahead; percentage point responses to a 1 percentage point change in actual inflation)

Inflation expectations appear significantly better anchored in advanced economies than in emerging economies, especially those with a high share of food in the CPI. In emerging economies, inflation targeting seems to have recently been more effective than alternative monetary policy frameworks in anchoring expectations.



Sources: Consensus Forecasts; and IMF staff calculations.
[1]Based on statistically significant coefficients from panel regressions with fixed effects, using semiannual data since 2003. The measure of core inflation is net of food and fuel inflation.

IMF WEO (2008)

Are expectations becoming unanchored?

Figure 3: Risk-neutral distributions of US inflation, 10-year horizon

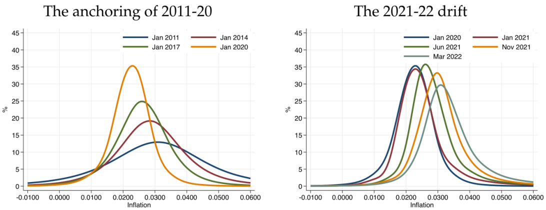
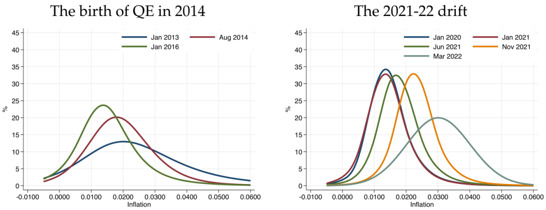


Figure 4: Risk-neutral distributions of EZ inflation, 10-year horizon



Hilscher et al. (2022)