University of Warsaw
Faculty of Economic Sciences

# Consumption

## Advanced Macroeconomics: Lecture 3

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# Intertemporal consumption choice

# Utility Maximization Problem

The household maximizes utility from consumption in two periods

$$\max_{c_1, c_2, a_1} \quad U = \boldsymbol{\ln c_1 + \beta \ln c_2}$$

$$\text{subject to} \quad c_1 + a_1 = y_1$$

$$c_2 = y_2 + (1 + r)\, a_1$$

Logarithmic utility for easy derivations, discount factor $\beta \in [0, 1]$

Exogenous variables: incomes $y_1$, $y_2$ and the real interest rate $r$

Choice variables: consumption $c_1$, $c_2$ and assets at the end of period 1 $a_1$

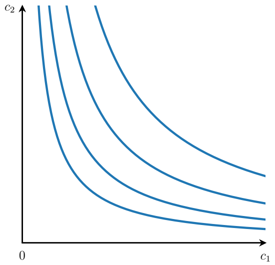
Lifetime budget constraint:

$$a_1 = \frac{c_2 - y_2}{1 + r} \quad \rightarrow \quad c_1 + \frac{c_2 - y_2}{1 + r} = y_1 \quad \rightarrow \quad \boldsymbol{c_1 + \frac{c_2}{1 + r} = y_1 + \frac{y_2}{1 + r}}$$

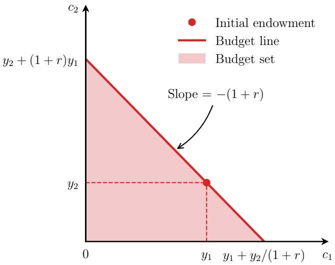
# Utility Maximization Problem: graphical interpretation

We are looking for a specific **indifference curve** that is just tangent to the **budget line**. The point of tangency is the **optimal consumption** choice:

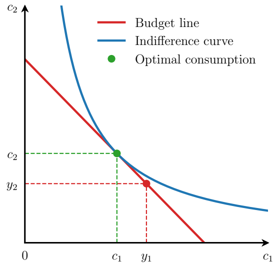
Indifference curve map




Lifetime budget constraint




Graphical solution



# Method of Lagrange multipliers

Set up the Lagrangian

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left[ y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right]$$

Derive the first order conditions (FOCs)

$$c_1 \quad : \quad \frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} + \lambda \left[ -1 \right] = 0 \qquad \rightarrow \qquad \lambda = \frac{1}{c_1}$$

$$c_2 \quad : \quad \frac{\partial \mathcal{L}}{\partial c_2} = \beta \cdot \frac{1}{c_2} + \lambda \left[ -\frac{1}{1+r} \right] = 0 \qquad \rightarrow \qquad \lambda = \beta \left( 1+r \right) \frac{1}{c_2}$$

Obtain the optimality condition (**Euler equation**)

$$\frac{1}{c_1} = \beta \left( 1+r \right) \frac{1}{c_2} \qquad \rightarrow \qquad \boldsymbol{c_2 = \beta \left( 1+r \right) c_1}$$

# Utility Maximization Problem: solution

Plug the Euler equation into the lifetime budget constraint

$$c_2 = \beta\left(1+r\right)c_1$$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$c_1 + \beta c_1 = y_1 + \frac{y_2}{1+r}$$

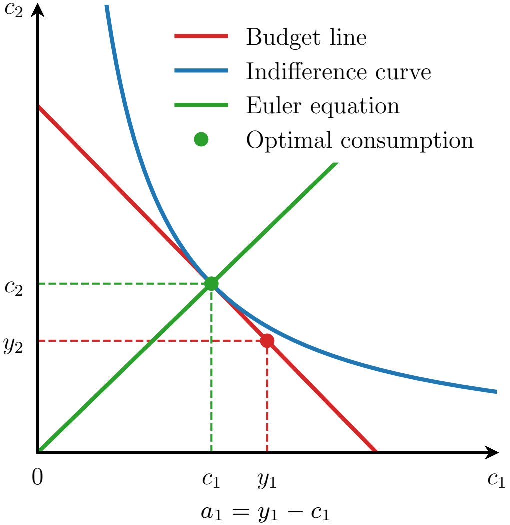
Optimal levels of consumption and assets

$$c_1 = \frac{1}{1+\beta}\left[y_1 + \frac{y_2}{1+r}\right]$$

$$c_2 = \frac{\beta}{1+\beta}\left[\left(1+r\right)y_1 + y_2\right]$$

$$a_1 = y_1 - c_1 = \frac{1}{1+\beta}\left[\beta y_1 - \frac{y_2}{1+r}\right]$$

# Utility Maximization Problem solution: graphical interpretation



# Comparative Statics

Consumer is more patient (higher $\beta$):

$$\frac{\partial c_1}{\partial \beta} < 0, \quad \frac{\partial c_2}{\partial \beta} > 0, \quad \frac{\partial a_1}{\partial \beta} > 0$$

Higher income in the first period

$$\frac{\partial c_1}{\partial y_1} > 0, \quad \frac{\partial c_2}{\partial y_1} > 0, \quad \frac{\partial a_1}{\partial y_1} > 0$$

Higher (expected) income in the second period

$$\frac{\partial c_1}{\partial y_2} > 0, \quad \frac{\partial c_2}{\partial y_2} > 0, \quad \frac{\partial a_1}{\partial y_2} < 0$$

# Comparative Statics: changes in real interest rate $r$

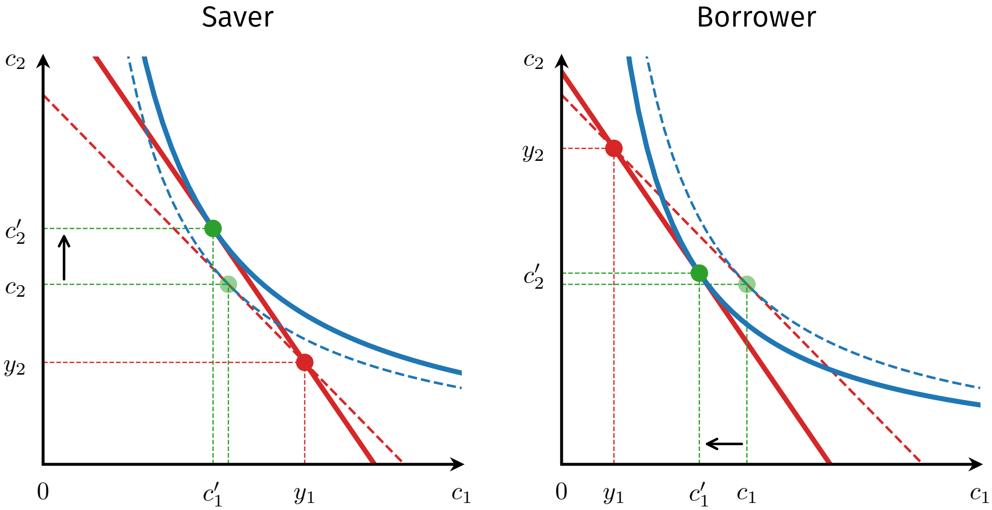
Substitution effect: as consumption in the future gets „cheaper", induces the agent to consume more in the second period and less in the first period

Income effect depends on the desired assets prior to interest rate change:

- Saver ($a_1 > 0$): an increase of the budget set induces increases in consumption in both periods
- Borrower ($a_1 < 0$): a decrease of the budget set induces decreases in consumption in both periods

| Effects of an | Saver | | | Borrower | | |
|---|---|---|---|---|---|---|
| increase in $r$ | $c_1$ | $c_2$ | $a_1$ | $c_1$ | $c_2$ | $a_1$ |
| Substitution | $-$ | $+$ | $+$ | $-$ | $+$ | $+$ |
| Income | $+$ | $+$ | $-$ | $-$ | $-$ | $+$ |
| Net | ? | $+$ | ? | $-$ | ? | $+$ |

# Comparative Statics: changes in real interest rate $r$



# Effects of changes in interest rate in the data

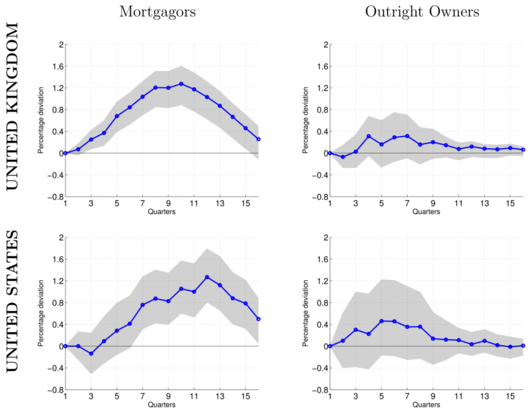



Figure 5: Dynamic effects of a 25 basis point unanticipated interest rate cut on the expenditure of durable goods by housing tenure group. Grey areas are bootstrapped 90% confidence bands. Top row: UK (FES/LCFS data). Bottom row: US (CEX data).

Cloyne, Ferreira, Surico (2016) *Monetary policy when households have debt*

# Additional constraints

# Borrowing constraint

Now the agent cannot have negative assets

$$\max_{c_1, c_2, a_1} \quad U = \ln c_1 + \beta \ln c_2$$

$$\text{subject to} \quad c_1 + a_1 = y_1$$

$$c_2 = y_2 + (1 + r)\, a_1$$

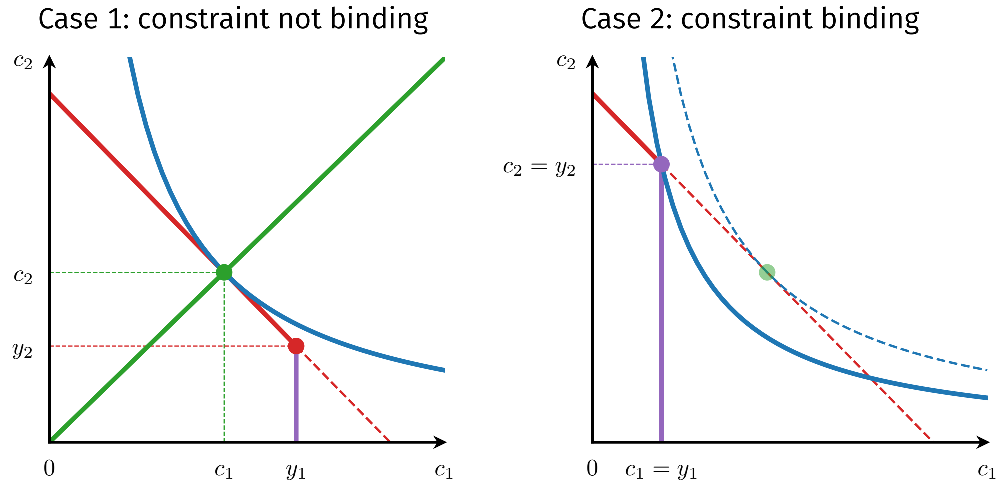
$$\boldsymbol{a_1 \geq 0}$$

Either the agent would choose $a_1 > 0$ and the constraint is not binding

Or they would like to choose $a_1 < 0$ and the constraint is binding:

$$a_1 = 0, \quad c_1 = y_1, \quad c_2 = y_2$$

# Borrowing constraint: graphical interpretation




In Case 2 the agent changes current consumption following any change in income

# Borrowing constraint: formal approach

Lagrangian (two separate budget constraints and the borrowing constraint)

$$\mathcal{L} = \ln \boldsymbol{c_1} + \beta \ln \boldsymbol{c_2} + \lambda_1 \left[ y_1 - \boldsymbol{c_1} - \boldsymbol{a_1} \right] + \lambda_2 \left[ y_2 + (1+r)\, \boldsymbol{a_1} - \boldsymbol{c_2} \right] + \mu \boldsymbol{a_1}$$

Derive the first order conditions (FOCs)

$$\begin{aligned}
c_1 &: \quad \frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} + \lambda_1 [-1] = 0 && \rightarrow \quad \lambda_1 = \frac{1}{c_1} \\
c_2 &: \quad \frac{\partial \mathcal{L}}{\partial c_2} = \beta \cdot \frac{1}{c_2} + \lambda_2 [-1] = 0 && \rightarrow \quad \lambda_2 = \frac{\beta}{c_2} \\
a_1 &: \quad \frac{\partial \mathcal{L}}{\partial a_1} = \lambda_1 [-1] + \lambda_2 [1+r] + \mu = 0 && \rightarrow \quad \lambda_1 = \lambda_2 (1+r) + \mu \\
\text{CS} &: \quad \mu a_1 = 0 \quad \text{and} \quad \mu \geq 0 \quad \text{and} \quad a_1 \geq 0
\end{aligned}$$

The last line is the complementary slackness condition: either $a_1 = 0$ or the constraint is not binding ($\mu = 0$)

## Case 1: constraint not binding, $\mu = 0$ and $a_1 \geq 0$

FOC for assets simplifies to

$$\lambda_1 = \lambda_2 (1 + r) + \mu \quad \text{and} \quad \mu = 0 \quad \rightarrow \quad \lambda_1 = \lambda_2 (1 + r)$$

By using remaining FOCs we get the Euler equation

$$\lambda_1 = \frac{1}{c_1} \quad \text{and} \quad \lambda_2 = \frac{\beta}{c_2} \quad \rightarrow \quad \frac{1}{c_1} = \frac{\beta}{c_2} (1 + r) \quad \rightarrow \quad c_2 = \beta (1 + r) c_1$$

Optimal levels of consumption and assets

$$c_1 = \frac{1}{1+\beta} \left[ y_1 + \frac{y_2}{1+r} \right] \quad \text{and} \quad c_2 = \frac{\beta}{1+\beta} \left[ (1+r) y_1 + y_2 \right]$$

$$a_1 = \frac{1}{1+\beta} \left[ \beta y_1 - \frac{y_2}{1+r} \right]$$

We should check if indeed $a_1 \geq 0$

# Case 2: constraint binding, $\mu > 0$ and $a_1 = 0$

The solution follows from the budget constraints alone

$$c_1 + a_1 = y_1, \quad c_2 = y_2 + (1+r)\,a_1, \quad a_1 = 0 \quad \rightarrow \quad c_1 = y_1 \quad \text{and} \quad c_2 = y_2$$

We should check if indeed $\mu > 0$

$$\mu = \lambda_1 - \lambda_2\,(1+r) = \frac{1}{y_1} - \frac{\beta}{y_2}\,(1+r)$$

The value of $\mu$ tells by how much the agent's utility would increase
if the constraint was marginally relaxed (the agent could borrow „one euro")

It is the higher, the higher is future income relative to current income

# Two interest rates

A similar, more realistic set-up is when the agent can freely borrow amount $b$, but at a higher interest rate $r^b > r$

$$\max_{c_1, c_2, a_1, b_1} \quad U = \ln c_1 + \beta \ln c_2$$

$$\text{subject to} \quad c_1 + a_1 = y_1 + b_1$$

$$c_2 + (1 + r^b) b_1 = y_2 + (1 + r)\, a_1$$

$$\boldsymbol{a_1 \geq 0}$$

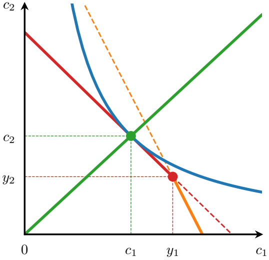
$$\boldsymbol{b_1 \geq 0}$$

We now have three (sensible) cases:

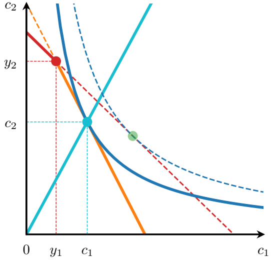
1. Saver: ($a_1 > 0,\ b_1 = 0$)
2. Borrower: ($a_1 = 0$, $b_1 > 0$)
3. Doubly constrained: ($a_1 = 0$, $b_1 = 0$)

# Two interest rates: graphical interpretation

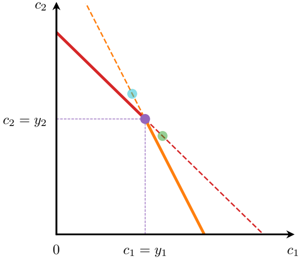
Saver




Borrower




Doubly constrained




In the third case the agent behaves (locally) as if borrowing constrained

# Ricardian Equivalence (and how to break it)

# Government

Government budget constraints

$$g_1 = \tau_1 + b_1$$

$$g_2 + (1 + r)\, b_1 = \tau_2$$

where $g_1$ and $g_2$ are public expenditure (per person) in periods 1 i 2, $\tau_1$ and $\tau_2$ are lump-sum taxes, and $b_1$ is issuance of government bonds (per person) financing deficit in period 1 and bought back in period 2

It's a simplified version of the full dynamic problem:

$$\sum_{t=1}^{\infty} \frac{g_t - \tau_t}{(1+r)^t} = b_0 + \lim_{t \to \infty} \frac{b_t}{(1+r)^t}$$

assuming the government does not go bankrupt: $\lim_{t \to \infty} \left[ b_t / (1+r)^t \right] = 0$

# Households' problem

Households solve their problem

$$\max_{c_1, c_2, a_1} \quad U = \ln c_1 + \beta \ln c_2$$

$$\text{subject to} \quad c_1 + a_1 = y_1 - \tau_1$$

$$c_2 = y_2 - \tau_2 + (1 + r)\, a_1$$

where assets $a_1$ comprise of bonds $b_1$ and other assets $\tilde{a}_1$

Lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 - \tau_1 + \frac{y_2 - \tau_2}{1+r}$$

# Households' problem: solution

Set up the Lagrangian

$$\mathcal{L} = \ln \boldsymbol{c_1} + \beta \ln \boldsymbol{c_2} + \lambda \left[ y_1 - \tau_1 + \frac{y_2 - \tau_2}{1+r} - \boldsymbol{c_1} - \frac{\boldsymbol{c_2}}{1+r} \right]$$

Derive the first order conditions (FOCs)

$$\begin{aligned} c_1 &: \quad \frac{1}{c_1} - \lambda = 0 && \rightarrow \quad \lambda = \frac{1}{c_1} \\ c_2 &: \quad \beta \frac{1}{c_2} - \frac{\lambda}{1+r} = 0 && \rightarrow \quad \lambda = \beta (1+r) \frac{1}{c_2} \end{aligned}$$

Optimality condition (Euler equation)

$$c_2 = \beta (1+r) c_1$$

# Households' problem: solution

Budget constraints once again

$$c_1 + b_1 + \tilde{a}_1 = y_1 - \tau_1 \quad \text{i} \quad b_1 = g_1 - \tau_1 \quad \rightarrow \quad \tilde{a}_1 = y_1 - g_1 - c_1$$

$$c_2 = y_2 - \tau_2 + (1+r)(b_1 + \tilde{a}_1) \quad \text{i} \quad b_1 = \frac{\tau_2 - g_2}{1+r} \quad \rightarrow \quad c_2 = y_2 - g_2 + (1+r)\tilde{a}_1$$

Lifetime budget constraint

$$c_2 = y_2 - g_2 + (1+r)(y_1 - g_1 - c_1) \quad \rightarrow \quad c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r}$$

After plugging in the Euler equation

$$c_1 = \frac{1}{1+\beta}\left[y_1 - g_1 + \frac{y_2 - g_2}{1+r}\right] \quad \text{and} \quad c_2 = \frac{\beta}{1+\beta}\left[(1+r)(y_1 - g_1) + (y_2 - g_2)\right]$$

$$a_1 = y_1 - \tau_1 - c_1 \quad \text{and} \quad \tilde{a}_1 = y_1 - g_1 - c_1 \quad \text{and} \quad b_1 = g_1 - \tau_1$$

Changes in sequence of taxes do not influence consumption choices!

Additionally, assets change 1:1 with changes in supply of government bonds

# Assumptions behind the Ricardian Equivalence result

All assets have the same rate of return (in expectation)

Taxes are non-distortionary

Changes in taxes are symmetric across households (no redistribution)

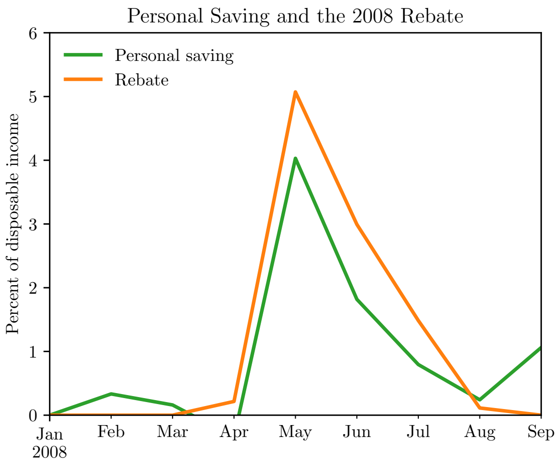
New public debt is repaid within current households' lifetime

Households are aware of the government budget constraints

Households are not borrowing constrained

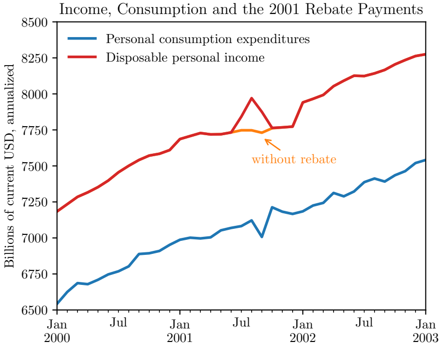
Households have „time-consistent" preferences

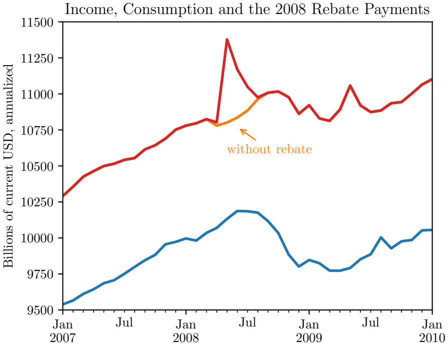
# 2008 tax rebates and savings




Taylor (2009), US Bureau of Economic Analysis.

# 2001 and 2008 tax rebates and consumption




Taylor (2009), US Bureau of Economic Analysis.

# 2001 and 2008 tax rebates and consumption

PCE Regressions with Rebate Payments

| | | |
|---|---|---|
| Lagged PCE | 0.794 | 0.832 |
| | (0.057) | (0.056) |
| **Rebate payments** | **0.048** | **0.081** |
| | **(0.055)** | **(0.054)** |
| Disposable personal income (w/o rebate) | 0.206 | 0.188 |
| | (0.056) | (0.055) |
| Oil price ($/bbl lagged 3 months) | | -1.007 |
| | | (0.325) |
| $R^2$ | 0.999 | 0.999 |

Taylor (2009)

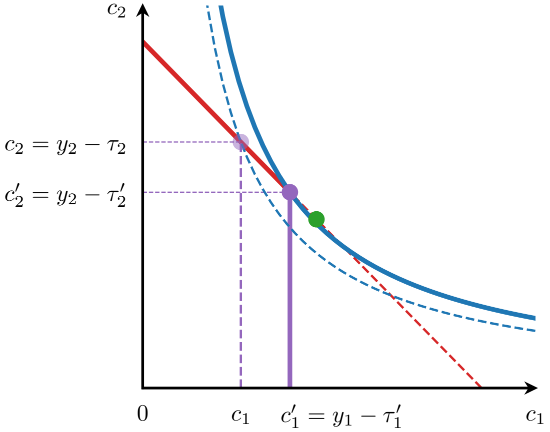
# Heterogeneous reaction to tax rebates

Responses to 2001 and 2008 Rebate Surveys

| | 2001 | | 2008 | |
|---|---|---|---|---|
| | Number | Percent | Number | Percent |
| Mostly spend | 256 | **21.8** | 447 | **19.9** |
| Mostly save | 376 | 32.0 | 715 | 31.8 |
| Mostly pay off debt | 544 | 46.2 | 1083 | 48.2 |
| Will not get rebate | 223 | | 212 | |
| Don't know / refused | 45 | | 61 | |
| Total | 1444 | 100 | 2518 | 100 |

Shapiro and Slemrod (2003), Shapiro and Slemrod (2009)

# Borrowing constrained consumers




Until disposable income moves beyond the green point, consumption increases 1:1 due to tax rebates / extra transfers

# Households with low liquid assets

Households with current consumption almost equal to current income and with almost no liquid assets are „hand-to-mouth"

Lusardi et al. (2011), Broda and Parker (2012): 30-40% US households have liquid assets below two months' income. But these are not necessarily „poor" people!

Kaplan and Violante (2014): in US microdata around 10% of households are „poor hand-to-mouth", but around 33% are „wealthy hand-to-mouth": with positive net worth allocated into illiquid assets (houses, pension funds, etc.)

They construct a model with two types of assets (low-return liquid and high-return illiquid), with transaction costs between them

In their model around 25% households spends immediately a small unforeseen extra income transfer, but if the transfer is large enough, they convert it into illiquid assets, behaving as „standard" consumers

# Finite planning horizon

Older households might expect that the higher future taxes will affect the economy after they die

Spending the 2008 Rebate, by Age

| Age group | Percent mostly spending |
| --- | --- |
| 29 or less | 11.7 |
| 30–39 | 14.2 |
| 40–49 | 16.9 |
| 50–64 | 19.9 |
| 65 or over | 28.4 |

Shapiro and Slemrod (2009)