Uniwersytet Warszawski, Wydział Nauk Ekonomicznych

Makroekonomia zaawansowana, rynek pracy

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Zadanie 8

$$\begin{aligned} \pi &= \frac{(eL)^\alpha}{\alpha} - wL \\ U &= w - x \\ e &= 1 \end{aligned}$$

a)

$$\begin{aligned} \frac{\partial \pi}{\partial L} &= L^{\alpha-1} - w = 0 \\ L(w) &= w^{\frac{1}{\alpha-1}} \\ \pi(w) &= \frac{1}{\alpha} w^{\frac{\alpha}{\alpha-1}} - w^{1+\frac{1}{\alpha-1}} = \left(\frac{1}{\alpha} - 1\right) w^{\frac{\alpha}{\alpha-1}} = \frac{1-\alpha}{\alpha} w^{\frac{\alpha}{\alpha-1}} \end{aligned}$$

b)

Iloczyn Nasha

$$\begin{aligned} \Omega &= (w-x)^\gamma \left(\frac{1-\alpha}{\alpha} w^{\frac{\alpha}{\alpha-1}}\right)^{1-\gamma} \\ \ln \Omega &= \gamma \ln(w-x) + (1-\gamma)\left[\ln\left(\frac{1-\alpha}{\alpha}\right) - \frac{\alpha}{1-\alpha} \ln w\right] \\ \frac{\partial \ln \Omega}{\partial w} &= \gamma \frac{1}{w-x} - (1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{w} = 0 \\ \gamma \frac{1}{w-x} &= (1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{w} \\ \gamma w &= (1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)(w-x) \end{aligned}$$

$$w\left[\gamma - (1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)\right] = (1-\gamma)\left(\frac{\alpha}{1-\alpha}\right)(-x)$$

$$\frac{\gamma - \alpha\gamma - \alpha + \alpha\gamma}{1-\alpha} w = \frac{\alpha - \alpha\gamma}{1-\alpha}(-x)$$

$$(\gamma - \alpha) w = (\alpha - \alpha\gamma)(-x)$$

$$w = \frac{\alpha - \alpha\gamma}{\alpha - \gamma} x$$

$$0 < \gamma < \alpha < 1 \quad \rightarrow \quad \frac{\alpha - \alpha\gamma}{\alpha - \gamma} > 1$$

c)

$$\begin{aligned}
\ln w &= \ln x + \ln(\alpha - \alpha\gamma) - \ln(\alpha - \gamma) \\
\frac{\partial(\ln w)}{\partial\gamma} &= \frac{-\alpha}{\alpha - \alpha\gamma} - \frac{-1}{\alpha - \gamma} = \frac{-1}{1-\gamma} + \frac{1}{\alpha - \gamma} > 0 \\
\frac{\partial(\ln w)}{\partial\gamma}\Big|_{\gamma=0} &= -1 + \frac{1}{\alpha} = \frac{1-\alpha}{\alpha} > 0
\end{aligned}$$

d)

$$\begin{aligned}
\pi &= \frac{\left[\left((w-x)/x\right)^{\beta} L\right]^{\alpha}}{\alpha} - wL \\
&= \frac{(w-x)^{\alpha\beta}}{\alpha x^{\alpha\beta}} L^{\alpha} - wL \\
\frac{\partial\pi}{\partial L} &= \frac{(w-x)^{\alpha\beta}}{x^{\alpha\beta}} L^{\alpha-1} - w = 0 \\
L^{\alpha-1} &= \frac{w \cdot x^{\alpha\beta}}{(w-x)^{\alpha\beta}} \\
L(w) &= \left[\frac{w \cdot x^{\alpha\beta}}{(w-x)^{\alpha\beta}}\right]^{\frac{1}{\alpha-1}} = w^{\frac{1}{\alpha-1}} \left(\frac{x}{w-x}\right)^{\frac{\alpha\beta}{\alpha-1}} \\
\pi(w) &= \frac{1}{\alpha}\left(\frac{w-x}{x}\right)^{\alpha\beta} w^{\frac{\alpha}{\alpha-1}} \left(\frac{x}{w-x}\right)^{\frac{\alpha^2\beta}{\alpha-1}} - w^{1+\frac{1}{\alpha-1}} \left(\frac{x}{w-x}\right)^{\frac{\alpha\beta}{\alpha-1}} \\
&= \frac{1}{\alpha} w^{\frac{\alpha}{\alpha-1}} \left(\frac{w-x}{x}\right)^{\alpha\beta + \frac{\alpha^2\beta}{1-\alpha}} - w^{\frac{\alpha}{\alpha-1}} \left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}} \\
&\left\{\alpha\beta + \frac{\alpha^2\beta}{1-\alpha} = \frac{\alpha\beta}{1-\alpha}\right\} \\
&= w^{\frac{\alpha}{\alpha-1}} \left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) = \frac{1-\alpha}{\alpha} w^{\frac{\alpha}{\alpha-1}} \left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}}
\end{aligned}$$

e)

$$
\begin{aligned}
\Omega &= (w-x)^{\gamma}\left(\frac{1-\alpha}{\alpha}w^{\frac{\alpha}{\alpha-1}}\left(\frac{w-x}{x}\right)^{\frac{\alpha\beta}{1-\alpha}}\right)^{1-\gamma} \\
\ln\Omega &= \gamma\ln(w-x)+(1-\gamma)\left[\ln\left(\frac{1-\alpha}{\alpha}\right)-\frac{\alpha}{1-\alpha}\ln w+\frac{\alpha\beta}{1-\alpha}\ln\left(\frac{1}{x}\right)+\frac{\alpha\beta}{1-\alpha}\ln(w-x)\right] \\
\frac{\partial\ln\Omega}{\partial w} &= \gamma\frac{1}{w-x}+(1-\gamma)\left[-\frac{\alpha}{1-\alpha}\frac{1}{w}+\frac{\alpha\beta}{1-\alpha}\frac{1}{w-x}\right]=0 \\
& (1-\gamma)\frac{\alpha}{1-\alpha}\frac{1}{w}=\frac{1}{w-x}\left[\gamma+(1-\gamma)\frac{\alpha\beta}{1-\alpha}\right] \\
& \left\{\gamma+(1-\gamma)\frac{\alpha\beta}{1-\alpha}=\frac{\gamma-\alpha\gamma+\alpha\beta-\alpha\beta\gamma}{1-\alpha}\right\} \\
& (\alpha-\alpha\gamma)\frac{1}{w}=\frac{1}{w-x}(\gamma-\alpha\gamma+\alpha\beta-\alpha\beta\gamma) \\
& (\alpha-\alpha\gamma)(w-x)=(\gamma-\alpha\gamma+\alpha\beta-\alpha\beta\gamma)w \\
& w(\alpha-\gamma-\alpha\beta+\alpha\beta\gamma)=\alpha(1-\gamma)x \\
& w=\frac{\alpha(1-\gamma)}{\alpha-\gamma-\alpha\beta+\alpha\beta\gamma}x=\{\beta=0\}=\frac{\alpha-\alpha\gamma}{\alpha-\gamma}x
\end{aligned}
$$

f)

$$
\begin{aligned}
\ln w &= \ln x+\ln(\alpha-\alpha\gamma)-\ln(\alpha-\gamma-\alpha\beta+\alpha\beta\gamma) \\
\frac{\partial(\ln w)}{\partial\gamma} &= \frac{-\alpha}{\alpha-\alpha\gamma}-\frac{-1+\alpha\beta}{\alpha-\gamma-\alpha\beta+\alpha\beta\gamma} \\
\frac{\partial(\ln w)}{\partial\gamma}|_{\gamma=0} &= -1+\frac{1-\alpha\beta}{\alpha-\alpha\beta}=\frac{1-\alpha\beta-\alpha+\alpha\beta}{\alpha-\alpha\beta}=\frac{1-\alpha}{\alpha(1-\beta)}>0
\end{aligned}
$$

$$
\frac{1-\alpha}{\alpha(1-\beta)}>\frac{1-\alpha}{\alpha}
$$

Zadanie 9

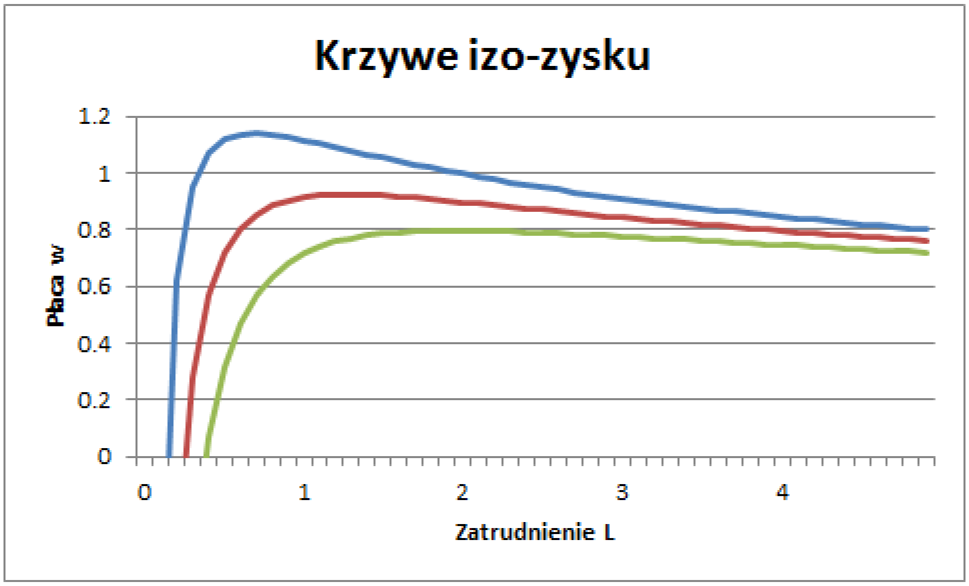
a)

Funkcja popytu

$$
\begin{aligned}
\pi &= \frac{L^{\alpha}}{\alpha}-wL \\
\frac{\partial\pi}{\partial L} &= L^{\alpha-1}-w=0 \\
L^{D} &= w^{\frac{1}{\alpha-1}}
\end{aligned}
$$

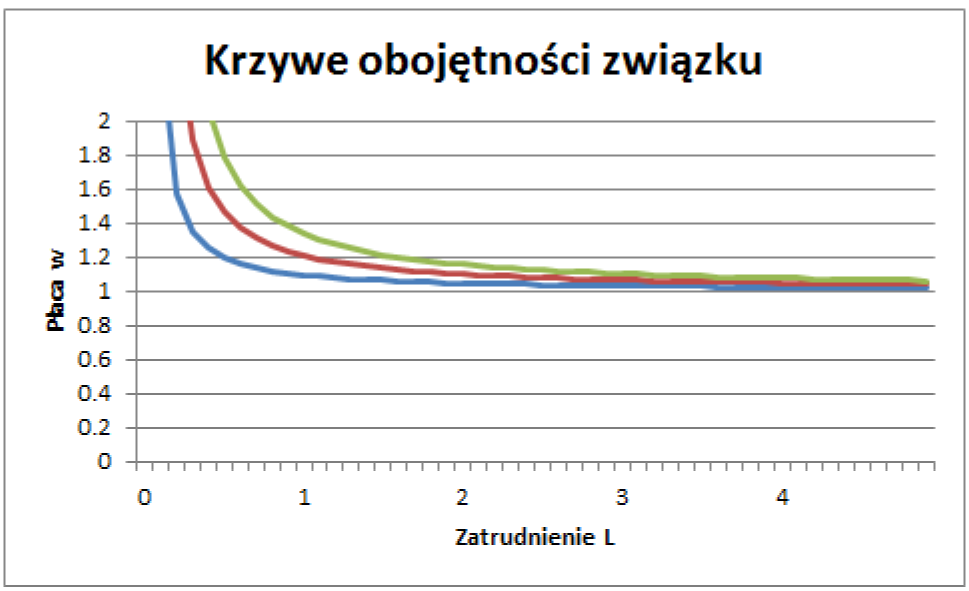
Izo-zysk

$$
\begin{aligned}
\bar{\pi} &= \frac{L^{\alpha}}{\alpha}-wL \\
wL &= \frac{L^{\alpha}}{\alpha}-\bar{\pi} \\
w &= \frac{L^{\alpha-1}}{\alpha}-\frac{\bar{\pi}}{L}
\end{aligned}
$$



b)

$$\begin{aligned}\bar{V} &= \frac{L}{N}\ln(w) + \left(1-\frac{L}{N}\right)\ln(B) \\ \exp(\bar{V}) &= w^{\frac{L}{N}}B^{1-\frac{L}{N}} \\ w &= \left(\frac{\exp(\bar{V})}{B^{1-\frac{L}{N}}}\right)^{\frac{N}{L}}\end{aligned}$$



c)

Zysk firmy

$$
\begin{aligned}
\pi &= \frac{L^{\alpha}}{\alpha} - wL \\
&= \frac{\left(w^{\frac{1}{\alpha-1}}\right)^{\alpha}}{\alpha} - w\left(w^{\frac{1}{\alpha-1}}\right) \\
&= \left(\frac{1-\alpha}{\alpha}\right) w^{\frac{\alpha}{\alpha-1}}
\end{aligned}
$$

Iloczyn Nasha

$$
\begin{aligned}
\Omega &= \left[V - \ln(B)\right]^{\beta} \left[\pi - 0\right]^{1-\beta} \\
&= \left[\frac{L}{N}\ln(w) + \left(1 - \frac{L}{N}\right)\ln(B) - \ln(B)\right]^{\beta} \left[\left(\frac{1-\alpha}{\alpha}\right) w^{\frac{\alpha}{\alpha-1}}\right]^{1-\beta} \\
&= \left[\frac{L}{N}\left(\ln(w) - \ln(B)\right)\right]^{\beta} \left[\left(\frac{1-\alpha}{\alpha}\right) w^{\frac{\alpha}{\alpha-1}}\right]^{1-\beta} \\
\ln(\Omega) &= \beta\ln[L] - \beta\ln[N] + \beta\ln\left[\ln(w) - \ln(B)\right] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right] + (1-\beta)\left(\frac{\alpha}{\alpha-1}\right)\ln[w] \\
&= \beta\ln\left[w^{\frac{1}{\alpha-1}}\right] - \beta\ln[N] + \beta\ln\left[\ln(w) - \ln(B)\right] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right] + (1-\beta)\left(\frac{\alpha}{\alpha-1}\right)\ln[w] \\
&= \left(\frac{\beta}{\alpha-1}\right)\ln[w] - \beta\ln[N] + \beta\ln\left[\ln(w) - \ln(B)\right] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right] + (1-\beta)\left(\frac{\alpha}{\alpha-1}\right)\ln[w] \\
&= \left(\frac{\alpha+\beta-\alpha\beta}{\alpha-1}\right)\ln[w] + \beta\ln\left[\ln(w) - \ln(B)\right] - \beta\ln[N] + (1-\beta)\ln\left[\frac{1-\alpha}{\alpha}\right]
\end{aligned}
$$

d)

$$
\begin{aligned}
\frac{\partial\ln(\Omega)}{\partial\ln(w)} &= \left(\frac{\alpha+\beta-\alpha\beta}{\alpha-1}\right) + \frac{\beta}{\ln(w) - \ln(B)} = 0 \\
& \frac{\alpha+\beta-\alpha\beta}{1-\alpha} = \frac{\beta}{\ln(w) - \ln(B)} \\
& \ln(w) - \ln(B) = \frac{\beta(1-\alpha)}{\alpha+\beta-\alpha\beta} \\
& \ln(w) = \ln(B) + \frac{\beta(1-\alpha)}{\alpha+\beta-\alpha\beta}
\end{aligned}
$$

e)

$$
\begin{aligned}
\frac{\partial \ln(w)}{\partial \beta} &= \frac{(1-\alpha)(\alpha+\beta-\alpha\beta) - \beta(1-\alpha)(1-\alpha)}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{(1-\alpha)(\alpha+\beta-\alpha\beta-\beta+\alpha\beta)}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{(1-\alpha)\alpha}{(\alpha+\beta-\alpha\beta)^2} > 0 \\
\frac{\partial L}{\partial \beta} &= \frac{\partial L^D}{\partial w}\frac{\partial w}{\partial \ln(w)}\frac{\partial \ln(w)}{\partial \beta} \\
&= \left(\frac{1}{\alpha-1}\right) w^{\frac{1}{\alpha-1}-1} \cdot w \cdot \frac{(1-\alpha)\alpha}{(\alpha+\beta-\alpha\beta)^2} < 0
\end{aligned}
$$

f)

$$
\begin{aligned}
|\varepsilon_D| &= -\frac{\partial L^D}{\partial w}\frac{w}{L^D} \\
&= -\left(\frac{1}{\alpha-1}\right) w^{\frac{1}{\alpha-1}-1} \cdot \frac{w}{w^{\frac{1}{\alpha-1}}} \\
&= \frac{1}{1-\alpha} \\
\frac{\partial |\varepsilon_D|}{\partial \alpha} &= \frac{1}{(1-\alpha)^2} > 0
\end{aligned}
$$

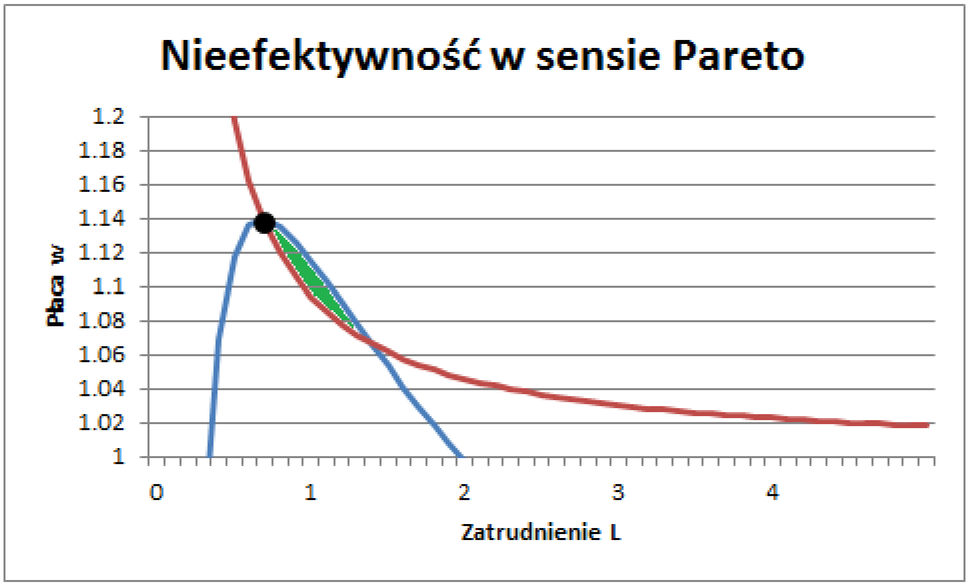
Im α jest większa, tym bardziej płaska staje się krzywa popytu na pracę (większa elastyczność)
Policzmy $\partial \ln(w) / \partial \alpha$

$$
\begin{aligned}
\frac{\partial \ln(w)}{\partial \alpha} &= \frac{-(\alpha+\beta-\alpha\beta) - \beta(1-\alpha)(1-\beta)}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{-\alpha-\beta+\alpha\beta-\beta(1-\alpha-\beta+\alpha\beta)}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{-\alpha-\beta+\alpha\beta-\beta+\alpha\beta+\beta^2-\alpha\beta^2}{(\alpha+\beta-\alpha\beta)^2} \\
&= \frac{\alpha\left(-1+\beta-\beta^2\right)+\beta(-2+\beta)}{(\alpha+\beta-\alpha\beta)^2} < 0 \\
\frac{\partial L}{\partial \alpha} &= \frac{\partial L^D}{\partial w}\frac{\partial w}{\partial \ln(w)}\frac{\partial \ln(w)}{\partial \alpha} \\
&= (-)\cdot(+)\cdot(-) > 0
\end{aligned}
$$

Zatrudnienie rośnie wraz ze wzrostem α, z kolei płace maleją.

g)

Równowaga jest nieefektywna w sensie Pareto, gdyż istnieją alokacje, które byłyby korzystniejsze jednocześnie dla firmy i związku zawodowego (obecna alokacja zaznaczona na czarno, lepsze alokacje zaznaczone na zielono).



Zadanie 13

a)

$$\begin{aligned}
m(u,v) &= \phi u^{\alpha} v^{1-\alpha} \\
p(\theta) &= \frac{m(u,v)}{u} = \phi u^{\alpha-1} v^{1-\alpha} = \phi \theta^{1-\alpha} \\
q(\theta) &= \frac{p(\theta)}{\theta} = \phi \theta^{-\alpha} \longrightarrow q'(\theta) = -\alpha \phi \theta^{-\alpha-1} \\
\dot{u} &= s(1-u) - p(\theta) u \\
\dot{u} = 0 &\Leftrightarrow u = \frac{s}{s+p(\theta)} = \frac{s}{s+\phi\theta^{1-\alpha}}
\end{aligned}$$

Dla u powyżej krzywej $\dot{u} = 0$ stopa bezrobocia maleje w czasie ($\dot{u} < 0$), a dla u powyżej tej krzywej stopa bezrobocia rośnie w czasie.

b)

$$\begin{aligned}
rV(t) &= -c + q(\theta(t))[J(t) - V(t)] + \dot{V}(t) \\
rJ(t) &= [y - w(t)] + s[V(t) - J(t)] + \dot{J}(t)
\end{aligned}$$

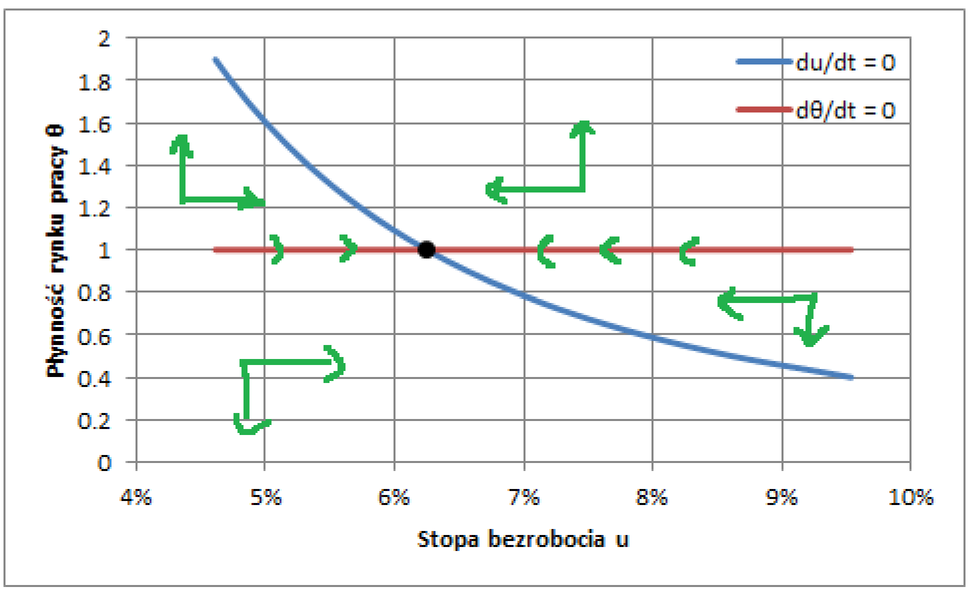
Warunek wolnego tworzenia wakatów zapewnia $V = 0$ w równowadze, także poza stanem ustalonym

$$
\begin{aligned}
0 &= -c + q(\theta(t))\, J(t) \longrightarrow J(t) = \frac{c}{q(\theta(t))} \\
rJ(t) &= [y - w(t)] - sJ(t) + \dot{J}(t) \longrightarrow \dot{J}(t) = (r+s)\, J(t) - [y - w(t)] \\
\dot{J}(t) &= -\frac{c \cdot q'(\theta(t)) \cdot \dot{\theta}(t)}{[q(\theta(t))]^2} = \frac{c\alpha q(\theta)/\theta}{[q(\theta(t))]^2} \dot{\theta}(t) = \frac{c\alpha}{q(\theta(t))} \frac{\dot{\theta}(t)}{\theta(t)} \\
\frac{c\alpha}{q(\theta(t))} \frac{\dot{\theta}(t)}{\theta(t)} &= (r+s) \frac{c}{q(\theta(t))} - [y - w(t)] \\
\frac{\dot{\theta}(t)}{\theta(t)} &= \frac{r+s}{\alpha} - \frac{q(\theta(t))}{c\alpha} [y - w(t)] \\
w(t) &= (1-\beta) z + \beta (y + c\theta) \\
\frac{\dot{\theta}(t)}{\theta(t)} &= \frac{r+s}{\alpha} - \frac{q(\theta(t))}{c\alpha} [(1-\beta)(y-z) + \beta c\theta] \\
\dot{\theta} &= \frac{\theta}{\alpha} \left[(r+s) - (1-\beta)(y-z) \frac{q(\theta)}{c} + \beta p(\theta) \right]
\end{aligned}
$$

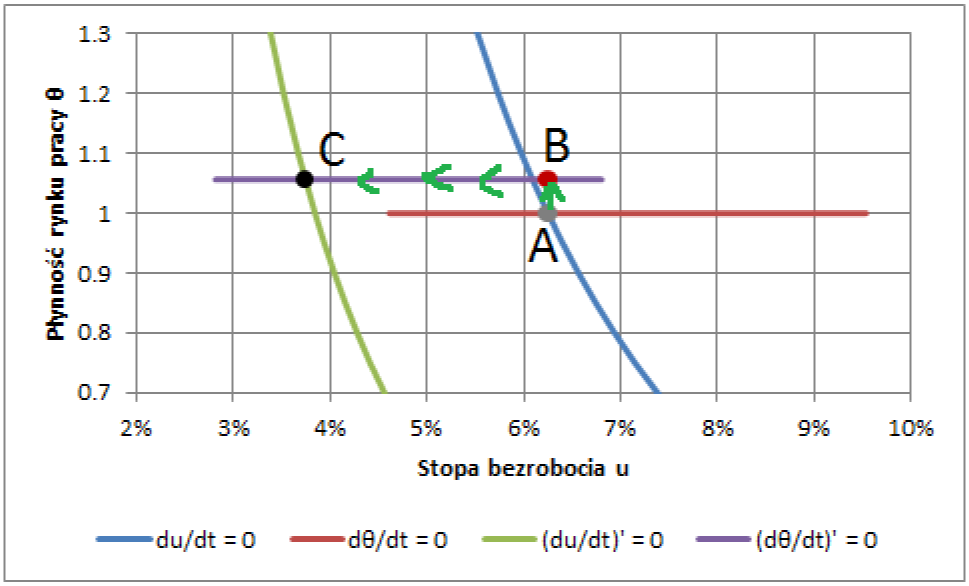
Powyższe równanie nie zależy od u, zatem linia $\dot{\theta} = 0$ w przestrzeni (u, θ) jest płaska na poziomie θ^*. Dla $\theta > \theta^*$ powyżej linii $\dot{\theta} = 0$ płynność rynku pracy θ wzrasta, a maleje dla $\theta < \theta^*$.

c)

Stan ustalony systemu jest określony przez poziom $\theta = \theta^*$ oraz $u = s/\left(s + \phi(\theta^*)^{1-\alpha}\right)$. Jest to równowaga siodłowa, przy czym ścieżka siodłowa pokrywa się z linią $\dot{\theta} = 0$:



Wzrost ϕ poprawia efektywność połączeń, co poprawia płynność rynku pracy i obniża stopę bezrobocia w stanie ustalonym. Płynność rynku pracy θ rośnie natychmiastowo ($A \to B$), a następnie stopa bezrobocia u obniża się stopniowo ($B \to C$):



Zachowanie się stopy bezrobocia w czasie po wzroście ϕ: