University of Warsaw
Faculty of Economic Sciences

New Keynesian model

Applied Macroeconomics: Lecture 12

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Fall 2019

University of Warsaw

Plan of the lecture

New Keynesian model

1. Nominal rigidities and sticky prices
2. New Keynesian Phillips Curve
3. New Keynesian IS Curve
4. Monetary policy in the New Keynesian model
5. Monetary policy at the Effective Lower Bound

Nominal rigidities and sticky prices

Models with price-setting firms

Under perfect competition firms are selling homogeneous goods and are price-takers

In reality many firms sell differentiated products and are able to set their own prices

Convenient framework: “monopolistic competition”

- Households enjoy consuming many different goods
- One sector: all firms “compete” with each other
- Elasticity of substitution determines market power
- Perfect competition: infinitely high elasticity

Monopolistic competition: setup

Households have "love for variety" utility function

Demand function for the i-th good

$$Y_i^d = (P/P_i)^{(1+\mu_i)/\mu_i} \, Y$$

where μ_i is a parameter related to the elasticity of substitution

If production function exhibits Constant Returns to Scale then marginal cost MC_i is independent of quantity produced

Profit maximization problem ▸ full solution

$$\begin{aligned} \max \quad & \Pi_i = (P_i - MC_i)\, Y_i \\ \text{subject to} \quad & Y_i = (P/P_i)^{(1+\mu_i)/\mu_i} \, Y \end{aligned}$$

Monopolistic competition: solution

"Markup pricing" is the profit-maximizing strategy

$$P_i^* = (1 + \mu_i) \cdot MC_i$$

Taking logarithms

$$\ln P_i^* = \ln (1 + \mu_i) + \ln MC_i$$

$$p_i^* = \mu_i + mc_i$$

Empirical evidence on markups in US and EA

Perfect competition can be rejected for almost all sectors in all countries

Markups are generally higher in services than manufacturing

Table 1. Weighted average markup, 1981-2004

Country	Manufacturing & Construction		Market Services		All (Manufacturing, Construction & Market Services)	
Germany	1.16	(0.01)*	1.54	(0.03)*	1.33	(0.01)*
France	1.15	(0.01)*	1.26	(0.02)*	1.21	(0.01)*
Italy	1.23	(0.01)*	1.87	(0.02)*	1.61	(0.01)*
Spain	1.18	(0.00)*	1.37	(0.01)*	1.26	(0.01)*
Netherlands	1.13	(0.01)*	1.31	(0.02)*	1.22	(0.01)*
Belgium	1.14	(0.00)*	1.29	(0.01)*	1.22	(0.01)*
Austria	1.20	(0.02)*	1.45	(0.03)*	1.31	(0.02)*
Finland	1.22	(0.01)*	1.39	(0.02)*	1.28	(0.01)*
Euro Area	1.18	(0.01)*	1.56	(0.01)*	1.37	(0.01)*
USA	1.28	(0.02)*	1.36	(0.03)*	1.32	(0.02)*

Christopoulou and Vermeulen (2008)

Prices do not change every period

Survey about price setting practices carried out by the Banco de Portugal

Firms in the sample are generally quicker to react to cost shocks,
in particular when they are positive, than to demand shocks

TABLE 1

Distribution of the price responses to demand and cost shocks

	Cost shocks		*Demand shocks*	
Price adjustment lag	*Positive*	*Negative*	*Positive*	*Negative*
1 – less than one week	4.7	3.5	2.8	4.8
2 – from one week to one month	16.8	15.2	12.2	16.8
3 – from one month to three months	25.0	25.7	19.3	23.4
4 – from three to six months	17.6	14.9	13.4	13.6
5 – from six months to one year	26.3	21.2	17.7	14.0
6 – more than one year	9.6	19.5	34.6	27.4
Total	100.0	100.0	100.0	100.0

Dias et al. (2014)

Stylized facts on price stickiness

Price duration

- US: average time between price changes is 2-4 quarters Blinder et al. (1998), Klenow and Kryvstov (2008), Nakamura and Steinsson (2008)
- EA: average time between price changes is 4-5 quarters Dhyne et al. (2005), Altissimo et al. (2006)
- PL: average time between price changes is 4 quarters Macias and Makarski (2013)

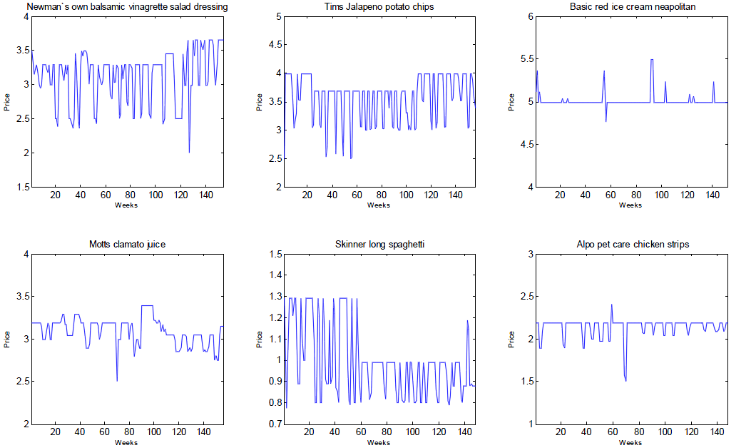
Cross-industry heterogeneity

- Prices of tradables less sticky than those of nontradables
- Retail prices usually more sticky than producer prices

Gagnon (2009): for inflation above 10-15% prices change more frequently with higher inflation

Example retail prices behavior

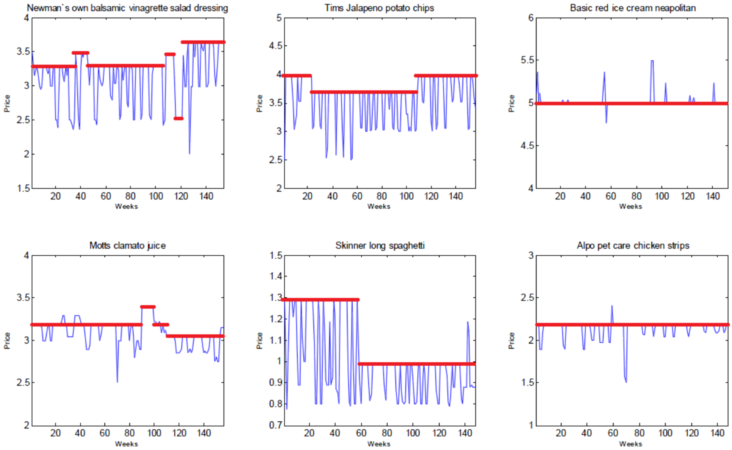
Raw retail scanner data



Koning (2015)

Example retail prices behavior

After “controlling” for short-lived sales prices: reference prices



Price stickiness depends on sector

Table 4.1 Frequency of consumer price changes by product type, in %

Country	Unprocessed food	Processed food	Energy (oil products)	Non-energy industrial goods	Services	Total, country weights	Total, Euro area weights
Belgium	31.5	19.1	81.6	5.9	3.0	17.6	15.6
Germany	25.2	8.9	91.4	5.4	4.3	13.5	15.0
Spain	50.9	17.7	n.a.	6.1	4.6	13.3	11.5
France	24.7	20.3	76.9	18.0	7.4	20.9	20.4
Italy	19.3	9.4	61.6	5.8	4.6	10.0	12.0
Luxembourg	54.6	10.5	73.9	14.5	4.8	23.0	19.2
The Netherlands	30.8	17.3	72.6	14.2	7.9	16.2	19.0
Austria	37.5	15.5	72.3	8.4	7.1	15.4	17.1
Portugal	55.3	24.5	15.9	14.3	13.6	21.1	18.7
Finland	52.7	12.8	89.3	18.1	11.6	20.3	-
Euro Area	28.3	13.7	78.0	9.2	5.6	15.1	15.8

Source: Dhyne et al. (2005). Figures presented in this table are computed on the basis of the 50 product sample, with the only exception of Finland for which figures based on the entire CPI are presented. The total with country weights is calculated using country-specific weights for each item, the total with euro area weights using common euro area weights for each sub-index. No figures are provided for Finland because of a lack of comparability of the sample of products used in this country.

Altissimo, Ehrmann and Smets (2006)

Factors behind price stickiness

Why prices are sticky?

- Implicit/explicit contracts
- Rigid marginal costs
- Coordination failure
- Judging quality by price
- Menu costs
- Costly information
- Pricing thresholds

Survey assessment of price stickiness theories

Table 4.6 Ranking of theories explaining price stickiness

	Belgium	Germany	Spain	France	Italy	Luxem-bourg	Nether-lands	Austria	Portugal	Euro Area
Implicit contracts	2.5		2.6	2.2		2.7	2.7	3.0	3.1	2.7
Explicit contracts	2.4	2.4	2.3	2.7	2.6	2.8	2.5	3.0	2.6	2.6
Cost-based pricing	2.4			2.5		2.7		2.6	2.7	2.6
Co-ordination failure	2.2	2.2	2.4	3.0	2.6	2.1	2.2	2.3	2.8	2.4
Judging quality by price	1.9		1.8			2.2	2.4	1.9	2.3	2.1
Temporary shocks	1.8	1.9	1.8	2.1	2.0	1.7	2.4	1.5	2.5	2.0
Change non-price factors	1.7		1.3			1.9	1.9	1.7		1.7
Menu costs	1.5	1.4	1.4	1.4	1.6	1.8	1.7	1.5	1.9	1.6
Costly information	1.6		1.3			1.8		1.6	1.7	1.6
Pricing thresholds	1.7		1.5	1.6	1.4	1.8	1.8	1.3	1.8	1.6

Source: Fabiani et al. (2005). Euro area figures are unweighted averages of country scores.

Altissimo, Ehrmann and Smets (2006)

New Keynesian Phillips Curve

Costs of price changes

Based on Rotemberg (1982)

Different take on “menu costs”

Assumes that bigger price changes are more costly, e.g. due to losses in consumer loyalty

Zbaracki et al. (2004):

- Many components of the managerial and customer costs are convex, whereas the menu costs are not
- Evidence of managers’ fear of antagonizing customers

Managerial and consumer costs

TABLE 1.—MANAGERIAL COSTS OF CHANGING PRICES

C. Total Cost

Task	Cost
Information-gathering and decision-making (from panel A)	$251,250
Internal communication (from panel B)	$28,900
Total cost	$280,150

TABLE 3.—CUSTOMER COST OF CHANGING PRICES (CONTINUED)

D. Total Negotiation Costs

Customer Group	Total Cost
Top 25 customers (from panel A)	$253,300
Middle 250 customers (from panel B)	$172,500
Bottom 1,100 customers (from panel C)	$98,175
Total negotiation costs for all customers	$523,975

E. Total Customer Costs of Changing Prices

Task	Total Cost
Total communication cost (from table 2D)	$368,940
Total negotiation cost (from panel D)	$523,975
Total customer costs	$892,915

Zbaracki et al. (2004)

Physical “menu” costs

TABLE 4.—PHYSICAL COSTS OF CHANGING PRICES (MENU COSTS)

Item	Measured Hours	Cost[a]	Source
Price list:			
Marketing communication	80 hours (from time sheets and marketing communication measures)	$4,000	Self-report (meets internal accounting standards)
Printing cost	N/A	$15,180	Data from bid sheets
Distribution cost:			
Time	8 hours	$400	
Postage		$2,200	
Create customer-specific price lists	7 hours	$350	Measure based on pricing analyst report
Supplemental price lists (monthly):			
Marketing communication	55 hours	$2,750	Measure based on annual costs
Printing	—	$10,000	Measure based on annual costs
Mailing	—	$4,000	
Create master file, cut prices, format disks and EDI	90 hours	$4,500	Measure based on pricing analyst reports
Total		$43,380	

Zbaracki et al. (2004)

Rotemberg model: setup

The dynamic profit maximizing problem can be recast as a simpler problem of minimizing a loss function

$$L_t = \sum_{k=0}^{\infty} \beta^k \cdot \mathrm{E}_t \left[\left(p_{t+k} - p^*_{t+k}\right)^2 + c\left(p_{t+k} - p_{t+k-1}\right)^2 \right]$$

- $\sum_{k=0}^{\infty}$ means that the firm considers the implications of the price set today for all possible future periods
- $\beta < 1$ implies that the firm places less weight on future losses than on today's losses
- $\mathrm{E}_t[(p_{t+k} - p^*_{t+k})^2]$ is the loss of profit due to setting price at a level different than would be optimal in a frictionless setting
- $\mathrm{E}_t[c(p_{t+k} - p_{t+k-1})^2]$ reflects the costs of changing prices: the bigger the change, the higher the associated cost

Rotemberg model: solution

Expand the loss function for convenience

$$L_t = (p_t - p_t^*)^2 + c\,(p_t - p_{t-1})^2 + \beta \cdot \mathrm{E}_t\left[(p_{t+1} - p_{t+1}^*)^2 + c\,(p_{t+1} - p_t)^2\right] + \dots$$

First order condition with respect to p_t

$$2\,(p_t - p_t^*) + 2c\,(p_t - p_{t-1}) + \beta \cdot \mathrm{E}_t\left[2c\,(p_{t+1} - p_t)\right] \cdot (-1) = 0$$

Divide by 2 and move stuff to the right hand side

$$c\,(p_t - p_{t-1}) = \beta c \cdot \mathrm{E}_t\left[p_{t+1} - p_t\right] - (p_t - p_t^*)$$

Use the definition of the inflation rate $\pi_t \equiv p_t - p_{t-1}$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{c}\,(p_t^* - p_t)$$

Staggered price adjustment

Based on Calvo (1983)

In Rotemberg firms make many small price changes

In Calvo firms are not "allowed" to do so

- Firms can change their price only if they receive a "signal"
- Price remains unchanged with probability θ
- If a firm set the price in period t, then the price remains unchanged in period $t + k$ with probability θ^k
- Denote price set in period t with $\tilde{p}_t$ ("reset" price)

Calvo model: setup

The dynamic profit maximizing problem can be recast as a simpler problem of minimizing a loss function

$$L\left(\tilde{p}_t\right) = \sum_{k=0}^{\infty} (\beta\theta)^k \cdot \mathrm{E}_t\left[\left(\tilde{p}_t - p^*_{t+k}\right)^2\right]$$

- $\sum_{k=0}^{\infty}$ means that the firm considers the implications of the price set today for all possible future periods
- $\beta < 1$ implies that the firm places less weight on future losses than on today's losses
- Price set in period t remains unchanged until period $t+k$ with probability θ^k
- $\mathrm{E}_t[(\tilde{p}_t - p^*_{t+k})^2]$ is the loss of profit due to setting price at a level different than would be optimal in a frictionless setting

Calvo model: solution

First order condition

$$\sum_{k=0}^{\infty} (\beta\theta)^k \cdot \mathrm{E}_t \left[2\left(\tilde{p}_t - p^*_{t+k}\right)\right] = 0$$

Divide by 2 and move the part with p^*_{t+k} to the right hand side

$$\sum_{k=0}^{\infty} (\beta\theta)^k \cdot \tilde{p}_t = \sum_{k=0}^{\infty} (\beta\theta)^k \cdot \mathrm{E}_t p^*_{t+k}$$

$$\frac{1}{1-\beta\theta} \cdot \tilde{p}_t = \sum_{k=0}^{\infty} (\beta\theta)^k \cdot \mathrm{E}_t p^*_{t+k}$$

$$\tilde{p}_t = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \cdot \mathrm{E}_t p^*_{t+k}$$

Reset price $\tilde{p}_t$ is the weighted average of today's and future prices that would be optimal in a frictionless setting

Simple numerical example and generalization

Assume $\beta = 1$ and $\theta = 1/2$

$$\tilde{p}_t = \frac{1}{2}\left(p_t^* + \frac{1}{2}\mathrm{E}_t p_{t+1}^* + \frac{1}{4}\mathrm{E}_t p_{t+2}^* + \dots\right)$$

$$\mathrm{E}_t\tilde{p}_{t+1} = \frac{1}{2}\left(\mathrm{E}_t p_{t+1}^* + \frac{1}{2}\mathrm{E}_t p_{t+2}^* + \frac{1}{4}\mathrm{E}_t p_{t+3}^* + \dots\right)$$

Reset price can be expressed recursively

$$\tilde{p}_t = \frac{1}{2}p_t^* + \frac{1}{2}\mathrm{E}_t\tilde{p}_{t+1}$$

Generalization

$$\tilde{p}_t = (1-\beta\theta)\,p_t^* + \beta\theta\mathrm{E}_t\tilde{p}_{t+1}$$

Reset price $\tilde{p}_t$ is the weighted average of today's "optimal" price p_t^* and next period's expected reset price $\mathrm{E}_t\tilde{p}_{t+1}$

Dynamics of prices and inflation

Within each period a fraction θ of firms keeps prices unchanged, the remaining $1-\theta$ fraction resets prices to $\tilde{p}_t$

$$p_t = \theta p_{t-1} + (1-\theta)\,\tilde{p}_t$$

After a series of algebraic manipulations ▸ here we get that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(p_t^* - p_t\right)$$

Compare to Rotemberg's outcome

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{1}{c}\left(p_t^* - p_t\right)$$

In both settings, inflation is forward-looking

Expectations on future inflation affect inflation today!

Comparing Rotemberg and Calvo

Identical (up to the first-order approximation) functional form, different economic conclusions

See Lombardo and Vestin (2007) and Ascari and Rossi (2012)

- Rotemberg: inflation is "costly" due to costs of changing prices, relative prices across firms are unaffected
- Calvo: inflation is "costly" since not every firm adjusts prices within each period, price dispersion arises
- Price dispersion introduces inefficiencies into the economy ($P_i/P_j \neq MC_i/MC_j$)
- Welfare costs of inflation are higher in the Calvo schemes

New Keynesian Phillips Curve

Under both schemes we have that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \gamma \left(p_t^* - p_t \right)$$

Relate it to the expression for "optimal" price

$$p_t^* = \mu + mc_t$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \gamma \left(mc_t - p_t - (-\mu) \right) \equiv \beta \mathrm{E}_t \pi_{t+1} + \gamma \tilde{mc}_t^r$$

where $\tilde{mc}_t^r$ is the "real marginal cost gap":
a difference between the actual real marginal cost $mc_t - p_t$
and its value in the frictionless world $-\mu$

$\tilde{mc}_t^r$ is not directly observable, but it can be shown that

$$\tilde{mc}_t^r = \phi \left(y_t - y_t^* \right) \equiv \phi x_t \quad \rightarrow \quad \pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t$$

where x_t is the measure of output gap and $\kappa \equiv \gamma \phi$

Empirical tests of NKPC

Gali and Gertler (1999) (quarterly US data, 1960Q1–1997Q4):

- Using detrended log GDP as proxy for output gap x_t

$$\pi_t = 0.988\,\mathrm{E}_t\pi_{t+1} - 0.016\,x_t$$

- Using a proxy for the real marginal cost gap $\tilde{mc}_t^r$

$$Y_t = A_t \bar{K}_t^{\alpha} N_t^{1-\alpha}$$

$$mc_t^r = \frac{W_t}{P_t} / \frac{\partial Y_t}{\partial N_t} = \frac{W_t N_t}{P_t Y_t} \cdot \frac{1}{1-\alpha} \equiv \text{labor share}\,(t) \cdot \frac{1}{1-\alpha}$$

$$\pi_t = 0.942\,\mathrm{E}_t\pi_{t+1} + 0.023\,\tilde{mc}_t^r$$

Hybrid NKPC

Not all firms reset their prices optimally

- Fraction $1-\omega$ is **forward looking** and set $p_t^f = \tilde{p}_t$
- Fraction ω is **backward looking** and reset prices basing on what their competitors did in $t-1$

$$p_t^b = \tilde{p}_{t-1} + \pi_{t-1}$$

- "Error" not very costly in low inflation environment

$$p_t^b - \tilde{p}_t = \theta / \left(1-\theta\right) \pi_t$$

Results in Hybrid NKPC

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f \mathrm{E}_t \pi_{t+1} + \lambda \tilde{mc}_t^r$$

Empirical tests of Hybrid NKPC: US

Table 2

Estimates of the New Hybrid Phillips Curve

	ω	θ	β	γ_b	γ_f	λ
GDP Deflator						
(1)	0.265 (0.031)	0.808 (0.015)	0.885 (0.030)	0.252 (0.023)	0.682 (0.020)	0.037 (0.007)
(2)	0.486 (0.040)	0.834 (0.020)	0.909 (0.031)	0.378 (0.020)	0.591 (0.016)	0.015 (0.004)
Restricted β						
(1)	0.244 (0.030)	0.803 (0.017)	1.000	0.233 (0.023)	0.766 (0.015)	0.027 (0.005)
(2)	0.522 (0.043)	0.838 (0.027)	1.000	0.383 (0.020)	0.616 (0.016)	0.009 (0.003)
NFB Deflator						
(1)	0.077 (0.030)	0.830 (0.016)	0.949 (0.019)	0.085 (0.031)	0.871 (0.018)	0.036 (0.008)
(2)	0.239 (0.043)	0.866 (0.025)	0.957 (0.021)	0.218 (0.031)	0.755 (0.016)	0.015 (0.006)

Note: Table 2 reports GMM estimates of parameters of equation (26). Rows (1) and (2) correspond to the two specifications of the orthogonality conditions found in equations (27) and (28) in the text, respectively. Estimates are based on quarterly data and cover the sample period 1960:1-1997:4. Instruments used include four lags of inflation, labor income share, long-short interest rate spread, output gap, wage inflation, and commodity price inflation. A 12 lag Newey-West estimate of the covariance matrix was used. Standard errors are shown in brackets.

Gali and Gertler (1999)

Empirical tests of Hybrid NKPC: EA

Phillips curves - Estimation output
(endogenous variable: core inflation)

	I Baseline	II Flattened	III Exponential	IV With oil price
Sample period	*1991q1 - 2010q4*	*2000q1 - 2010q4*	*1991q1 - 2010q4*	*1991q1 - 2010q4*
Core inflation lagged one quarter	0.830***	0.881***	0.799***	0.776***
Inflation expectations	0.164*	0.143**	0.179***	0.199***
Output gap	0.071***	0.038***		0.033*
EXP (output gap)			0.057***	
Constant			-0.072	
Oil price change lagged four quarters				0.002***
R^2	0.97	0.88	0.97	0.95
J statistic	0.046	0.09	0.049	0.030

Estimation by GMM. Dependent variable: core inflation. Instruments: First lag of output gap; first and second lags of inflation expectations; short-term interest rates and their first lag; change in the rate of capacity utilisation; oil price change; fifth lag of oil price change (the latter two for panel IV only).

*, **, *** denote significance at 5, 2 and 1% confidence level

Source: Commission services

European Commission (2011)

New Keynesian IS curve

Household's dynamic consumption choice: setup

Household "lives" for two time periods: t and $t+1$
(enough to capture logic of a multi-period problem)

Receives incomes Y_t and Y_{t+1}

Chooses consumption C_t and C_{t+1}

Utility maximization problem

$$\begin{aligned} \max \quad & U\left(C_t,\, C_{t+1}\right) = u\left(C_t\right) + \beta\, u\left(C_{t+1}\right) \\ \text{subject to} \quad & C_t + \frac{C_{t+1}}{1+r_{t+1}} = Y_t + \frac{Y_{t+1}}{1+r_{t+1}} \end{aligned}$$

where $\beta = 1/\left(1+\rho\right)$ is the discount factor

Lagrangian

$$\mathcal{L} = u\left(C_t\right) + \beta\, u\left(C_{t+1}\right) + \lambda \left[Y_t + \frac{Y_{t+1}}{1+r_{t+1}} - C_t - \frac{C_{t+1}}{1+r_{t+1}}\right]$$

Household's dynamic consumption choice: solution

First order conditions

$$\frac{\partial \mathcal{L}}{\partial C_t} = u'\left(C_t\right) - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} = \beta\, u'\left(C_{t+1}\right) - \frac{\lambda}{1 + r_{t+1}} = 0$$

Rewrite and simplify

$$\lambda = u'\left(C_t\right)$$

$$\lambda = \beta\left(1 + r_{t+1}\right)\, u'\left(C_{t+1}\right)$$

Euler equation

$$u'\left(C_t\right) = \beta\left(1 + r_{t+1}\right)\, u'\left(C_{t+1}\right)$$

$$u'\left(C_t\right) = \frac{1 + r_{t+1}}{1 + \rho}\, u'\left(C_{t+1}\right)$$

Assume functional form for utility

For the CRRA function

$$u\left(C\right)=\frac{C^{1-\sigma}}{1-\sigma}\quad\rightarrow\quad u'\left(C\right)=C^{-\sigma}$$

We have

$$C_t^{-\sigma}=\frac{1+r_{t+1}}{1+\rho}\,C_{t+1}^{-\sigma}$$

Take logarithms

$$-\sigma\ln C_t=-\sigma\ln C_{t+1}+\ln\left(1+r_{t+1}\right)-\ln\left(1+\rho\right)$$

$$c_t\approx c_{t+1}-\frac{1}{\sigma}\left(r_{t+1}-\rho\right)$$

Add a few assumptions

Assume no investment and govt. spending, so that $c_t = y_t$

$$y_t = y_{t+1} - \frac{1}{\sigma}\left(r_{t+1} - \rho\right)$$

In the flexible price world output is always at natural level y_t^*

$$y_t^* = y_{t+1}^* - \frac{1}{\sigma}\left(r_t^* - \rho\right)$$

where r_t^* is the natural real interest rate

Subtract the "natural" from the "actual" equation

$$y_t - y_t^* = y_{t+1} - y_{t+1}^* - \frac{1}{\sigma}\left(r_{t+1} - r_t^*\right)$$

Use definition of output gap $x_t \equiv y_t - y_t^*$
"Add" uncertainty about the future

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(\mathrm{E}_t r_{t+1} - r_t^*\right)$$

New Keynesian IS curve

Expected real interest rate is the difference between known nominal interest rate and expected inflation

$$\mathrm{E}_t r_{t+1} = i_t - \mathrm{E}_t \pi_{t+1}$$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^* \right)$$

In general case where $c_t \neq y_t$, one can capture the influence of other expenditures as a demand shock u

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^* \right) + u_t$$

Holding expectations costant, higher nominal interest rates translate to lower output gaps

Key equations of the New Keynesian model

New Keynesian Phillips Curve (e_t is a cost-push shock)

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^*\right) + u_t$$

A forward-looking system of three variables:
output gap, inflation and nominal interest rate

Need an additional equation to close the system
$\hookrightarrow$ need to specify monetary policy rule

Monetary policy in the New Keynesian model

Optimal policy: long run

Two distortions in the model

1. Monopolistic competition: $P > MC \quad \rightarrow \quad Y < \bar{Y}$
2. Price dispersion: $P_i/P_j \neq MC_i/MC_j \quad \rightarrow \quad Y < \bar{Y}$

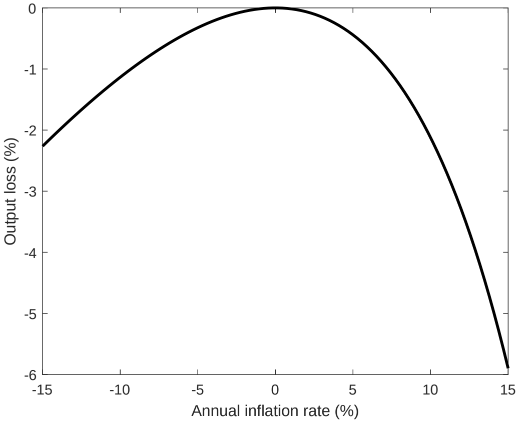
The first distortion cannot be eliminated by monetary policy

But the second can, by keeping inflation rate at 0%

Welfare losses of annual inflation in 2-3% range
are very small, and other considerations matter

Costs of inflation under Calvo scheme

Output decreases (non-linearly) with price dispersion



Assumed parameter values: $\mu = 1.33$, $\theta = 0.75$

Optimal policy: short run

If there are no cost-push shocks and sticky prices are the only distortion then optimal monetary policy in the short run is to stabilize inflation perfectly

This would also perfectly stabilize output gap
$\hookrightarrow$ divine coincidence, see Blanchard and Gali (2007)

If there are other distortions, e.g. sticky wages, then optimal policy becomes more complicated, e.g. has to also stabilize wage inflation

Divine coincidence no longer holds
(impossible to stabilize all variables at the same time)

Optimal policy: loss function

Due to many distortions optimal policy involves trade-offs

Rotemberg and Woodford (1998): when real imperfections are present, the second order approximation to social welfare is

$$W = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\lambda x_t^2 + \pi_t^2 \right) \right]$$

Consistent with behavior of central banks,
who aim to stabilize both inflation and output gaps

Question arises whether policy should be
conducted discretionary or under commitment

Optimal policy under discretion

Under optimal discretionary policy (ODP) the central bank is not able to influence expectations about future policy

Optimization boils down to solving a series of static problems

$$\begin{aligned} \min \quad & \lambda x_t^2 + \pi_t^2 \\ \text{subject to} \quad & \pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t \end{aligned}$$

Note that expectation terms are taken as given, since the CB is assumed not to influence them

Solution:

$$\pi_t = -\frac{\lambda}{\kappa} x_t$$

This is a targeting rule, without specifying instruments

After an inflationary cost-push shock the central bank allows the output gap to become negative

Optimal policy under commitment

Under (credible) commitment the CB is able
to influence expectations about future policy

The problem is now dynamic

$$\min \quad \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\,\mathrm{E}_0\left[\lambda x_t^2+\pi_t^2\right]$$

$$\text{subject to} \quad \pi_t=\beta\mathrm{E}_t\pi_{t+1}+\kappa x_t+e_t$$

Lagrangian

$$\mathcal{L}=\sum_{t=0}^{\infty}\beta^t\,\mathrm{E}_0\left[\frac{1}{2}\left(\lambda x_t^2+\pi_t^2\right)+\mu_t\left(\beta\pi_{t+1}+\kappa x_t+e_t-\pi_t\right)\right]$$

First order conditions

$$\begin{aligned} x_t \quad &: \quad \beta\mathrm{E}_0\left[\lambda x_t+\mu_t\kappa\right]=0 && \rightarrow && \mu_t=-\frac{\lambda}{\kappa}x_t \\ \pi_t \quad &: \quad \beta\mathrm{E}_0\left[\pi_t-\mu_t+\mu_{t-1}\beta\right]=0 && \rightarrow && \pi_t=\mu_t-\mu_{t-1} \end{aligned}$$

Optimal policy under commitment

For the current period the past is not a constraint ($\mu_{-1} = 0$)

$$\pi_0 = \mu_0 = -\frac{\lambda}{\kappa} x_0$$

Same as under discretion

For the future periods ($t \geq 1$) we get

$$\pi_t = \mu_t - \mu_{t-1} = -\frac{\lambda}{\kappa} (x_t - x_{t-1})$$

Different than for today: will take the past into account

Optimal commitment policy (OCP) means pursuing
a discretionary policy today, but promising
a non-discretionary policy from tomorrow on!

But when we'll arrive in the next period, we will be tempted
to act as in the current period: time inconsistency

Optimal policy under commitment

OCP is time inconsistent – solutions?

1. Appoint very credible central bankers
2. To build credibility, adopt a timeless perspective: pretend that OCP has been applied long ago and apply the formula for $t \geq 1$ from the beginning

Which is better: OCP or ODP?

- Neither invokes inflation bias
- ODP generates stabilization bias, making economy more volatile

The superiority of commitment calls for a credible, long-term arrangement for the central bank

Stabilization bias: discretion vs commitment

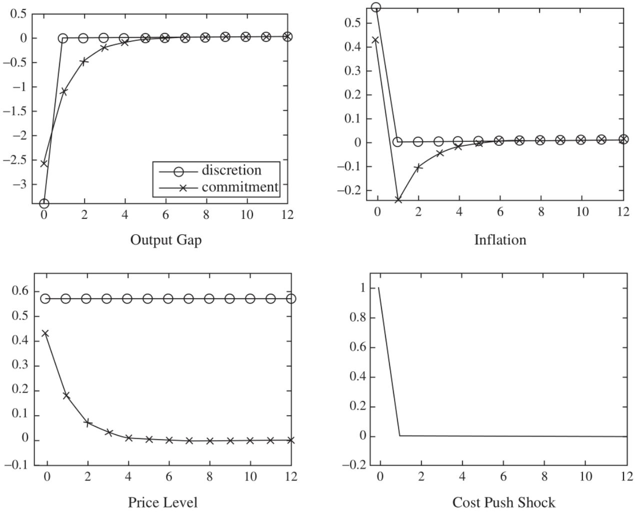


Figure 5.1 Optimal Responses to a Transitory Cost Push Shock

Gali (2008)

Stabilization bias: discretion vs commitment

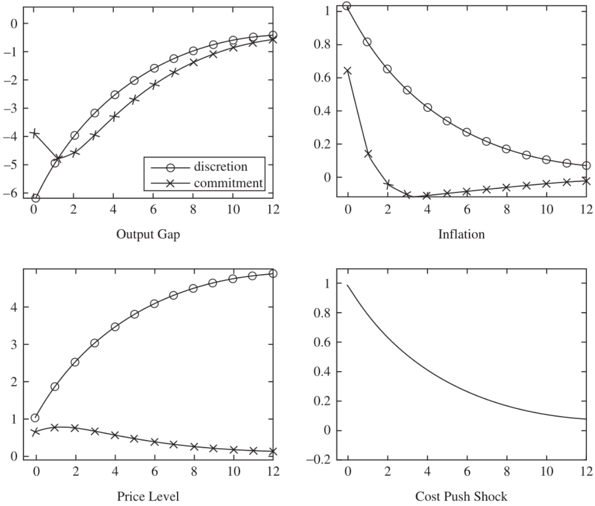


Figure 5.2 Optimal Responses to a Persistent Cost Push Shock

Gali (2008)

Digression: problems with identifying the NKPC

Recently, many papers have claimed that either:

1. NKPC has become more "flat", or
2. NKPC relationship has disappeared at all

McLeay and Tenreyro (2019): Under ODP, observed inflation will be unrelated to the measure of slack in the economy!

Assume the cost-push shock follows an AR(1) process

$$e_t = \rho_e \, e_{t-1} + \epsilon_t$$

Combining above with NKPC and ODP rule, we get

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda \, (1 - \beta \rho_e)} e_t \equiv \lambda \alpha \, e_t = \lambda \alpha \, (\rho_e \, e_{t-1} + \epsilon_t)$$

Digression: problems with identifying the NKPC

Under ODP inflation is a function of cost-push shock

$$\pi_t = \lambda\alpha\, e_t = \lambda\alpha\,(\rho_e\, e_{t-1} + \epsilon_t)$$

But this means that current inflation will be very well forecastable using past inflation

$$\pi_t = \rho_e\, \pi_{t-1} + \lambda\alpha\epsilon_t$$

Realized inflation and output gap will be negatively correlated

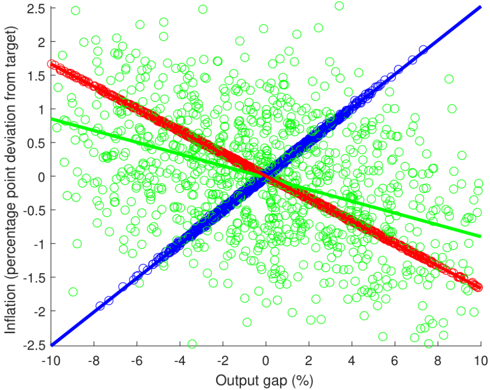
$$\pi_t = -\frac{\lambda}{\kappa} x_t \quad \rightarrow \quad \pi_t = b x_t + \varepsilon_t$$

$$\hat{b} < 0 \quad \text{and} \quad \mathrm{Corr}\,(x_t,\, \varepsilon_t) \neq 0$$

Additionally, if there are shocks to monetary policy rule, any estimate of $\hat{b}$ is possible, including $\hat{b} > 0$ and $\hat{b} \approx 0$!

Estimate for $\hat{b}$ depends on relative magnitude of shocks

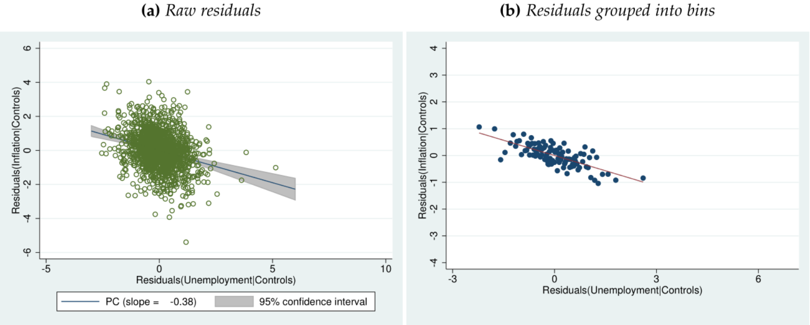
Figure 5: *Inflation/output gap correlation in model-simulated data: optimal discretion with shocks to the targeting rule*



McLeay and Tenreyro (2019)

One solution: Identification using regional data

Figure 12: *Year and metro area fixed effects: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*



Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (4) in table 3. See the notes to Figures 10a and 10b for details.

McLeay and Tenreyro (2019)

Instrument rules

We have been assuming that the CB can "choose" x_t and π_t

In reality, the CB can influence these variables indirectly, by e. g. changing the nominal interest rate

Recall the NKIS curve

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^*\right)$$

where the CB can affect output gap by varying i

Note that monetary policy by changing i
affects output gap "first" and inflation rate "second"

Determinacy concerns

Under ODP inflation and output gap react to cost-push shock

$$\pi_t = \lambda\alpha\, e_t \quad \text{and} \quad x_t = -\kappa\alpha\, e_t$$

The CB could vary the level of i to try implementing the ODP

$$-\kappa\alpha\, e_t = -\kappa\alpha\rho_e\, e_t - \frac{1}{\sigma}\left(i_t - \lambda\alpha\rho_e\, e_t - r_t^*\right)$$
$$i_t = r_t^* + \alpha\left(\kappa\sigma\left(1-\rho_e\right) + \lambda\rho_e\right) e_t$$

But then our forward-looking system would have multiple solutions, only one of which is consistent with ODP

Such instrument rule would be “too weak”

And would require observing e_t perfectly in real-time!

Taylor rules

Instead of constructing the instrument rule as function of shocks, construct the rule as function of endogenous variables

$$i_t = r_t^* + \gamma_\pi \, \pi_t$$

where $\gamma_\pi = (1 - \rho_e)\, \kappa\sigma/\alpha + \rho_e$

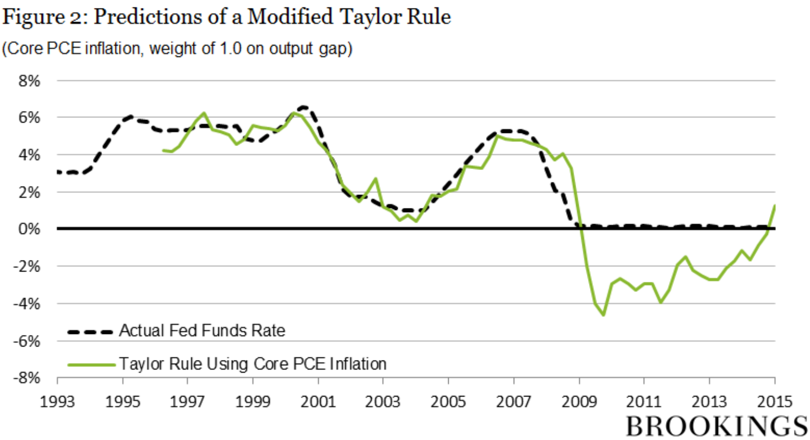
It can be shown that if only $\gamma_\pi > 1$, the system has a unique solution, and the CB can "select" the ODP equilibrium

A more general Taylor rule allows for reactions to output gap and smoothing of policy rate changes

$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i)\,(r + \gamma_\pi \, \pi_t + \gamma_x \, x_t)$$

A CB should raise the interest rate when inflation
is above target and / or output gap is positive

A Taylor rule can capture Fed's actual policy prior to 2009



Bernanke (2015)

Taylor principle and stability of inflation

Taylor principle: when inflation increases by 1 p.p.,
the CB should raise the interest rate by $\gamma_\pi > 1$ p.p.

Failure to do so results in inflation instability

The estimate for the pre-Volcker rule is significantly less than unity.

Monetary policy over this period was accommodating increases in expected inflation, in clear violation of the [Taylor principle – MB].

TABLE 1
ESTIMATES OF POLICY REACTION FUNCTION

	γ_π	γ_x	ρ
Pre-Volcker	0.83	0.27	0.68
	(0.07)	(0.08)	(0.05)
Volcker–Greenspan	2.15	0.93	0.79
	(0.40)	(0.42)	(0.04)

Clarida, Gali and Gertler (1999)

Role of expectations

Modern monetary policy: management of expectations

Woodford (2005, p. 3):
For not only do expectations about policy matter, but,
at least under current conditions, very little *else* matters

Basic three-equation New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

New Keynesian IS curve

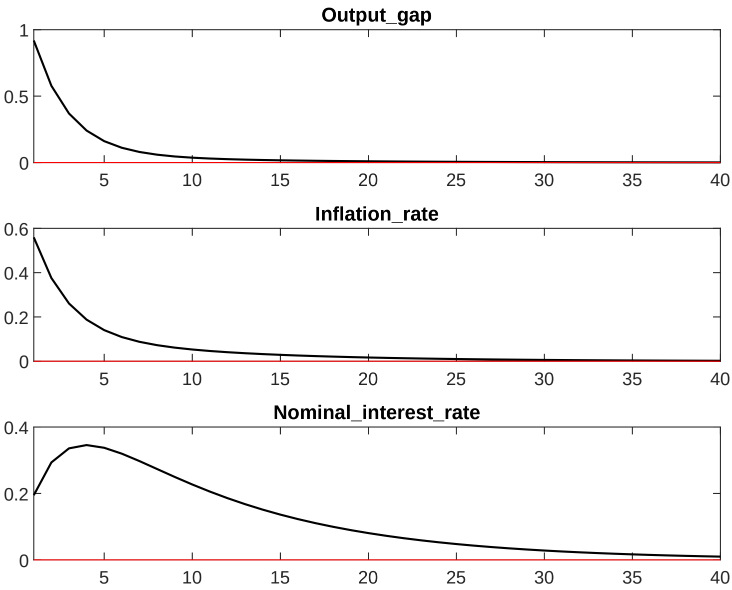
$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathrm{E}_t \pi_{t+1} - r^* \right) + u_t$$

Taylor rule

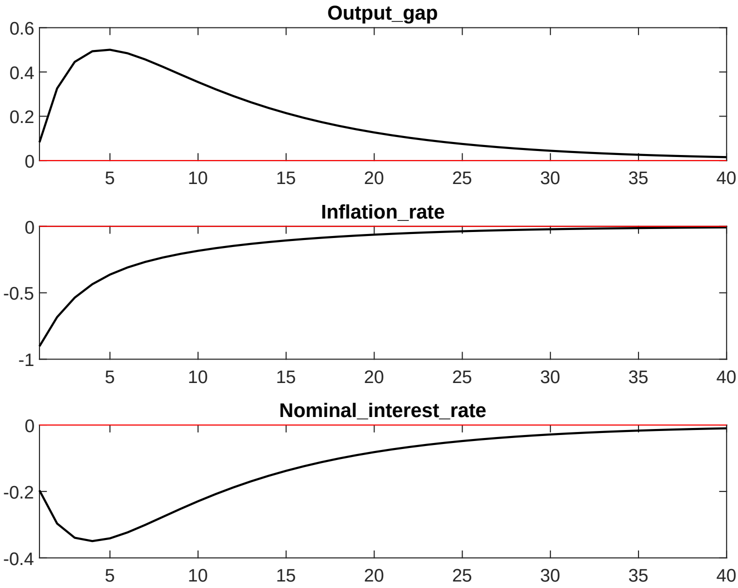
$$i_t = \rho_i \, i_{t-1} + (1 - \rho_i) \left(r^* + \gamma_\pi \, \pi_t + \gamma_x \, x_t \right) + v_t$$

where v is an exogenous monetary policy shock

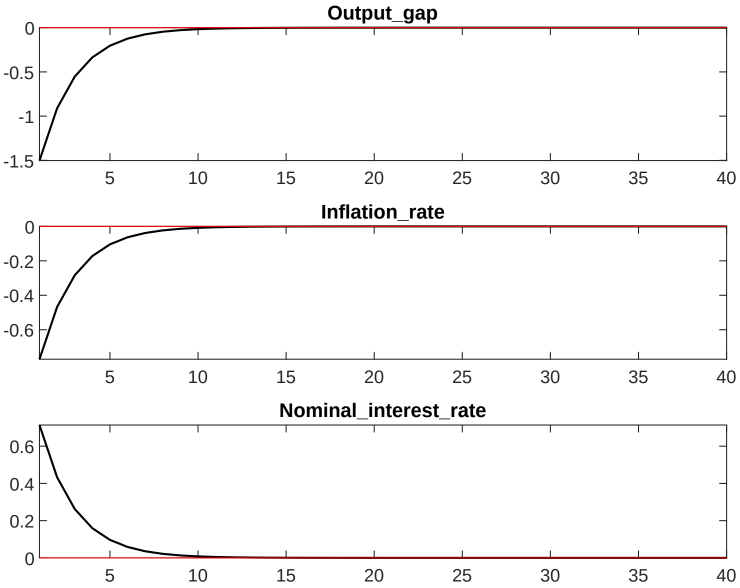
Positive demand shock



Positive cost-push shock



Negative monetary shock



Empirical performance of NK

All three shocks affect the economy in the desired direction

Basic model is too stylized to take it directly to data

Some standard extensions introduced to applied NK models:

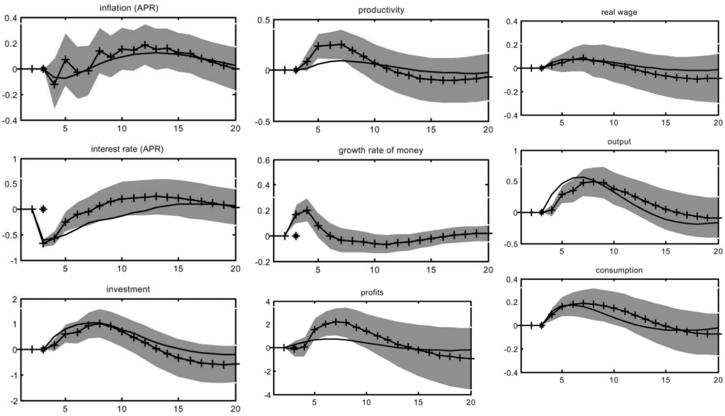
- Habits in the utility function add a backward-looking term to the IS equation
- Various mechanisms that generate Hybrid NKPC
- Investment adjustment costs delay the response of investment to shocks

More complicated extensions:

- Financial frictions (Bernanke, Gertler and Gilchrist 1999, Kiyotaki and Moore 1997, Iacoviello 2005)
- Unemployment (Gertler, Sala and Trigari 2008; Gali 2010)

Empirical fit of a medium-scale NK model

Model- and VAR-based impulse responses. Solid lines are benchmark model impulse responses; solid lines with plus signs are VAR-based impulse responses. Grey areas are 95 percent confidence intervals about VAR-based estimates.



Christiano, Eichenbaum and Evans (2005)

Monetary policy at the Effective Lower Bound

Liquidity trap

In response to the 2007-2008 financial crisis many central banks have cut interest rates to around 0%

But the desired level of interest rates was then negative

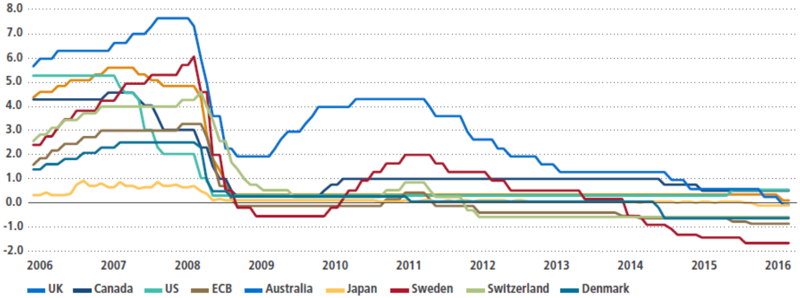
Since 2012 more and more central banks have set negative interest rates

These events raise several questions:

- Why (and where) is the Effective Lower Bound (ELB)
- How to conduct monetary policy in a liquidity trap
- What is then the optimal inflation target

Nominal interest rates in advanced economies

FIGURE 1: GLOBAL CENTRAL BANK RATES



Source: Bloomberg as of 17 October 2016

PIMCO

Zero / Effective Lower Bound (ZLB / ELB)

Intuition: lower bound for nominal interest rates is 0%

The nominal rate of return from cash is 0%

Households are not willing to hold assets that yield negative nominal return, prefer cash

In reality the ELB is not at 0, since there are costs of storing and securing cash

Ideas to eliminate the ZLB / ELB (Buiter 2009):

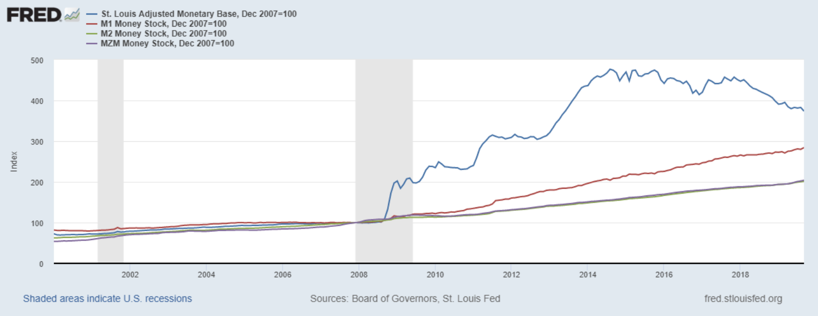
- Abandon cash (100% electronic money)
- Tax cash (need to extend its validity)
- Floating exchange rate between cash and electronic money

General public vehemently opposes these ideas

Monetary policy at the ELB

Reducing the nominal interest rate no longer available

Expansion in monetary base (higher demand for reserves) does not translate to increases in money supply aggregates



Federal Reserve Economic Database

Monetary policy at the ELB

What works then?

- Forward guidance
  Central bank manages expectations on future nominal interest rates to indirectly affect long-term interest rates
- Quantitative easing
  Central bank purchases certain assets to directly affect long-term interest rates

Both implemented in practice

Forward guidance

CB promises to maintain rates “lower for longer” than would be indicated by the monetary policy rule

Raises inflation expectations

Reduces expected real interest rates

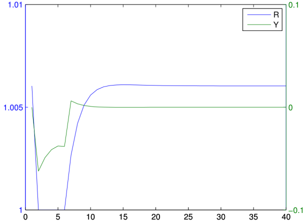
Expansionary effect on the economy

To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens. In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.

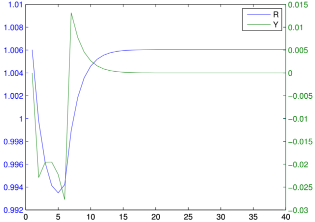
US FOMC (2012)

Effects of forward guidance in the New Keynesian model

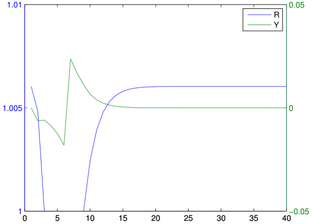
Zero Lower Bound



No bound



ZLB + forward guidance



Quantitative easing

Purchasing assets of longer maturities

Reduces rate of return on those assets

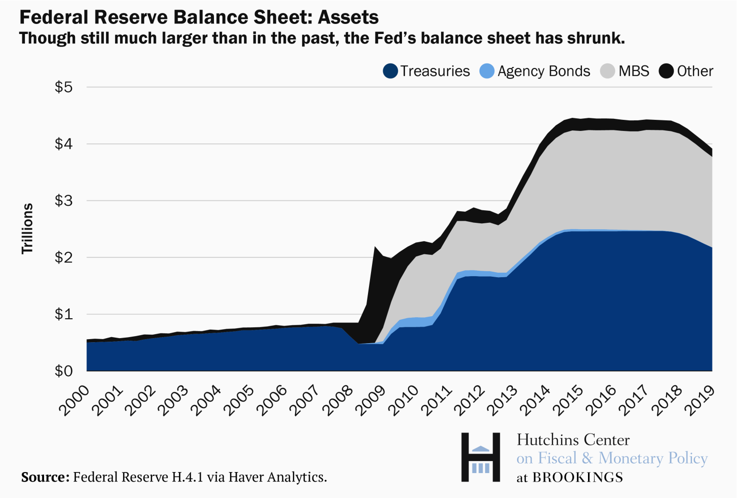
Reduces expected real interest rates

Expansionary effect on the economy

To support a stronger economic recovery and to help ensure that inflation, over time, is at the rate most consistent with its dual mandate, the Committee will continue purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of Treasury securities, and it is maintaining its existing policy of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities. These actions, which together will increase the Committee's holdings of longer-term securities by about $85 billion each month through the end of the year, should put downward pressure on longer-term interest rates, support mortgage markets, and help to make broader financial conditions more accommodative.

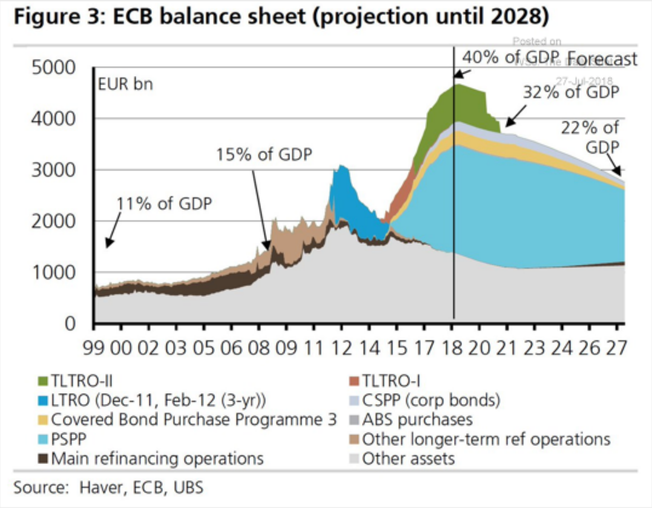
US FOMC (2012)

Quantitative easing: balance sheet of Fed



Ng and Wessel (2019)

Quantitative easing: balance sheet of ECB



WSJ: Bodorovsky (2018)

Limits to effectiveness of unconventional policies

In the New Keynesian model forward guidance works "too well" (Del Negro et al. 2015)

In reality effectiveness of these policies depends on frictions and imperfections in the financial markets

Information frictions: private agents have less than perfect information on future monetary policy (Campbell et al. 2019)

Market frictions: imperfect degrees of substitutability between different classes of assets (Haldane et al. 2016)

Four-equation New Keynesian model

Based on Friedman (2013)

Loan contracts have maturity longer that 1 period

Interest rates that households and businesses face are determined by the commercial banks

Central bank affects them only indirectly

Simple modification of the three-equation model:

- NKPC $\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t$
- NKIS $x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t^p - \mathrm{E}_t \pi_{t+} - r^*\right)$
- Taylor rule $i_t = r^* + \gamma_x x_t + \gamma_\pi \pi_t$

where $i_t^p \neq i_t$ is the interest rate relevant to the private spending decisions and $\mathrm{E}_t \pi_{t+}$ are expectations of inflation over the horizon corresponding to the maturity of loans

Fourth equation

The relation between the central bank interest rate and the private sector interest rate takes into account:

1. the default risk of private obligations
2. their longer maturity

$$i_t^p = (1 - \delta)\, i_t + \delta i_{t+}^e + \phi\, (R_t / A_t) + z_t$$

where:

- δ reflects maturity of the private sector assets
- R_t / A_t is the ratio of risky assets to total, $\phi' > 0$
- z_t is a shock in the financial markets

Unconventional policies in the four-equation model

Forward guidance

- Communicating low interest rates in the future lowers i^e_{t+} and i^p_t

Quantitative easing

- Buying risky assets from commercial banks lowers R_t/A_t and i^p_t
- Buying longer maturity assets increases δ and (under forward guidance) lowers i^p_t

Drop in the natural real interest rate

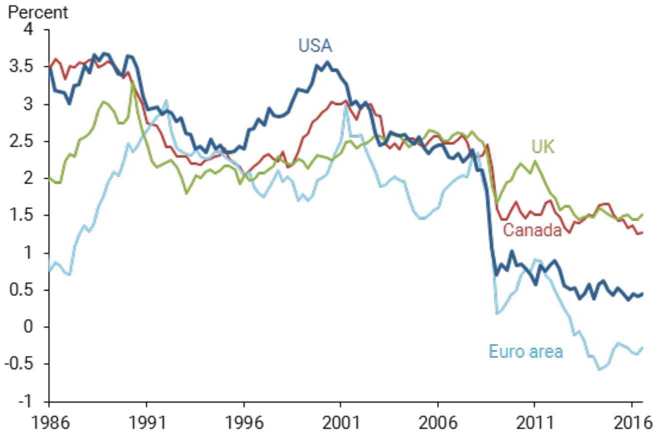
The ELB binds more often when the average nominal interest rates level is lower

Average nominal rate is the sum of inflation target and the natural real rate of interest

$$i = \pi^* + r^*$$

Due to demographics (aging populations) and lower rate of growth of productivity r^* has been falling in the advanced countries (Brand et al. 2018)

Drop in the natural real interest rate



Williams (2017)

Optimal inflation target

Without adjusting inflation targets advanced economies will experience ELB more often

- Blanchard et al. (2010) and Ball (2014): raise inflation target 4%: still low costs of inflation, ELB less often
- Coibon et al. (2012): even accounting for ELB costs the inflation target should be low
- Andrade et al. (2018): inflation target should be raised by 0.9 p.p. in response to decrease of r^* by 1 p.p.
- Ascari and Sbordone (2014): higher inflation leads to indeterminacy, caution advised

Question still remains open

Monopolistic competition: full solution

Derivative of Y_i w.r.t. P_i

$$\frac{\partial Y_i}{\partial P_i} = -\frac{1+\mu}{\mu}\left(P/P_i\right)^{(1+\mu)/\mu} Y/P_i = -\frac{1+\mu}{\mu}\frac{Y_i}{P_i}$$

First order condition

$$\frac{\partial \Pi_i}{\partial P_i} = Y_i + (P_i - MC)\cdot\frac{\partial Y_i}{\partial P_i} = 0$$

$$Y_i + (P_i - MC)\cdot -\frac{1+\mu}{\mu}\frac{Y_i}{P_i} = 0$$

$$Y_i = \frac{1+\mu}{\mu}(P_i - MC)\frac{Y_i}{P_i} \quad \rightarrow \quad \frac{\mu}{1+\mu}P_i = P_i - MC$$

$$MC = P_i\left(1 - \frac{\mu}{1+\mu}\right) = \frac{1}{1+\mu}P_i$$

back

Dynamics of prices and inflation: algebra

Price level dynamics

$$p_t = \theta p_{t-1} + (1-\theta)\,\tilde{p}_t$$
$$\tilde{p}_t = \frac{1}{1-\theta}\,(p_t - \theta p_{t-1})$$

The new reset price is given by

$$\tilde{p}_t = (1-\beta\theta)\,p_t^* + \beta\theta \mathrm{E}_t \tilde{p}_{t+1}$$
$$\tilde{p}_t = (1-\beta\theta)\,p_t^* + \beta\theta \mathrm{E}_t \left[\frac{1}{1-\theta}\,(p_{t+1} - \theta p_t)\right]$$
$$\tilde{p}_t = (1-\beta\theta)\,p_t^* + \beta\theta \mathrm{E}_t \left[\frac{1}{1-\theta}\,(p_{t+1} - p_t + (1-\theta)\,p_t)\right]$$
$$\tilde{p}_t = (1-\beta\theta)\,p_t^* + \frac{\beta\theta}{1-\theta}\mathrm{E}_t\left[p_{t+1} - p_t\right] + \beta\theta p_t$$
$$\tilde{p}_t = (1-\beta\theta)\,(p_t^* - p_t) + p_t + \frac{\beta\theta}{1-\theta}\mathrm{E}_t \pi_{t+1}$$

Dynamics of prices and inflation: algebra

Inflation rate

$$
\begin{aligned}
p_t &= \theta p_{t-1} + (1-\theta)\,\tilde{p}_t \\
\theta p_t + (1-\theta)\,p_t &= \theta p_{t-1} + (1-\theta)\,\tilde{p}_t \\
\theta\,(p_t - p_{t-1}) &= (1-\theta)\,(\tilde{p}_t - p_t)
\end{aligned}
$$

$$
\begin{aligned}
\theta \pi_t &= (1-\theta)\left[(1-\beta\theta)\,(p_t^* - p_t) + \frac{\beta\theta}{1-\theta}\mathrm{E}_t \pi_{t+1}\right] \\
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\theta)\,(1-\beta\theta)}{\theta}\,(p_t^* - p_t)
\end{aligned}
$$

back