



UNIVERSITY OF WARSAW

Faculty of Economic Sciences

Financial Frictions

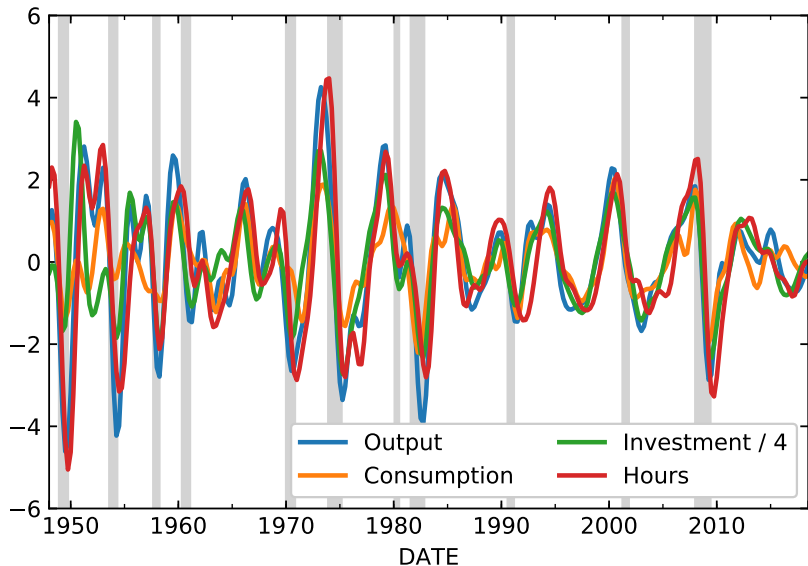
Advanced Macroeconomics QF: Lecture 13

Marcin Bielecki

Fall 2018

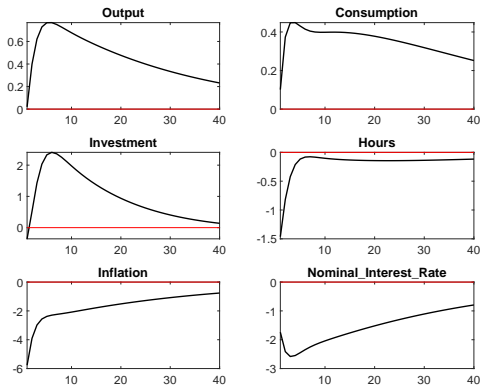
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Comovement of four main variables over business cycle



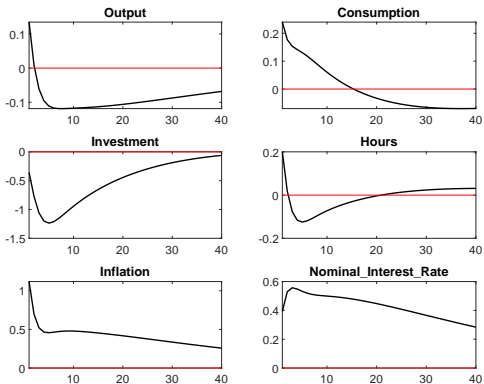
Effects of shocks in New Keynesian Model

Productivity shock



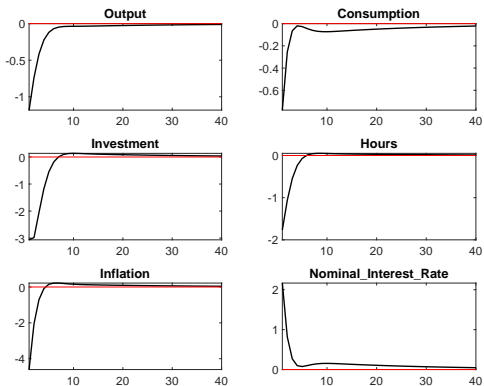
Effects of shocks in New Keynesian Model

Demand (“impatience”) shock



Effects of shocks in New Keynesian Model

Monetary shock

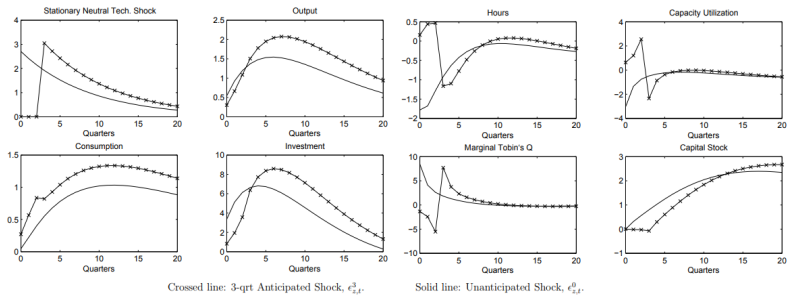


Effects of shocks in New Keynesian Model

- By introducing price stickiness to the model, we have lost the ability to generate comovement of output, consumption, investment and hours as a result of productivity shocks
- Logic: in the short runs firm “cannot” change prices while demand schedule remains quite stable
→ keep output constant & reduce employment (hours)
- Demand shocks do not solve the issue
→ consumption and investment move in opposite directions
- Monetary shocks affect all 4 variables in the desired directions, but are very small in the data → cannot be the main driver

News shocks

- Shocks do not have to always surprise agents at time t
- Imagine agents can get a “signal” on future government spending or future productivity
- While the fundamentals today haven't changed it is optimal to act on anticipated future conditions
- Both consumption (via Euler equations) and investment (via capital adjustment costs) go up \rightarrow output increases
- Today's productivity unchanged \rightarrow hours go up as well
- Even if expectations turn out to be incorrect, we have generated a boom (and subsequent bust)
 \rightarrow waves of optimism / pessimism can drive the business cycle



Schmitt-Grohe and Uribe (2008)

Liquidity demand (risk premium) shocks

- Imagine agent derive utility from holding riskless bonds b

$$U = E_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) + \varrho_t b_{t+1})$$

- Budget constraint (capital k has risky return)

$$c_t + q_t k_{t+1} + b_{t+1} = w_t + R_t^k q_{t-1} k_t + R_t b_t$$

- Two Euler equations: one for risky and one for riskless asset

$$u'(c_t) = \beta E_t [u'(c_{t+1}) R_{t+1}^k]$$

$$u'(c_t) = \beta E_t [u'(c_{t+1}) R_{t+1}] + \varrho_t$$

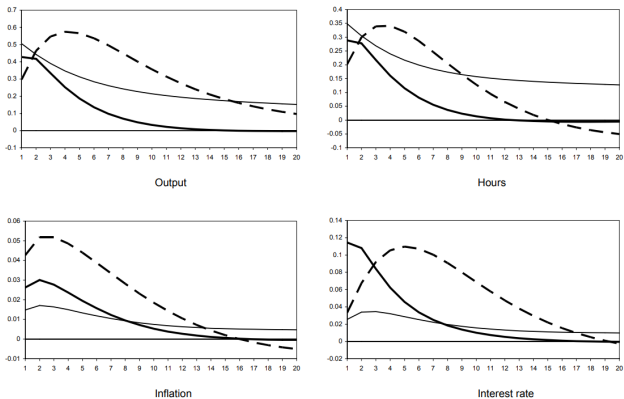
- Wedge between returns on assets \rightarrow generates risk premium

$$E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} (R_{t+1}^k - R_{t+1}) \right] = \frac{\varrho_t}{u'(c_t)}$$

- Increase in ϱ_t induces precautionary saving in riskless assets
 - drop in investment and consumption
 - drop in output and hours worked

Liquidity demand (risk premium) shocks

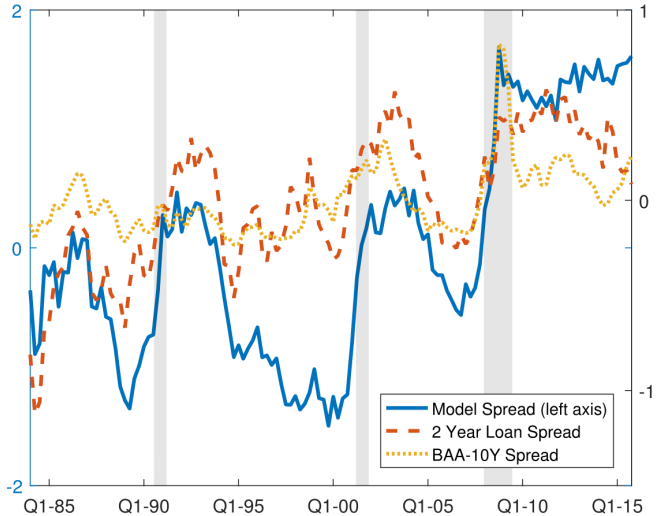
Figure 2: The estimated mean impulse responses to “demand” shocks



Notes: Bold solid line: risk premium shock; thin solid line: exogenous spending shock; dashed line: investment shock.

Smets and Wouters (2007)

Liquidity demand (risk premium) shocks

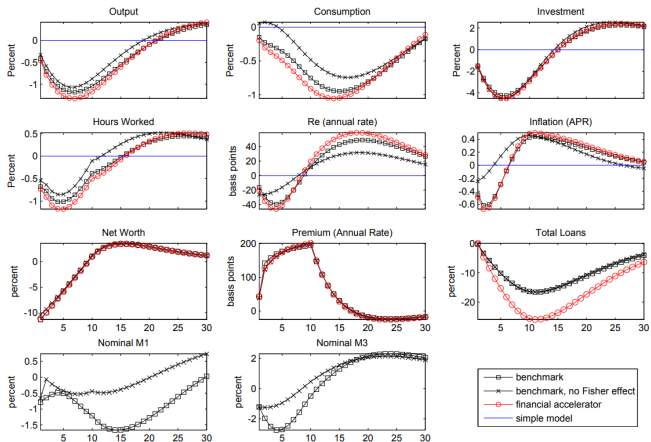


Anzoategui et al. (2016)

- Discussed later in the lecture

Firm riskiness shocks

Figure 12b: US, Impulse Response to a Riskiness Shock



Christiano, Motto and Rostagno (2009)

“To motivate interest in a paper on financial factors in business fluctuations it used to be necessary to appeal either to the Great Depression or to the experiences of many emerging market economies. This is no longer necessary.”

– Gertler and Kiyotaki (2010)

Financial frictions¹

- Crisis detected important channels of the monetary/credit transmission mechanism
- Matter for our understanding of driving forces in the economy
- Place discussion of financial variables within a consistent framework
- Paramount when dealing with issues related to financial stability
- Interactions between macroprudential and monetary policy
- Brunnermeier, Eisenbach and Sannikov (2012)
Macroeconomics with Financial Frictions: A Survey

¹The following slides were adapted from [Leif Brubakk's lecture](#)

Pre-crisis DSGE models

- General equilibrium models with financial frictions before the financial crisis: Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Iacoviello (2005)
- Financial frictions were largely absent from DSGE models used by central banks at the time of the financial crisis
- Usual simplifications of financial mechanisms
 - Modigliani-Miller theorem holds
 - balance sheet positions do not affect firms' decisions
 - Financial markets' state summarized by one interest rate
 - No systematic heterogeneity (in terms of discounting)
 - no borrowing and lending in equilibrium
- Feedback from financial markets to real economy hard to analyze
- Many central banks have now implemented financial frictions/spillovers in core DSGE models

Shortcomings of standard DSGE models

- “Linear” framework (Taylor expansions around steady state)
 - Abstracts from nonlinearities (e.g. ZLB constraint) and higher order effects of risk / uncertainty
 - Not suitable for analyzing precautionary saving / hoarding
 - Or movements in asset prices, where risk is an important factor
- Rational expectations
 - Make the existence of bubbles unlikely (but not impossible)
 - Analyzing non-fundamental developments is hard

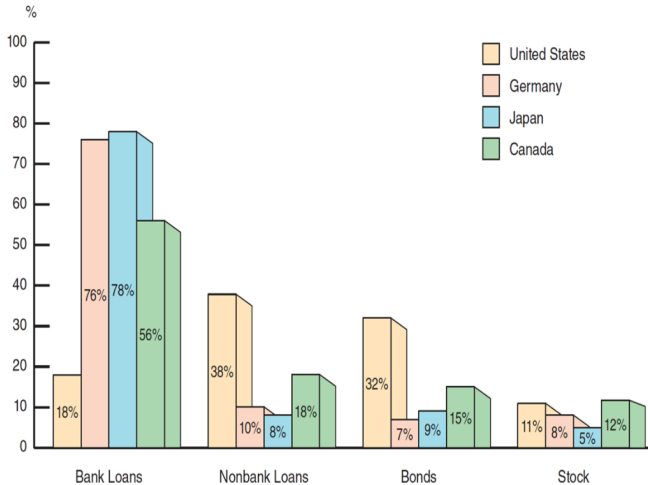
Financial market frictions

- Due to asymmetric information, lenders will generally require that borrowers post collateral and / or pay a credit premium to obtain funding
- There is a large literature on optimal financial contracts that link the net worth of borrowers to their access to credit
- Since net worth is determined by assets in place, movements in asset prices will determine agents' access to credit
- When asset values increase, the borrower's stake in the project increases, implying that the incentives to default decreases, leading in turn the lender to reduce the finance premium
- Establishes direct link between asset prices and credit
- Hence, financing costs become countercyclical, which will typically strengthen the effects of a given shock

- Firms finance assets with some mix of equity and debt (liabilities)
 - Non-financial firms: real assets (capital)
 - Financial firms: financial assets (contractual claims)
- Non-financial firms rely (at least partially) on external funds:
 - Direct: get funds directly from a lender or equity investor
 - Indirect: get funds from a financial intermediary (e.g. a bank)
- Indirect finance is mostly comprised of debt contracts, whereas direct finance could either be debt or equity
- Indirect finance is typically more important than direct finance, and bank loans are really important
- This is why financial structure is relevant for monetary policy
 - so much depends on banking

²The following slides were adapted from [Eric Sims' lecture](#)

Sources of External Funding



Eric Sims' lecture

Asymmetric Information

- Generally refers to a situation in which different parties to a transaction are not equally informed about characteristics or actions of the other parties to the transaction
- Two main kinds of asymmetric information:
 1. **Adverse Selection** occurs before a transaction takes place
 2. **Moral Hazard** occurs after a transaction takes place
- Both can help us understand the kind of financial structure we observe in the real world – in particular, why indirect finance is so important (and why financial intermediation is important)

Adverse Selection

- The buyer of a product (e.g. a car, a stock) doesn't know the true "type" of the product (e.g. good or bad, risky or safe)
- Buyer a priori knows only the average type
- Hence, buyer will only be willing to pay the average valuation, which is more than the bad type but less than the good type
- This tends to drive sellers of products that are a good type away and attract sellers of products that are a bad type
- But then buyer knows this, and entire market can fall apart
- Classic example: "lemons" in the market for used cars

Lemons Example

- Akerlof (1970): two types of used cars: lemons and peaches
- Sellers know whether they have a lemon or a peach, but buyers only know the fraction of lemons and peaches out there
- Suppose each type has the following valuations:

$$V_{\text{buyer}}(\text{peach}) = 20\,000 \quad V_{\text{buyer}}(\text{lemon}) = 15\,000$$

$$V_{\text{seller}}(\text{peach}) = 18\,000 \quad V_{\text{seller}}(\text{lemon}) = 13\,000$$

- Suppose that when meeting a seller the buyer only knows there is a 50% chance it's a peach, and 50% chance it's a lemon
- The average valuation for the buyer is 17 500, less than a peach owner's valuation \rightarrow peaches will not be sold
- But then the buyer will know only lemons are on the market
- Only lemons will sell for a price between 13 000 and 15 000
- "Good" cars driven out of the market by presence of "bad" cars!

Dealing with the Lemons Problem

- Most used car deals are through dealerships, not person-to-person transactions
- The dealership serves as an intermediary and helps solve the informational problem
- The dealership gets good at determining lemons vs. peaches, and can offer warranties to buyers to ensure that the buyer isn't dealing with a lemon
- Intermediaries who specialize in resolving informational asymmetry problem naturally arise in the car market
- Similarly in financial markets

Lemons Problem and Indirect Finance

- Lemons problem helps us understand why direct finance is not that important for external funds
- The firms who most want funds are probably those who are the worst type
- Savers know this and will not buy debt directly from firms
- Financial intermediaries (e.g. banks) can step in
- They can become experts in learning about firms and can alleviate the informational asymmetry problems
- Because these financial intermediaries make private loans they can avoid the free rider problem that arises when third party firms try to produce information about firms

Example: Risky and Safe Firms

- Two types of firms who need 1 unit to undertake a project
- Project succeeds or fails with a given probability
- Firm types and payoffs are:

	Safe Firm	Risky Firm
Payoff in "good" state	4	8
Payoff in "bad" state	0	0
Prob. of "good" state	1/2	1/4

- Expected return the same for both firms, but lender would prefer to loan to safe firm since it is less risky

Debt Contract

- Only one kind of debt contract: bank lends firm one unit, firm promises to repay R (gross) units if project succeeds, 0 otherwise (it can't pay back in event low state occurs)
- Borrower only has to pay back in the good state
- Borrower's expected payoffs are:

$$\text{Safe} = \frac{1}{2} (4 - R)$$

$$\text{Risky} = \frac{1}{4} (8 - R)$$

- Borrower takes a loan only if her expected payoff is non-negative
- If $R > 4$, safe firms won't take the loan
- If $R > 8$, both firms won't take the loan

Lender's Problem

- Lender's opportunity cost of funds is 1
→ needs to earn at least 1 in expectation
- If lender charges $R > 8$, she makes no loan and “earns” 1 (i.e. keeps its money)
- If she charges $R \leq 4$, both types of firms will take the loan
- If it charges $R > 4$, only risky firms will take the loan
- Suppose fraction q of firms are risky, and $1 - q$ are safe
- Expected lender payoff:

$$R \leq 4 \quad : \quad E[\text{payoff}] = (1 - q) \cdot \frac{1}{2}R + q \cdot \frac{1}{4}R$$

$$4 < R \leq 8 \quad : \quad E[\text{payoff}] = \frac{1}{4}R$$

Pooling vs. Separating Equilibrium

- A **pooling equilibrium** emerges for such R in which both types of firms take a loan
- A **separating equilibrium** emerges for such R in which only one type of firms gets a loan
- Look first for a pooling equilibrium to see if one exists
- Can write lender's expected payoff as:

$$E[\text{payoff}] = \left(\frac{1}{2} - \frac{1}{4}q \right) R$$

- Suppose $q = 0.8$, so most firms are risky. Expected payoff must be bigger or equal to 1. Solve for the “break-even” R :

$$R \geq 1 / (0.5 - 0.2) = 3\frac{1}{3}$$

- So $3\frac{1}{3} \leq R \leq 4$ would be a pooling equilibrium, while $4 < R \leq 8$ would be a separating equilibrium

Equilibrium

- A pooling equilibrium exists for $3\frac{1}{3} \leq R \leq 4$
- A separating equilibrium exists for $4 < R \leq 8$
- Don't know which equilibria we'll end up at
- But if it's separating equilibrium, safe firm doesn't get a loan, which is a bad outcome relative to symmetric information case
- If it's pooling equilibrium, interest rate charged to safe firm may be "too high" relative to symmetric information case and interest rate charged to risky firm is "too low"
- This will tend to over-attract risky firms and deter safe firms from getting loans

Adverse Selection and Collateral

- Collateral is an important feature of many debt contracts
- A firm receiving funds pledges some collateral that can be seized in the event that the firm defaults
- Banks can offer different kinds of contracts:
 - some require posting more collateral
 - some require less collateral but charge higher interest rates
- This offering different kinds of contracts can get firms to voluntarily reveal their type
- In mortgage finance: the more you put down (more collateral), the better the terms on the loan typically
- Collateral can be a useful way for financial contracts to deal with information asymmetry
- But when collateral loses value (“bubble bursting”) this can exacerbate information asymmetry problems

Loan Contracts with Collateral

- Lender requires borrower to post collateral C , which borrower has to pay in the event of project failure
- Borrower's expected payoffs are:

$$\text{Safe} = \frac{1}{2}(4 - R) - \frac{1}{2}C$$

$$\text{Risky} = \frac{1}{4}(8 - R) - \frac{3}{4}C$$

- R “hurts” the safe firm more, but C “hurts” the risky firm more
- Suppose that the lender posts two contracts, one without collateral ($R, C = 0$) and the other with collateral ($R_C, C > 0$)
- Safe firm chooses to post collateral and reveals itself as safe

Requirements

- If lender seizes collateral, a fraction d goes bad, so lender only recovers $(1 - d) C$
- Think of this as a “bankruptcy cost”
- For this to work, we must have the following hold:
 1. Risky firm prefers no collateral contract
 2. Safe firm prefers posting collateral
 3. Lender breaks even (or better) on both contracts

- Risky firm prefers no collateral

$$\frac{1}{4}(8 - R_C) - \frac{3}{4}C \leq \frac{1}{4}(8 - R)$$
$$R - R_C \leq 3C$$

1. $R > R_C$ (you get a lower interest rate if you post collateral)
 2. Collateral must be big enough to induce risky firm to take R , $C = 0$
- Safe firm prefers posting collateral

$$\frac{1}{2}(4 - R_C) - \frac{1}{2}C \geq \frac{1}{2}(4 - R)$$
$$R - R_C \geq C$$

1. $R > R_C$ again
2. Collateral can't be too big, otherwise safe firm won't post it

All Conditions

- Lender must at least break even on both contracts

$$\frac{1}{4}R \geq 1$$

$$\frac{1}{2}R_C + \frac{1}{2}(1-d)C \geq 1$$

- Everything together

$$R - R_C \leq 3C$$

$$R - R_C \geq C$$

$$R \geq 4$$

$$R_C + (1-d)C \geq 2$$

- Multiple equilibria (possibly depending on bargaining power)
- One possibility: lender just breaks even on both contracts (perfect competition among lenders) and risky firm is indifferent between contracts (weakly prefers the no-collateral contract)

Solution (perfect competition among lenders)

- Lender just breaks even:

$$R = 4 \quad \text{and} \quad R_C + (1 - d)C = 2$$

- Let $d = 1/4$:

$$3C = 8 - 4R_C$$

- Risky firm just breaks even:

$$4 - R_C = 3C = 8 - 4R_C$$

- Resulting in:

$$R_C = \frac{4}{3} \quad \text{and} \quad C = \frac{8}{9}$$

- Firms voluntarily separate into different loan contracts

Financial Accelerator and Business Cycles

- Posting of collateral allows safe firms to reveal their type
- Allows them to get loans (depending on market structure) and results in more efficient allocation
- Since collateral consists of assets, asset price fluctuations can affect ability of firms to get loans
- **Financial accelerator:**
 1. Decline in economic activity causes assets to lose value
 2. Declining asset values makes it harder for firms to post collateral
 3. Inability to post collateral → stronger adverse selection problem
→ less investment and/or a worse allocation of investment between safe and risky firms
 4. Less investment causes more declines in economic activity, and further falls in collateral values
 5. An adverse feedback loop! (although not without end)

Moral Hazard

- Information asymmetry occurring after a transaction takes place
- For example, someone lends you money, but then can't perfectly monitor what you do with the money
- Because of limited liability, you have an incentive to "gamble" with someone else's money
- Moral hazard can encourage excessive risk taking
- Can be applied to insurance markets too: once you have insurance, you have less incentive to behave safely
- Insurers know that, and may not sell you insurance in first place
- Just like adverse selection, markets can break down

Moral Hazard and Financial Structure

- Moral hazard can also help us understand why indirect finance is more important than direct finance
- Intermediaries (e.g. banks) become experts in monitoring the behavior of borrowers in a way that wouldn't be possible with direct finance: leads to less “gambling” and explains why loan contracts often include covenants restricting behavior
- Also helps make sense of preference of debt over equity
- With equity, lender needs to monitor profits all the time (since equity owner is due her share of profits)
- With debt, since payments are fixed, only need to monitor behavior of firm in event of default
- Lower monitoring costs (**costly state verification**) with debt over equity

Moral Hazard and Collateral

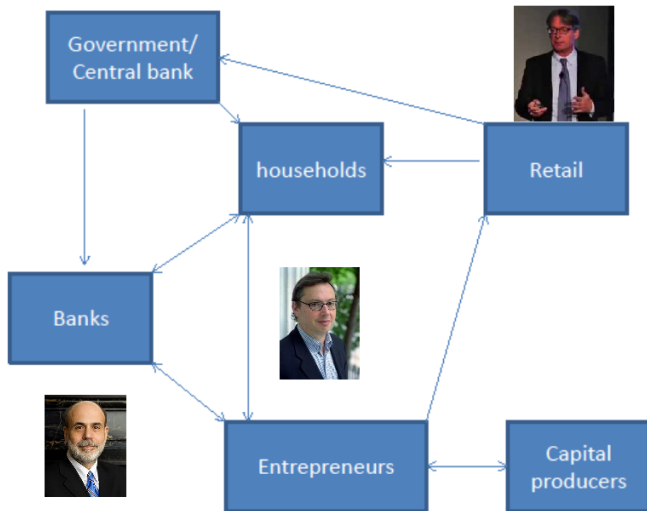
- Collateral also plays a role in mitigating moral hazard
- Requiring firms to post collateral gives them some “skin in the game” and encourages good behavior
- Without collateral, lenders may be reluctant to lend because they can't perfectly control what borrowers do
- Similarly to adverse selection, this importance of collateral can give rise to a financial accelerator mechanism
- Assets decline in value → harder to post collateral, → harder for firms to get loans

Costly state verification³

- Townsend (1979), Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2004, 2014)
- Entrepreneurs borrow funds to finance risky projects
- Project outcome is known ex post to the entrepreneur only
→ information asymmetry
- If the project outcome is low entrepreneur defaults on the loan
- But successful entrepreneurs are tempted to default as well
→ moral hazard
- Verification of the project success by outsiders is costly
- Optimal contract: fixed rate loan, verification in case of default
- Endogenous premium on loans

³The following slides were adapted from [Leif Brubakk's lectures](#)

Bernanke, Gertler and Gilchrist (1999) model



Bernanke, Gertler and Gilchrist (1999) model

- Bernanke, Gertler and Gilchrist (1999)
- New Keynesian model with entrepreneurial sector which requires external funding to invest in new projects
- Entrepreneurs identical up to an idiosyncratic productivity shock ω
- Funds provided by intermediary sector financed from household deposits
- Asymmetric information, costly state verification
- External finance premium (credit premium) is a decreasing function of the share of project financed by net worth (equity)
- Net worth equals retained profits by surviving entrepreneurs
- Gives rise to a financial accelerator mechanism

Utility maximization problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \phi \frac{h_t^{1+\eta}}{1+\eta} \right)$$

$$\text{subject to } P_t c_t + P_t g_t + D_t = W_t h_t + R_{t-1} D_{t-1} + Div_t$$

Lagrangian

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \phi \frac{h_t^{1+\eta}}{1+\eta} + \lambda_t [w_t n_t + (R_{t-1}/\Pi_t) d_{t-1} + div_t - c_t - d_t - g_t] \right]$$

FOCs

$$c_t : c_t^{-\sigma} = \lambda_t$$

$$h_t : \phi h_t^{\eta} = \lambda_t w_t$$

$$d_t : \lambda_t = \beta E_t [\lambda_{t+1} (R_t/\Pi_{t+1})]$$

Define $\Lambda_{t,t+j} = \lambda_{t+j}/\lambda_t$

Profit maximization problem under perfect competition

$$\begin{aligned} \max \quad & P_t y_t - \int_0^1 P_t(i) y_t(i) di \\ \text{subject to} \quad & y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\mu}} di \right)^{\mu} \end{aligned}$$

Resulting in

$$\begin{aligned} y_t(i) &= \left(\frac{P_t(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} y_t \\ P_t &= \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}} di \right)^{1-\mu} \end{aligned}$$

Cost minimization problem

$$\begin{aligned} \min \quad & w_t h_t(i) + r_t^k k_{t-1}(i) \\ \text{subject to} \quad & y_t(i) = z_t k_{t-1}(i)^\alpha h_t(i)^{1-\alpha} \end{aligned}$$

Resulting in

$$\begin{aligned} w_t &= mc_t (1 - \alpha) z_t k_{t-1}^\alpha h_t^{-\alpha} \\ r_t^k &= mc_t \alpha z_t k_{t-1}^{\alpha-1} h_t^{1-\alpha} \end{aligned}$$

Intermediate goods producers II

Profit maximization problem

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_{0,t} \left[\left(\frac{\tilde{p}_0(j)}{\Pi_{0,t}} - mc_t \right) y_t(j) \right] \\ \text{subject to} \quad & y_t(j) = \left(\frac{\tilde{p}_0(i)}{\Pi_{0,t}} \right)^{\frac{\mu}{1-\mu}} y_t \end{aligned}$$

Resulting in

$$\tilde{p}_0(j) = \mu \frac{E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \lambda_t mc_t y_t \Pi_{0,t}^{\frac{\mu}{\mu-1}}}{E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \lambda_t y_t \Pi_{0,t}^{\frac{1}{\mu-1}}}$$

$$\tilde{p}_t = \mu \frac{Num_t}{Den_t}$$

$$Num_t = \lambda_t mc_t y_t + \beta\theta E_t \Pi_{t+1}^{\frac{\mu}{\mu-1}} Num_{t+1}$$

$$Den_t = \lambda_t y_t + \beta\theta E_t \Pi_{t+1}^{\frac{1}{\mu-1}} Den_{t+1}$$

$$\Pi_t^{\frac{1}{1-\mu}} = \theta + (1-\theta) (\tilde{p}_t \Pi_t)^{\frac{1}{1-\mu}}$$

$$\Delta_t = \theta \Delta_{t-1} \Pi_t^{\frac{\mu}{\mu-1}} + (1-\theta) \tilde{p}_t^{\frac{\mu}{1-\mu}}$$

$$y_t = z_t k_t^\alpha h_t^{1-\alpha} / \Delta_t$$

$$\ln g_t = (1-\rho_g) \ln (\bar{g} / \bar{y}) + \rho_g \ln g_{t-1} + \varepsilon_{g,t}$$

$$R_t = R_{t-1}^{\gamma_R} \left(\bar{R} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\gamma_\pi} \left(\frac{y_t}{\bar{y}} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp(\varepsilon_{R,t})$$

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t}$$

Capital goods producers

Profit maximization problem

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t} [q_t (k_t - (1 - \delta) k_{t-1}) - i_t] \\ \text{subject to} \quad & k_t = (1 - \delta) k_{t-1} + \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right) i_t \end{aligned}$$

where $q_t = Q_t/P_t$ is the real price of capital goods
and κ measures cost of adjusting investment

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{0,t} \left[q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right) i_t - i_t \right]$$

Resulting in

$$\begin{aligned} 1 = & q_t \left(1 - \frac{\kappa}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \right) \\ & + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right] \end{aligned}$$

Take loans to finance purchases of raw capital

$$L_t(i) = Q_t k_t(i) - V_t(i) \geq 0$$

where $L_t(i)$ denotes loan and $V_t(i)$ net worth of i -th entrepreneur

Entrepreneurs transform raw capital $k_t(i)$ into productive capital $\omega_{t+1}(i) k_t(i)$, with pdf Φ of ω satisfying $E[\omega] = 1$

Gross nominal return on capital

$$R_{t+1}^E(i) = \frac{R_{t+1}^K \omega_{t+1}(i) k_t(i) + Q_{t+1} (1 - \delta) \omega_{t+1}(i) k_t(i)}{Q_t k_t(i)}$$

$$R_{t+1}^E = \frac{R_{t+1}^K + Q_{t+1} (1 - \delta)}{Q_t}$$

$$R_{t+1}^E(i) = \omega_{t+1}(i) R_{t+1}^E$$

Optimal contract: loan size $L_t(i)$ and gross non-default interest rate R_{t+1}^L

$$\bar{\omega}_{t+1} R_{t+1}^E Q_t k_t(i) = R_{t+1}^L L_t(i)$$

Entrepreneurs with ω below the threshold level $\bar{\omega}$ go bankrupt

All their resources are taken over by the banks,
after they pay proportional monitoring costs ψ

Perfect competition in banking sector \rightarrow zero profits (in expectation)

$$(1 - F_{t+1}) R_{t+1}^L L_t + (1 - \psi) G_{t+1} R_{t+1}^E Q_t k_t = R_t L_t$$

where

$$F_{t+1} = \int_0^{\bar{\omega}_{t+1}} d\Phi_{t+1}(\omega)$$
$$G_{t+1} = \int_0^{\bar{\omega}_{t+1}} \omega d\Phi_{t+1}(\omega)$$

F is the share of firms that go bankrupt, G is their average productivity

Equivalently

$$R_{t+1}^E Q_t k_t [\bar{\omega}_{t+1} (1 - F_{t+1}) + (1 - \psi) G_{t+1}] = R_t L_t$$

Optimal contract

Entrepreneur return on equity maximization (s.t. banks' zero profits)

$$\max E_t \left[\frac{\int_{\bar{\omega}_{t+1}}^{\infty} [R_{t+1}^E Q_t k_t(i) \omega - R_{t+1}^L L_t(i)] d\Phi_{t+1}(\omega)}{R_t V_t(i)} \right]$$

Resulting in

$$E_t \left[\frac{\frac{R_{t+1}^E}{R_t} [1 - \bar{\omega}_{t+1} (1 - F_{t+1}) - G_{t+1}] + \frac{1 - F_{t+1}}{1 - F_{t+1} - \psi \bar{\omega}_{t+1} F'_{t+1}} \left[\frac{R_{t+1}^E}{R_t} (\bar{\omega}_{t+1} (1 - F_{t+1}) + (1 - \psi) G_{t+1}) - 1 \right]}{R_t} \right] = 0$$

where if $\psi = 0$ then $E_t [R_{t+1}^E] = R_t \rightarrow$ frictionless financial markets

- Optimal contract is summarized by leverage ratio $\varrho_t = (Q_t k_t) / V_t$ and threshold value $\bar{\omega}_{t+1}$
- Resulting gross interest rate on loan is $R_{t+1}^L = \frac{\bar{\omega}_{t+1} R_{t+1}^E \varrho_t}{\varrho_t - 1}$

Entrepreneurs' net worth evolution

$$V_t = v \left[R_t^E Q_{t-1} k_{t-1} - \left(R_{t-1} + \frac{\psi G_t R_t^E Q_{t-1} k_{t-1}}{L_{t-1}} \right) L_{t-1} \right] + T_t^E$$

where $(1 - v)$ of ex-post profits finance entrepreneurs' consumption, while T_t^E are injections of funds from entrepreneurs to their firms

Output accounting

$$c_t + i_t + g_t + \psi G_t \frac{R_t^E}{\Pi_t} q_{t-1} k_{t-1} = y_t$$

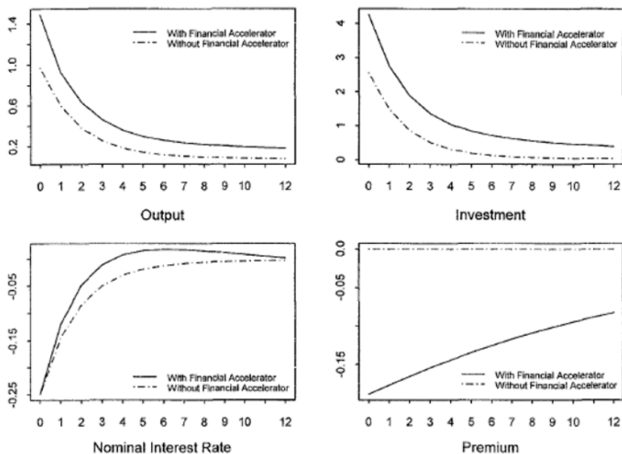
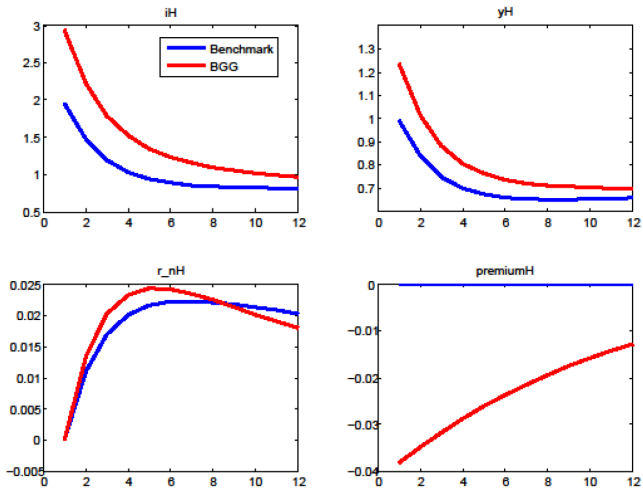
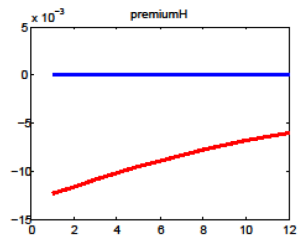
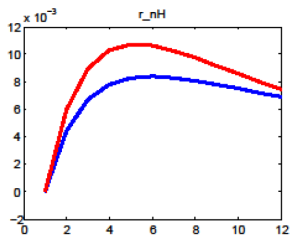
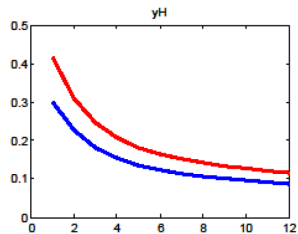
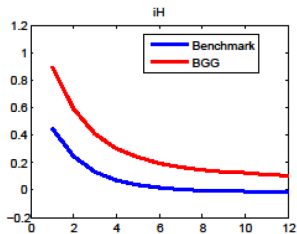


Fig. 3. Monetary shock – no investment delay. All panels: time horizon in quarters.

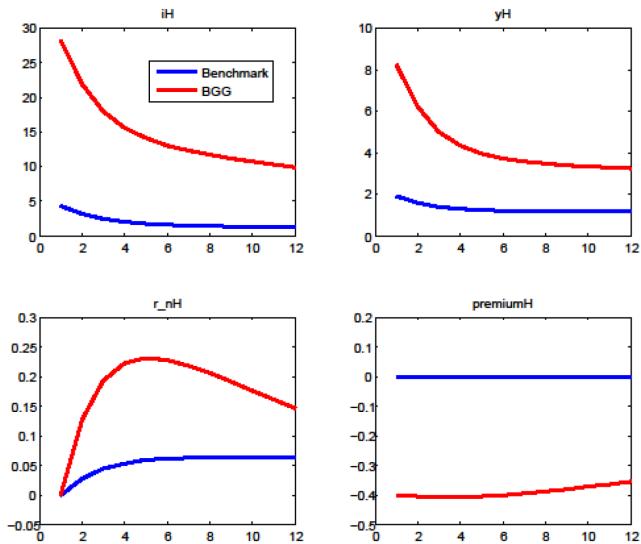
Productivity shock



Government spending shock



Net worth shock



Takeaway

- Financial intermediation (indirect finance) is much more important than direct finance for external funding for firms
- Asymmetric information (adverse selection and moral hazard) can help explain this phenomenon
- Asymmetric information also affords an important role to collateral in indirect finance
- Gives rise to the financial accelerator mechanism
- Relevance for monetary and macroprudential policy:
 1. Banking system (over which central banks have some control) really important for functioning of economy
 2. Fluctuating asset prices (e.g. bubbles) can impact ability of banking system to funnel savings into productive investments