Labor markets over the business cycle
Indivisible labor. Search and matching

Advanced Macroeconomics

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RBC model vs data comparison

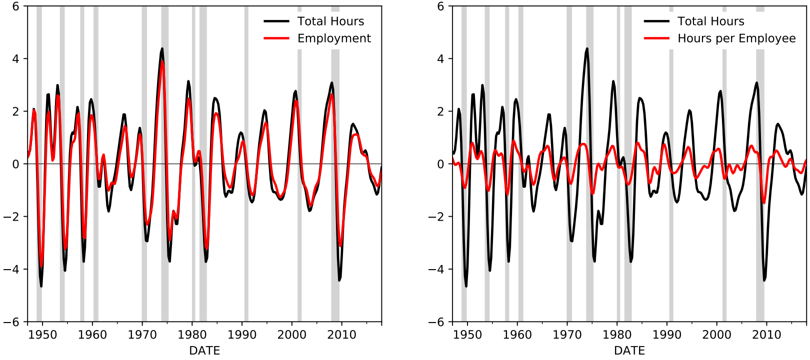
		Std. Dev.		Corr. w. y		Autocorr.	
		Data	Model	Data	Model	Data	Model
Output	y	1.60	1.60	1.00	1.00	0.85	0.72
Consumption	c	0.86	0.57	0.76	0.92	0.83	0.80
Investment	i	4.54	5.14	0.79	0.99	0.87	0.71
Capital	k	0.57	0.46	0.36	0.08	0.97	0.96
Hours	h	**1.60**	**0.73**	0.81	0.98	0.90	0.71
Wage	w	0.84	0.73	**0.10**	**0.99**	0.65	0.75
Interest rate	r	0.39	0.06	**-0.01**	**0.96**	0.40	0.71
TFP	z	1.00	1.15	**0.67**	**1.00**	0.71	0.72
Productivity	y/h	1.30	0.95	**0.51**	**0.99**	0.65	0.75

RBC model vs data comparison

- Model performance is quite good – it was a big surprise in the 1980s!
- There are some problems with it though
  - In the data, hours are just as volatile as output
  - In the model, hours are less than half as volatile as output
  - In the data, real wage can be either pro- or countercyclical
  - In the model, real wage is strongly procyclical
  - In the data TFP and productivity are mildly correlated with output
  - In the model both are 1:1 correlated with output
- These results suggest that
  - Need some room for nominal variables
  - More shocks than just TFP are needed
  - **We need to focus more on labor market**
    – should improve behavior of hours and real wage

Indivisible labor: introduction

Most of the variation in hours worked is on the *extensive* margin (employment-unemployment) rather than on the *intensive* margin (hours worked by individual employees)



Indivisible labor: introduction

Most of the variation in hours worked is on the *extensive* margin (employment-unemployment) rather than on the *intensive* margin (hours worked by individual employees)

$$H_t = L_t h_t \quad \longrightarrow \quad \log H_t = \log L_t + \log h_t$$

$$\text{Var}(\log H) = \text{Var}(\log L) + \text{Var}(\log h) + 2 \cdot \text{Cov}(\log N, \log h)$$

Variance-covariance matrix of Hodrick-Prescott deviations

	Total Hours	Employment	Hours per Employee
Total Hours	**3.52**		
Employment		**2.47**	**0.40**
Hours per Employee		**0.40**	**0.24**

About 70% of variance of total hours worked is accounted for by variance of employment level and only 7% is accounted for by variance of hours worked by individual employees (the rest is accounted for by covariance)

Indivisible labor: setup

- “Realistic” hours worked variation results from a two-step process
  - Decision between working and not working
  - Conditional on working, how much to work
- This is difficult to model – we’ll focus on the first step only

- Gary Hansen (1985) and Richard Rogerson (1988) invented a clever technical solution
- In the RBC model households choose how much to work
- Here they will choose the probability p of working $\bar{h}$ hours
  - All workers are identical
  - Each worker can work either 0 hours or a fixed number of hours $\bar{h}$
  - Each worker is a part of big family and consumes the same amount regardless of working or not
  - As a consequence all workers choose the same probability of working

Households' problem

Consider first a single-period problem

$$\max \quad U = \log c + E\left[\phi \log\left(1-h\right)|p\right]$$

Expand the expected term

$$E\left[\phi \log\left(1-h\right)|p\right] = p\phi \log\left(1-\bar{h}\right) + \left(1-p\right)\phi \log\left(1-0\right) = p\phi \log\left(1-\bar{h}\right)$$

Since all workers choose the same p, the average number of hours per worker household h is equal to probability p times working hours per employed $\bar{h}$

$$h = p\bar{h} \quad \longrightarrow \quad p = h/\bar{h}$$

Going back to the expected term

$$E\left[\phi \log\left(1-h\right)|p\right] = p\phi \log\left(1-\bar{h}\right) = h\frac{\phi \log\left(1-\bar{h}\right)}{\bar{h}} = -Bh$$

where $B = \left(-\phi \log\left(1-\bar{h}\right)/\bar{h}\right) > 0$. Utility becomes linear in h!

Households' solution I

A representative household solves expected utility maximization problem

$$\begin{aligned} \max \quad & U_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\log c_t - B h_t \right) \right] \\ \text{subject to} \quad & a_{t+1} + c_t = (1 + r_t)\, a_t + w_t h_t + div_t \end{aligned}$$

Lagrangian

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t E_0 \left[\log c_t - B h_t \right] \\ & + \sum_{t=0}^{\infty} \beta^t E_0 \left[\lambda_t \left[(1 + r_t)\, a_t + w_t h_t + div_t - a_{t+1} - c_t \right] \right] \end{aligned}$$

Households' solution II

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_0 \left[\log c_t - B h_t\right] + \sum_{t=0}^{\infty} \beta^t E_0 \left[\lambda_t \left[(1 + r_t)\, a_t + w_t h_t + div_t - a_{t+1} - c_t\right]\right]$$

First Order Conditions

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t E_0 \left[\frac{1}{c_t}\right] - \beta^t E_0 \left[\lambda_t\right] = 0 \quad \longrightarrow \quad \lambda_t = \frac{1}{c_t}$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = \beta^t \cdot E_0 \left[-B\right] + \beta^t E_0 \left[\lambda_t w_t\right] = 0 \quad \longrightarrow \quad \lambda_t = \frac{B}{w_t}$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -E_0 \left[\lambda_t\right] + \beta E_0 \left[\lambda_{t+1} \left(1 + r_{t+1}\right)\right] = 0$$

$$\longrightarrow \quad \lambda_t = \beta E_t \left[\lambda_{t+1} \left(1 + r_{t+1}\right)\right]$$

Households' solution III

First Order Conditions

$$\begin{aligned} c_t &: \quad \lambda_t = \frac{1}{c_t} \\ h_t &: \quad \lambda_t = \frac{B}{w_t} \\ a_{t+1} &: \quad \lambda_t = \beta E_t \left[\lambda_{t+1} \left(1 + r_{t+1}\right)\right] \end{aligned}$$

Resulting

$$\begin{aligned} \text{Intertemporal condition } (c+a) &: \quad 1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \left(1 + r_{t+1}\right)\right] \\ \text{Intratemporal condition } (c+h) &: \quad B = \frac{w_t}{c_t} \end{aligned}$$

Full set of equilibrium conditions

System of 8 equations and 8 unknowns: $\{c, h, y, r, w, k, i, z\}$

Euler equation	:	$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} (1 + r_{t+1}) \right]$
Consumption-hours choice	:	$B = \frac{w_t}{c_t}$
Production function	:	$y_t = z_t k_t^{\alpha} h_t^{1-\alpha}$
Real interest rate	:	$r_t = \alpha \frac{y_t}{k_t} - \delta$
Real hourly wage	:	$w_t = (1 - \alpha) \frac{y_t}{h_t}$
Investment	:	$i_t = k_{t+1} - (1 - \delta) k_t$
Output accounting	:	$y_t = c_t + i_t$
TFP AR(1) process	:	$\log z_t = \rho_z \log z_{t-1} + \varepsilon_t$

Steady state – closed form solution

Start with the Euler equation

$$1 = \beta\,(1 + r) \quad \longrightarrow \quad r = \frac{1}{\beta} - 1$$

From the interest rate equation obtain the k/h ratio

$$r = \alpha k^{\alpha-1} h^{1-\alpha} - \delta \quad \longrightarrow \quad \left(\frac{k}{h}\right)^{\alpha-1} = \frac{r+\delta}{\alpha} \quad \longrightarrow \quad \frac{k}{h} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}$$

From the production function obtain the y/h ratio and use it to get wage

$$y = k^{\alpha} h^{1-\alpha} \quad \longrightarrow \quad \frac{y}{h} = \left(\frac{k}{h}\right)^{\alpha} \quad \text{and} \quad w = (1-\alpha)\,\frac{y}{h}$$

From investment and output accounting equations obtain the c/h ratio

$$i = \delta k \quad \longrightarrow \quad y = c + \delta k \quad \longrightarrow \quad \frac{c}{h} = \frac{y}{h} - \delta \frac{k}{h}$$

Get c from the consumption-hours choice. Then obtain h.
The rest follows from h.

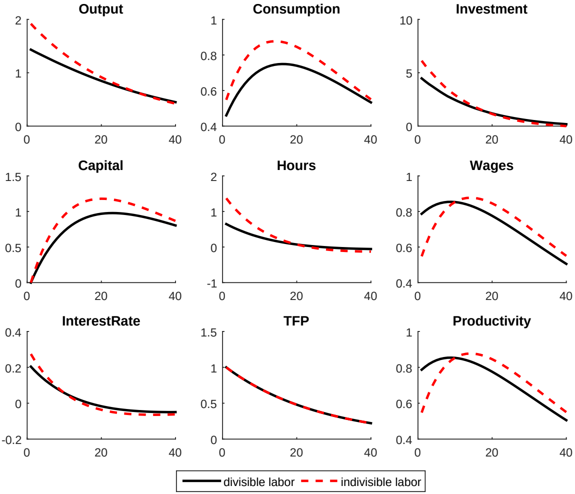
$$c = \frac{w}{B} \quad \text{and} \quad h = \frac{c}{c/h}$$

Parameters

- To best compare our two models,
  we need them to generate identical steady states
- We replace parameter ϕ with parameter B
- We choose the value for B so that it matches $h = 1/3$
- For this model $B = 2.63$

Model comparison: impulse response functions

RBC model IRF: black solid lines
Indivisible labor IRF: red dashed lines

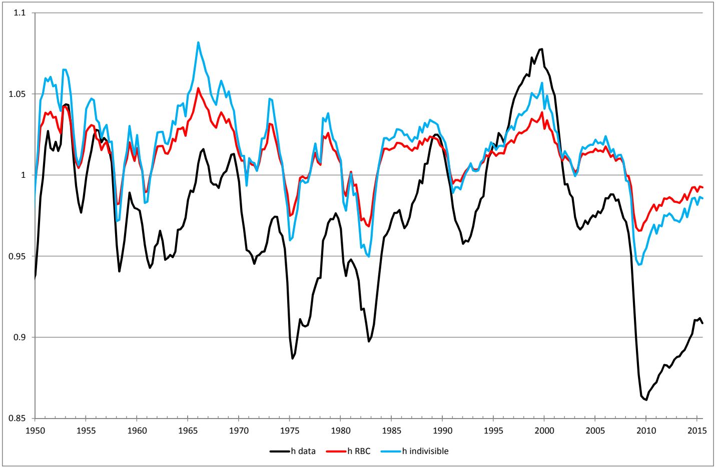


Percentage deviations from steady state (percentage points for r)

Model comparison: moments

	Std. Dev.			Corr. w. y			Autocorr.		
	Data	RBC	Ind	Data	RBC	Ind	Data	RBC	Ind
y	1.60	1.60	1.60	1.00	1.00	1.00	0.85	0.72	0.72
c	0.86	0.57	0.53	0.76	0.92	0.90	0.83	0.80	0.81
i	4.54	5.14	5.33	0.79	0.99	0.99	0.87	0.71	0.71
k	0.57	0.46	0.47	0.36	0.08	0.08	0.97	0.96	0.96
h	**1.60**	**0.73**	**1.15**	0.81	0.98	0.98	0.90	0.71	0.70
w	0.84	0.73	0.53	0.10	0.99	0.90	0.65	0.75	0.81
z	1.00	1.15	0.83	0.67	1.00	1.00	0.71	0.72	0.72
y/h	1.30	0.95	0.53	0.51	0.99	0.90	0.65	0.75	0.81

Model comparison: model-generated hours worked



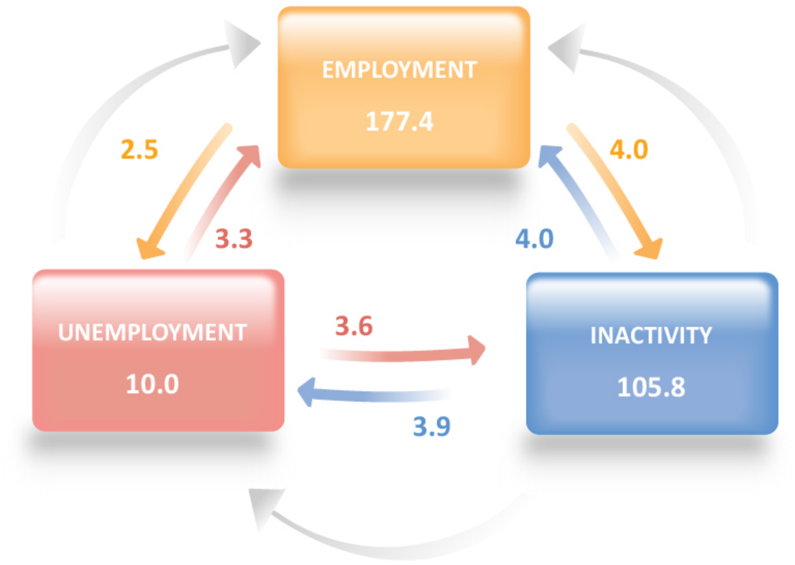
Indivisible labor: summary

- Model enhances hours volatility – but it's still too low
- Improves a bit correlation of wages and productivity with output
- Slightly decreases empirical match in other dimensions
- Technical advantage – requires smaller TFP shocks
- Philosophical advantage – more “realistic” labor market

Search and matching: introduction

- Labor markets are in a state of constant flux
- At the same time there are job-seeking workers and worker-seeking firms
- Labor markets are decentralized and thus active search is needed
- Search friction leads to unemployment even in the steady state

Labor market status and flows: EU 2017Q2-2017Q3

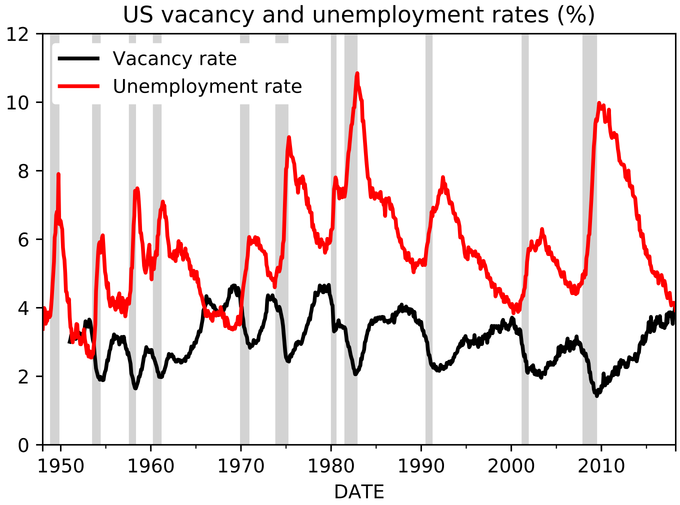


Labor market status change probabilities in EU

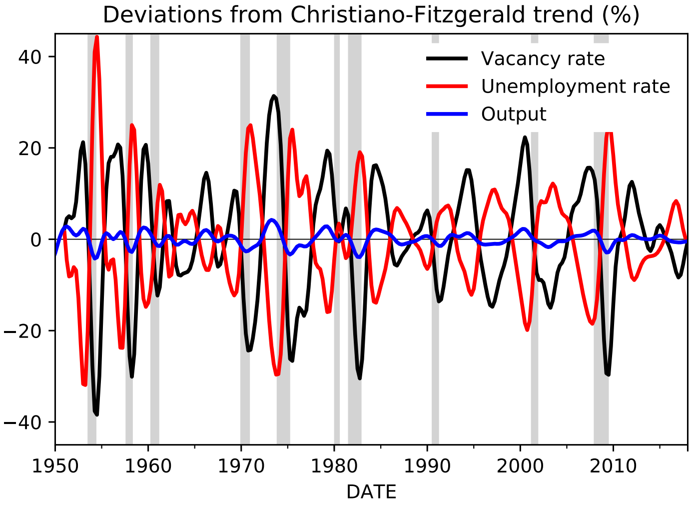
	EMPLOYMENT 2017Q3	UNEMPLOYMENT 2017Q3	INACTIVITY 2017Q3
EMPLOYMENT 2017Q2	96.5 %	1.3 %	2.2 %
UNEMPLOYMENT 2017Q2	19.8 %	59.0 %	21.2 %
INACTIVITY 2017Q2	3.5 %	3.5 %	93.0 %

Source:
http://ec.europa.eu/eurostat/statistics-explained/index.php/Labour_market_flow_statistics_in_the_EU

Unemployment and vacancy rates: USA 1948Q1-2018Q1



Labor market fluctuations: USA 1950Q1-2018Q1



Matching function

- Firms create open job positions (openings, vacancies)
- Workers search for jobs
- Both jobs and workers are heterogeneous
  – not every possible match is attractive
- Matching function captures this feature
- New matches M are a function of the pool of unemployed U and vacancies V
  $$M_t = \chi V_t^{\eta} U_t^{1-\eta}$$
- After normalizing labor force to unity, match probability m is a function of unemployment rate u and vacancy rate v
  $$m_t = \chi v_t^{\eta} u_t^{1-\eta}$$
  where $\chi > 0$ and $\eta \in (0, 1)$

Job finding and job filling probabilities

- Unemployed workers are interested in job finding probability p

$$p_t = \frac{m_t}{u_t} = \chi \left(\frac{v_t}{u_t} \right)^{\eta} = \chi \theta_t^{\eta} = q_t \theta_t$$

  where $\theta = v/u$ is called labor market tightness
- Firms with vacancies care about job filling probability q

$$q_t = \frac{m_t}{v_t} = \chi \left(\frac{v_t}{u_t} \right)^{\eta - 1} = \chi \theta_t^{\eta - 1} = \frac{p_t}{\theta_t}$$

- Dual externality from congestion
  - High unemployment rate decreases p and increases q
  - High vacancy rate increases p and decreases q

Employment dynamics

- Ignoring labor market inactivity, employment rate n and unemployment rate u sum to unity:

$$n_t + u_t = 1 \quad \longrightarrow \quad n_t = 1 - u_t$$

- Existing matches are destroyed with exogenous probability s
- New matches increase next period employment

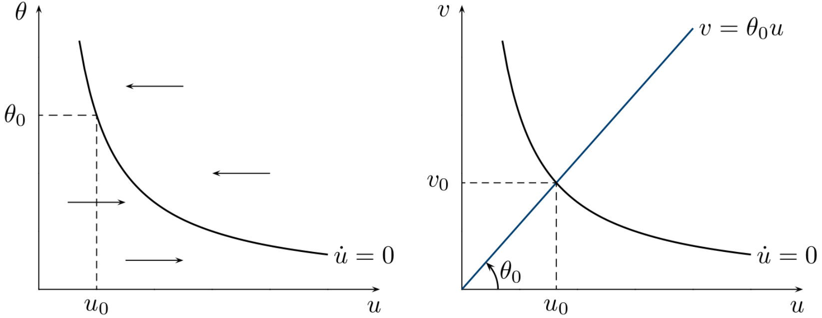
$$n_t = n_{t-1} - sn_{t-1} + m_{t-1}$$
$$u_t = u_{t-1} + sn_{t-1} - m_{t-1}$$

- We can find the steady state unemployment rate

$$u = u + s\left(1 - u\right) - p\left(\theta\right) u$$
$$u = \frac{s}{s + p\left(\theta\right)}$$

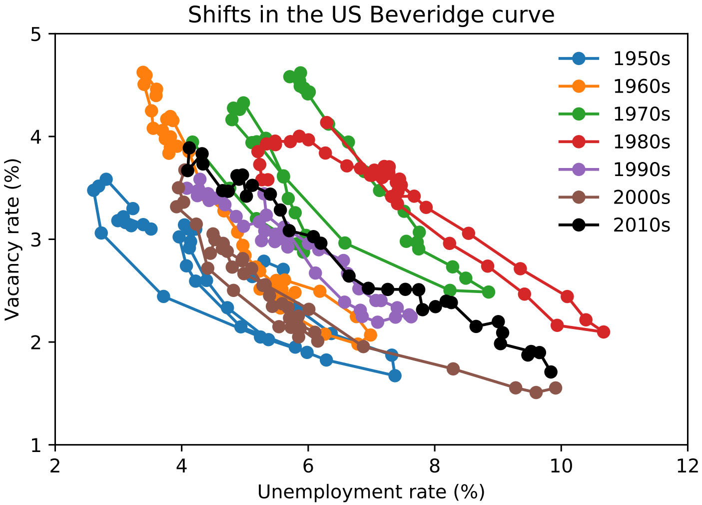
  as a function of separation and job finding probabilities
- If separation probability and matching function parameters do not change, then there exists a stable negative relationship between unemployment and vacancy rates known as the Beveridge curve

Beveridge curve: theory



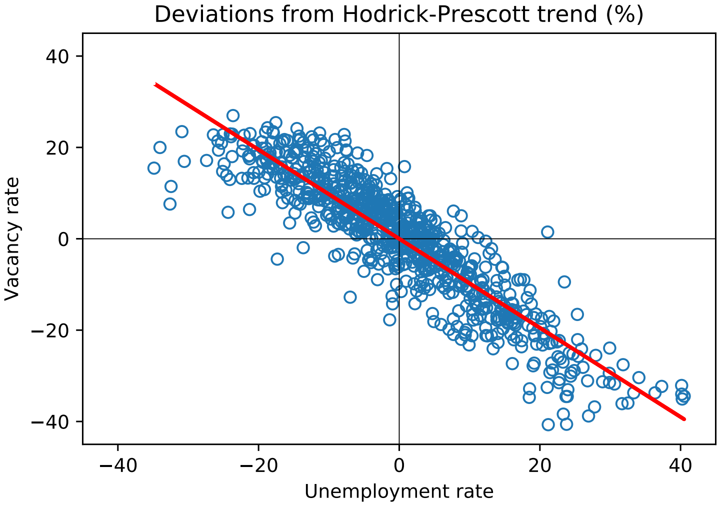
Graph by Leszek Wincenciak

Beveridge curve: data

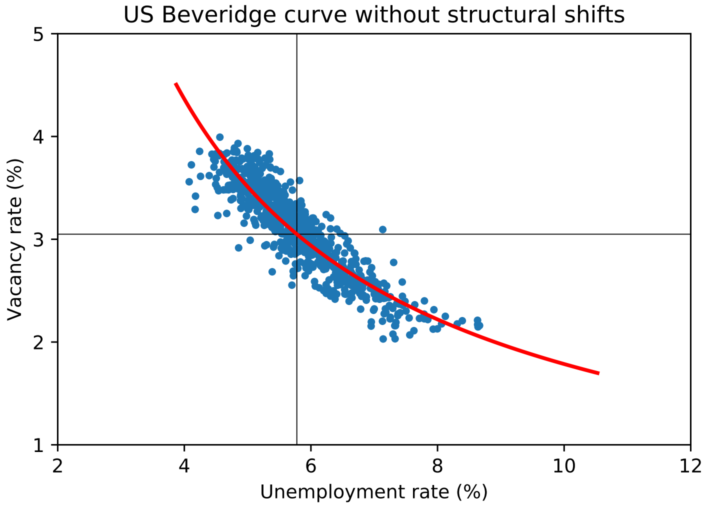


Beveridge curve: data

Detrending with Hodrick-Prescott filter takes out structural shifts



Beveridge curve: “estimation”



Firm side

- Assume firms and workers discount future with β
- Period net gain from a filled job equals
  marginal product of employee less wage
- With probability $(1 - s)$ the match will survive into the next period

$$\mathcal{J}_t = (mpn_t - w_t) + \beta E_t \left[(1 - s) \mathcal{J}_{t+1} + s \mathcal{V}_{t+1} \right]$$

- Period net loss from open vacancy is its cost κ
  (advertising, interviewing)
- With probability q the vacancy will be filled

$$\mathcal{V}_t = -\kappa + \beta E_t \left[q_t \mathcal{J}_{t+1} + (1 - q_t) \mathcal{V}_{t+1} \right]$$

- Free entry in vacancies ensures that always $\mathcal{V} = 0$

$$\frac{\kappa}{q_t} = \beta E_t \left[\mathcal{J}_{t+1} \right]$$
$$\mathcal{J}_t = (mpn_t - w_t) + \beta E_t \left[(1 - s) \mathcal{J}_{t+1} \right]$$

- In the steady state $(r = 1/\beta - 1)$

$$w = mpn - (r + s) \frac{\kappa}{q(\theta)}$$

Worker side

- Period net gain from employment equals wage
- With probability $(1 - s)$ the match will survive into the next period

$$\mathcal{E}_t = w_t + \beta E_t \left[(1 - s) \mathcal{E}_{t+1} + s \mathcal{U}_{t+1} \right]$$

- Period net gain from unemployment equals benefits (and possibly utility from leisure)
- With probability p unemployed finds a job

$$\mathcal{U}_t = b + \beta E_t \left[p_t \mathcal{E}_{t+1} + (1 - p_t) \mathcal{U}_{t+1} \right]$$

Wage setting I

- In principle, wage can be as low as gain from unemployment b or as high as marginal product of employee *mpn* plus match gain
- Negotiated wage will be somewhere between those two values
- An easy way to pin down wage is Nash bargaining
- Let $\gamma \in [0, 1]$ denote the relative bargaining power of firms
- Intuitively $w \rightarrow b$ if $\gamma \rightarrow 1$ and $w \rightarrow mpn + \kappa\theta$ if $\gamma \rightarrow 0$
- The negotiated wage is the solution of the problem

$$\max_{w_t} \quad \left(\mathcal{J}_t\left(w_t\right)\right)^{\gamma}\left(\mathcal{E}_t\left(w_t\right) - \mathcal{U}_t\right)^{1-\gamma}$$

- Solving the problem results in

$$\gamma\left(\mathcal{E}_t - \mathcal{U}_t\right) = \left(1 - \gamma\right)\mathcal{J}_t$$

- Alternatively: total match surplus $\mathcal{S}_t = \left(\mathcal{E}_t - \mathcal{U}_t\right) + \mathcal{J}_t$

$$\mathcal{E}_t - \mathcal{U}_t = \left(1 - \gamma\right)\mathcal{S}_t \quad \text{and} \quad \mathcal{J}_t = \gamma\mathcal{S}_t$$

Wage setting II

$$\gamma\left(\mathcal{E}_t - \mathcal{U}_t\right) = \left(1-\gamma\right)\mathcal{J}_t$$

Plug in expressions for $\mathcal{E}_t$, $\mathcal{U}_t$ and $\mathcal{J}_t$

$$\gamma\left\{\left(w_t - b\right) + \beta\left(1-s-p_t\right)E_t\left[\mathcal{E}_{t+1} - \mathcal{U}_{t+1}\right]\right\}$$
$$= \left(1-\gamma\right)\left\{\left(mpn_t - w_t\right) + \beta E_t\left[\left(1-s\right)\mathcal{J}_{t+1}\right]\right\}$$

$$w_t - \gamma b + \left(1-s-p_t\right)\beta E_t\left[\gamma\left(\mathcal{E}_{t+1} - \mathcal{U}_{t+1}\right)\right]$$
$$= \left(1-\gamma\right)mpn_t + \left(1-s\right)\beta E_t\left[\left(1-\gamma\right)\mathcal{J}_{t+1}\right]$$

$$w_t - \gamma b + \left(1-s-p_t\right)\beta E_t\left[\left(1-\gamma\right)\mathcal{J}_{t+1}\right]$$
$$= \left(1-\gamma\right)mpn_t + \left(1-s\right)\beta E_t\left[\left(1-\gamma\right)\mathcal{J}_{t+1}\right]$$

$$\begin{aligned}
w_t &= \gamma b + \left(1-\gamma\right)\left\{mpn_t + p_t\beta E_t\left[\mathcal{J}_{t+1}\right]\right\} \\
\kappa/q_t &= \beta E_t\left[\mathcal{J}_{t+1}\right] \\
w_t &= \gamma b + \left(1-\gamma\right)\left(mpn_t + p_t\kappa/q_t\right) \\
w_t &= \gamma b + \left(1-\gamma\right)\left(mpn_t + \kappa\theta_t\right)
\end{aligned}$$

Full set of equilibrium conditions

System of 9 equations and 9 unknowns: $\{w, mpn, \theta, \mathcal{J}, q, u, n, m, v\}$

$$
\begin{aligned}
w_t &= \gamma b + (1-\gamma)\left(mpn_t + \kappa\theta_t\right) \\
\mathcal{J}_t &= (mpn_t - w_t) + \beta E_t\left[(1-s)\,\mathcal{J}_{t+1}\right] \\
\frac{\kappa}{q_t} &= \beta E_t\left[\mathcal{J}_{t+1}\right] \\
u_t &= 1 - n_t \\
n_t &= (1-s)\,n_{t-1} + m_{t-1} \\
q_t &= \chi\theta_t^{\eta-1} \\
\theta_t &= \frac{v_t}{u_t} \\
m_t &= \chi v_t^{\eta} u_t^{1-\eta} \\
\ln mpn_t &= \rho_{mpn}\ln mpn_{t-1} + \varepsilon_t
\end{aligned}
$$

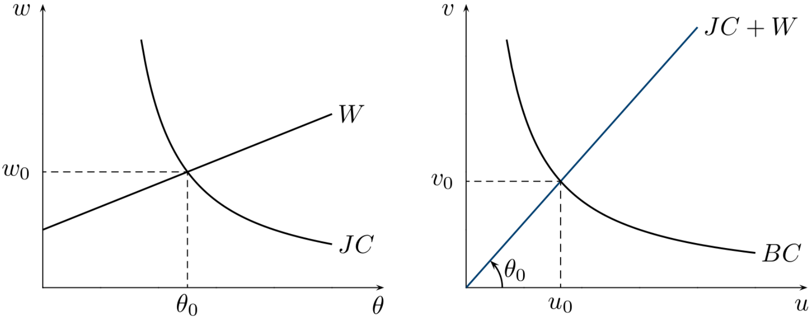
Steady state: key equations

In the steady state the model is fully summarized by the following three key equations:

$$\begin{aligned} \text{Beveridge curve (BC)} \quad &: \quad u = \frac{s}{s + p(\theta)} \\ \text{Job (vacancy) creation (JC)} \quad &: \quad w = mpn - (r + s)\frac{\kappa}{q(\theta)} \\ \text{Wage setting (W)} \quad &: \quad w = \gamma b + (1 - \gamma)(mpn + \kappa\theta) \end{aligned}$$

Can be even reduced further to equations in u and θ

Steady state: graphical solution



Graph by Leszek Wincenciak

Steady state: algebraic solution

- In this model the crucial variable is labor market tightness θ
- We can find it by solving the following system

$$w = \gamma b + (1-\gamma)(mpn + \kappa\theta)$$
$$w = mpn - (r+s)\frac{\kappa}{q(\theta)}$$

- After some rearrangement

$$(r+s)\frac{\kappa}{\chi}\theta^{1-\eta} = \gamma(mpn - b) - (1-\gamma)\kappa\theta$$

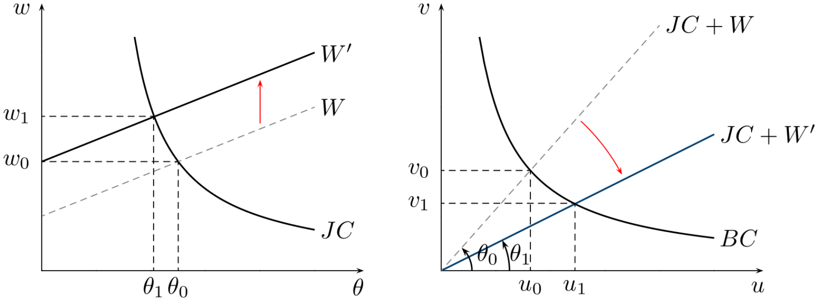
- The above equation does not have a closed form solution for θ
- We can solve it easily via numerical methods
- We can also use a trick – set $\theta = 1$ and solve for χ (but loose a degree of freedom for calibration)

$$\chi = [(r+s)\kappa] / [\gamma(mpn - b) - (1-\gamma)\kappa]$$

Comparative statics I

Effects of an increase in unemployment benefits ($b \uparrow$)
or in workers' bargaining power ($\gamma \downarrow$):

- Increase in real wage w
- Decrease in labor market tightness θ
- Decrease in vacancy rate v
- Increase in unemployment rate u

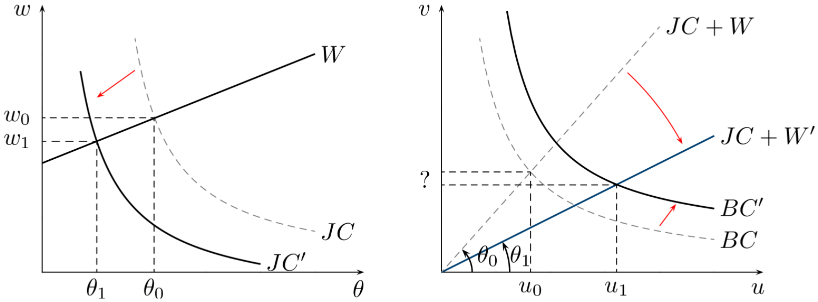


Graph by Leszek Wincenciak

Comparative statics II

Effects of an increase in separation rate ($s \uparrow$)
or a decrease in matching efficiency ($\chi \downarrow$):

- Decrease in real wage w
- Decrease in labor market tightness θ
- Ambiguous effect on vacancy rate v (depends on parameter values)
- Increase in unemployment rate u

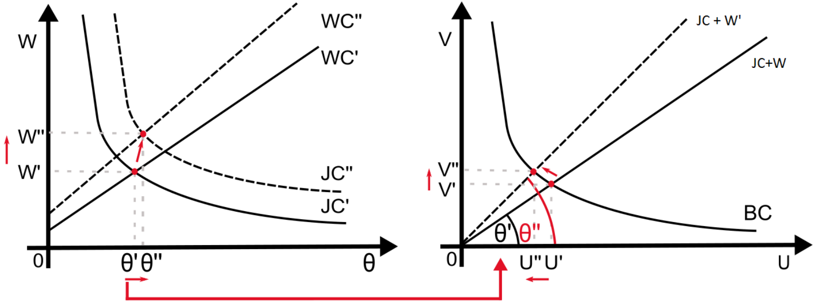


Graph by Leszek Wincenciak

Comparative statics III

Effects of an increase in labor productivity ($mpn \uparrow$):

- Increase in real wage w
- Increase in labor market tightness θ
- Increase in vacancy rate v
- Decrease in unemployment rate u

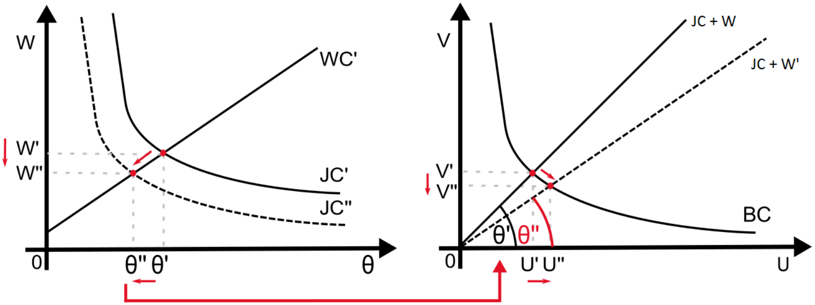


Graph by Matthias Hertweck

Comparative statics IV

Effects of an increase in interest rate ($r \uparrow$)
or an increase in impatience ($\rho \uparrow \rightarrow \beta \downarrow$):

- Decrease in real wage w
- Decrease in labor market tightness θ
- Decrease in vacancy rate v
- Increase in unemployment rate u



Graph by Matthias Hertweck

Transitional dynamics

Reduced form of the model:

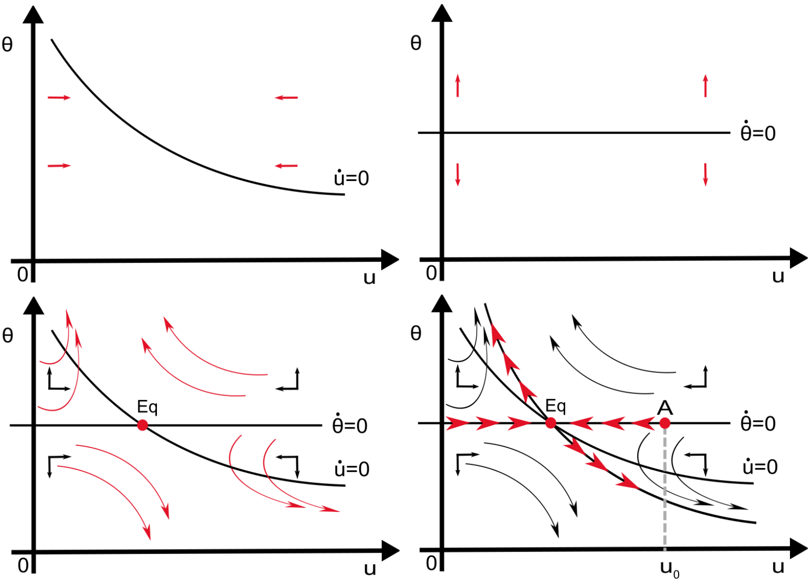
$$\Delta u = 0 \quad \longrightarrow \quad u = \frac{s}{s + \chi\theta^{\eta}}$$

$$\Delta\theta = \frac{\theta}{1-\eta}\left[(r+s) - \gamma\,(mpn - b)\,\frac{\chi\theta^{\eta-1}}{\kappa} + (1-\gamma)\,\chi\theta^{\eta}\right]$$

The dynamic equation for θ is independent of u

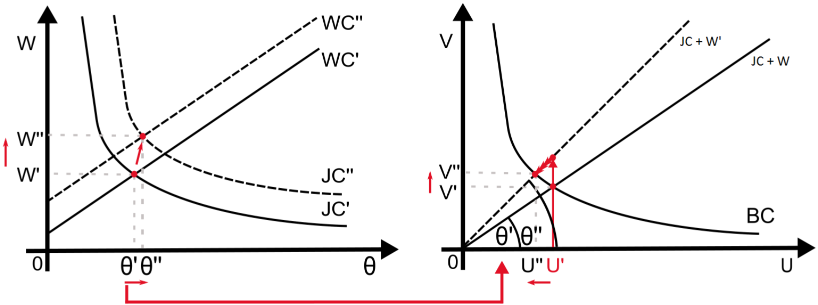
– $\Delta\theta = 0$ is a flat line in (u, θ) space

Transitional dynamics: phase diagram



Graphs by Matthias Hertweck

Transitional dynamics: positive productivity shock



Graph by Matthias Hertweck

Parameters

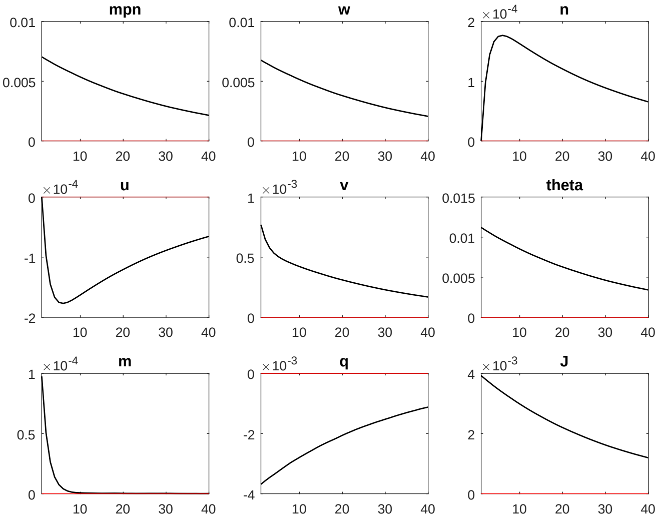
Values come from Shimer (2005, AER)

	Description	Value
χ	matching efficiency	0.45
η	matching elasticity of v	0.28
s	separation probability	0.033
β	discount factor	0.99
mpn	steady state marginal product	1
κ	vacancy cost	0.21
b	unemployment benefit	0.4
γ	firm bargaining power	0.28

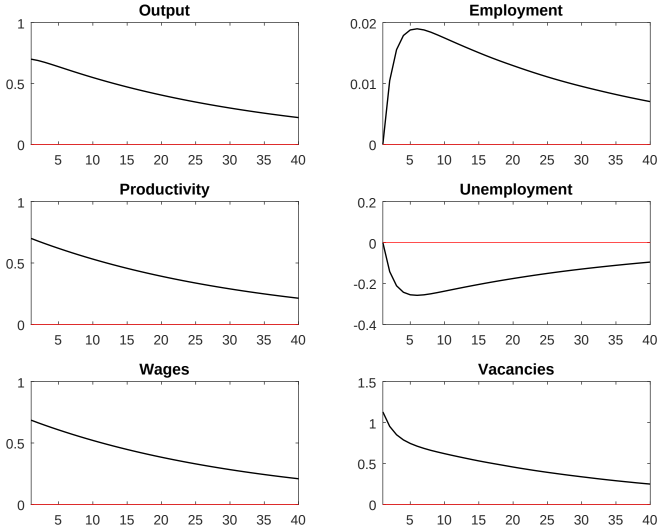
Implied steady state values

	Description	Value
u	unemployment rate	0.0687
v	vacancy rate	0.0674
m	new matches	0.031
θ	tightness	0.98
p	job finding probability	0.448
q	job filling probability	0.456
w	wage	0.98

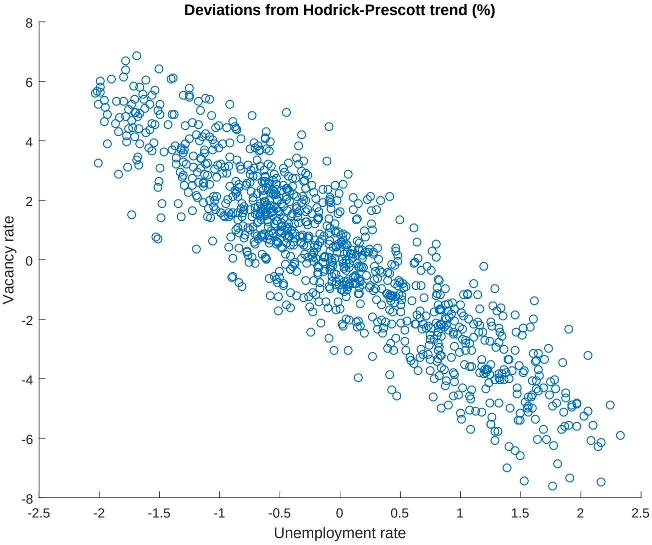
Impulse response functions I



Impulse response functions II



Model generated Beveridge curve



Summary

- We have a "realistic" model of the labor market
- Able to match both steady state (average)
  and some cyclical properties of the labor market
- Replicates the negative slope of the Beveridge curve
- Not enough variation in employment
- Beveridge curve too steep
- Too much variation in wages

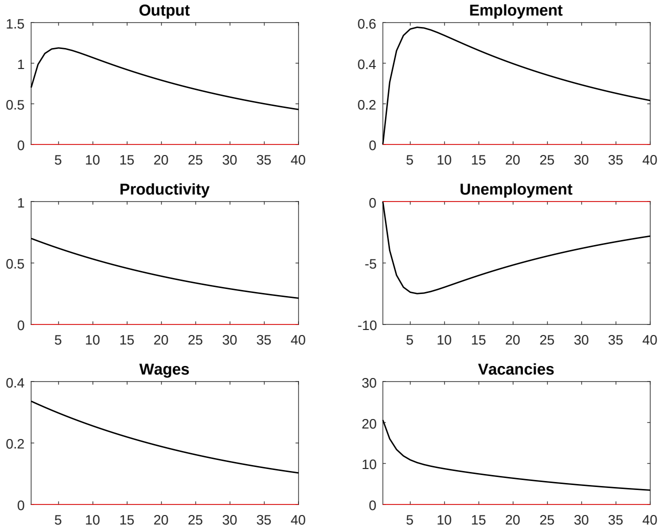
Alternative parametrization

Values come from Hagedorn & Manovskii (2008, AER)

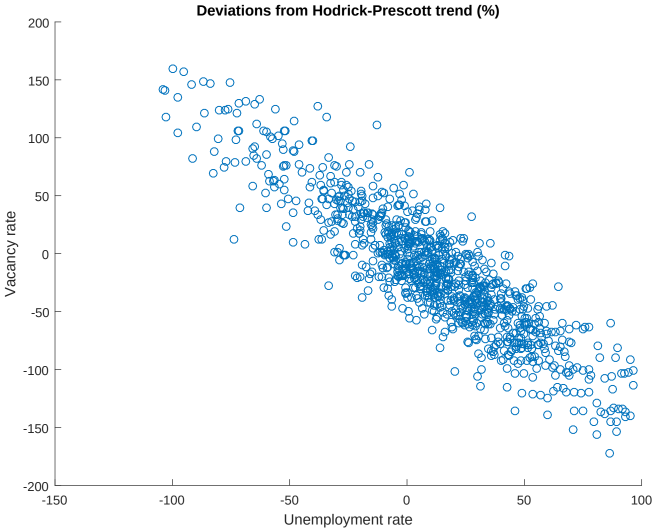
	Description	Value
η	matching elasticity of v	0.45
b	unemployment benefit	0.965
γ	firm bargaining power	0.928

- Firms have very strong bargaining position
- But unemployment gain includes leisure utility
- Steady state unchanged

Hagedorn & Manovskii: Impulse response functions

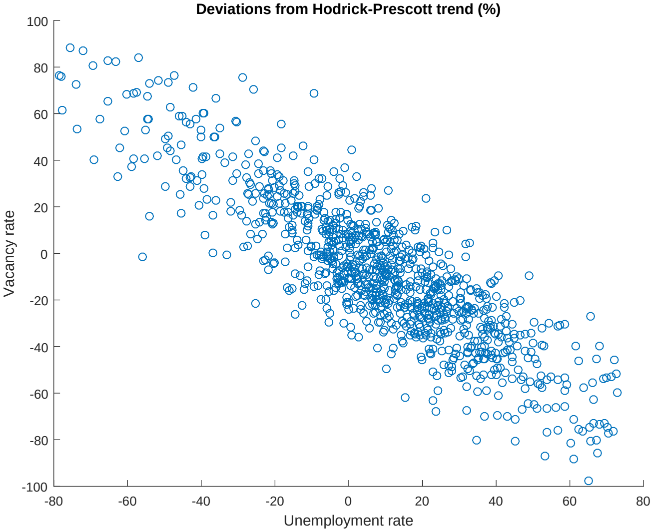


Hagedorn & Manovskii: Beveridge curve



Mortensen & Nagypal (2007): Beveridge curve

Set $\eta = 0.54$. Model BC replicates slope of the data BC



Summary

- Alternative parametrizations yield better results
- Both unemployment and employment become more volatile
- Volatility of wages is diminished
- Key problem for the search and matching model identified
  – period-by-period Nash bargaining
- Further extensions make alternative assumptions
  about the wage setting process

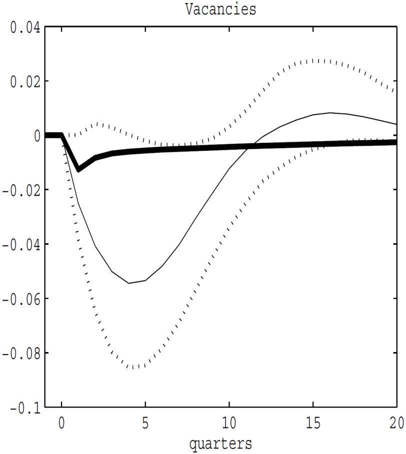
Integration with RBC framework

- Very easy
- Get *mpn* from the usual firm problem
- Adjust β for $\beta\frac{\lambda_{t+1}}{\lambda_t}$ in the firm's valuation since the latter is the correct stochastic discounting factor
- Solve for labor market variables
- Get back to the RBC part
- Remember to include vacancy costs in the national accounting equation

$$y_t = c_t + i_t + \kappa v_t$$

Observation of Fujita (2004)

Model IRF for vacancies is counterfactual



Alternative hiring cost function

- We assumed linear vacancy posting costs

$$\begin{aligned}\psi(v_t) &= \kappa v_t \\ w_t &= \gamma b + (1-\gamma)(mpn_t + \kappa\theta_t) \\ \frac{\kappa}{q_t} &= \beta E_t\left[mpn_{t+1} - w_{t+1} + (1-s)\frac{\kappa}{q_{t+1}}\right]\end{aligned}$$

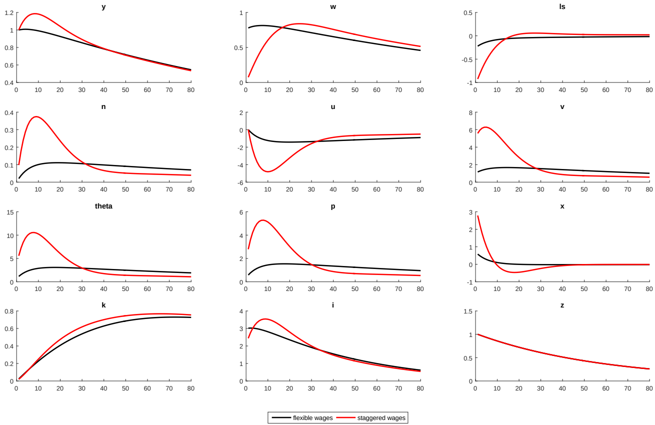
- Gertler & Trigari (2009, JPE) assume convex labor posting costs
- Define hiring rate x as the ratio of new hires to employed workers

$$\begin{aligned}x_t &= \frac{m_t}{n_t} \\ \psi(x_t) &= \frac{\kappa}{2}x_t^2 n_t \\ w_t &= \gamma b + (1-\gamma)\left(mpn_t + \frac{\kappa}{2}x_t^2 + p_t\kappa x_t\right) \\ \kappa x_t &= \beta E_t\left[mpn_{t+1} - w_{t+1} + (1-s)\kappa x_{t+1} + \frac{\kappa}{2}x_t^2\right]\end{aligned}$$

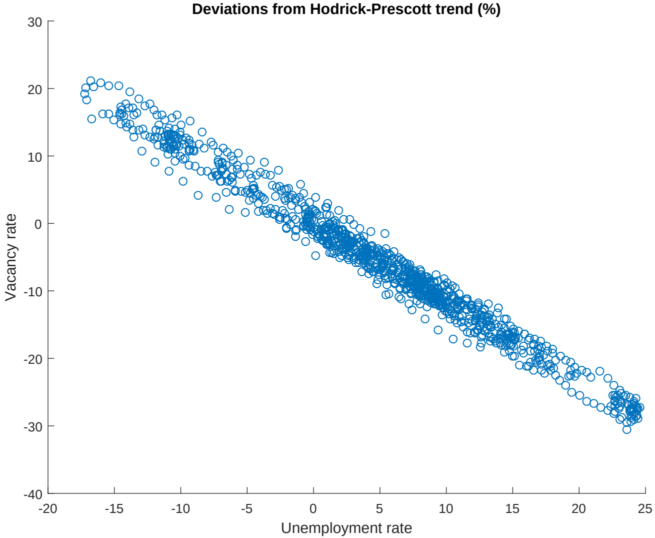
- They also consider staggered (multi-period) wage contracts where only a fraction of previous wage contracts are renegotiated

Gertler & Trigari: Impulse response functions

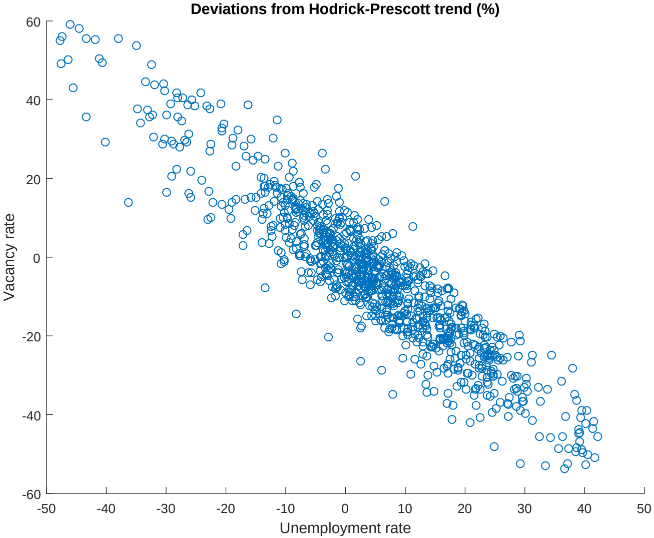
Monthly period frequency



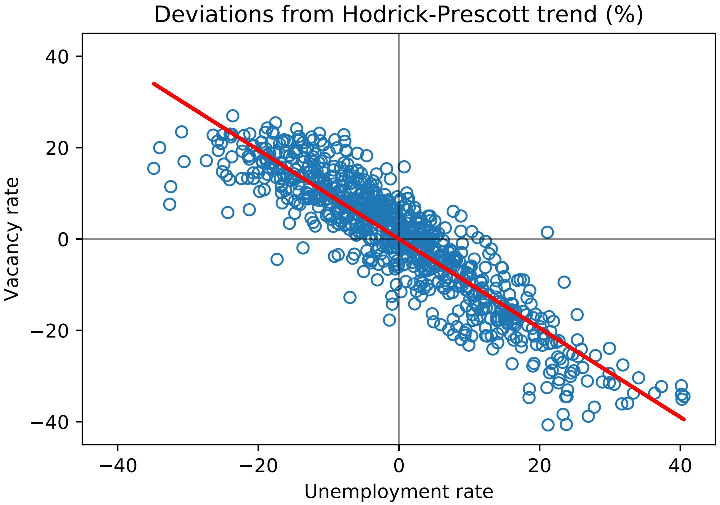
Gertler & Trigari: Beveridge curve (flexible wages)



Gertler & Trigari: Beveridge curve (staggered wages)



Beveridge curve: data



Gertler & Trigari: business cycle statistics

	y	w	ls	n	u	v	θ	a	i	c
	A. U.S. Economy, 1964:1–2005:1									
Relative standard deviation	1.00	.52	.51	.60	5.15	6.30	11.28	.61	2.71	.41
Autocorrelation	.87	.91	.73	.94	.91	.91	.91	.79	.85	.87
Correlation with y	1.00	.56	−.20	.78	−.86	.91	.90	.71	.94	.81
	B. Model Economy, $\lambda = 0$ (Flexible Wages)									
Relative standard deviation	1.00	.87	.09	.10	1.24	1.58	2.72	.93	3.11	.37
Autocorrelation	.81	.81	.58	.92	.92	.86	.90	.78	.80	.85
Correlation with y	1.00	1.00	−.54	.59	−.59	.98	.92	1.00	.99	.93
	C. Model Economy, $\lambda = 8/9$ (3 Quarters)									
Relative standard deviation	1.00	.56	.57	.35	4.44	5.81	9.84	.71	3.18	.35
Autocorrelation	.84	.95	.65	.90	.90	.82	.88	.76	.86	.86
Correlation with y	1.00	.66	−.56	.77	−.77	.91	.94	.97	.99	.90
	D. Model Economy, $\lambda = 11/12$ (4 Quarters)									
Relative standard deviation	1.00	.48	.58	.44	5.68	7.28	12.52	.64	3.18	.34
Autocorrelation	.85	.96	.68	.91	.91	.86	.90	.74	.88	.86
Correlation with y	1.00	.55	−.59	.78	−.78	.93	.95	.95	.99	.90

Summary

- After adding multi-period contracts, Gertler & Trigari obtain a very good empirical match of the RBC model with search & matching features
- This is one of the best matches for single-shock models
- Key to the success was
  - Convex vacancy posting
  - Staggered (multi-period) wage contracts

Possible further extensions

- Endogenous (non-constant) separation rate
- On-the-job search
- Hours per worker adjustments