Growth facts. Solow-Swan model

Advanced Macroeconomics QF: Lecture 4

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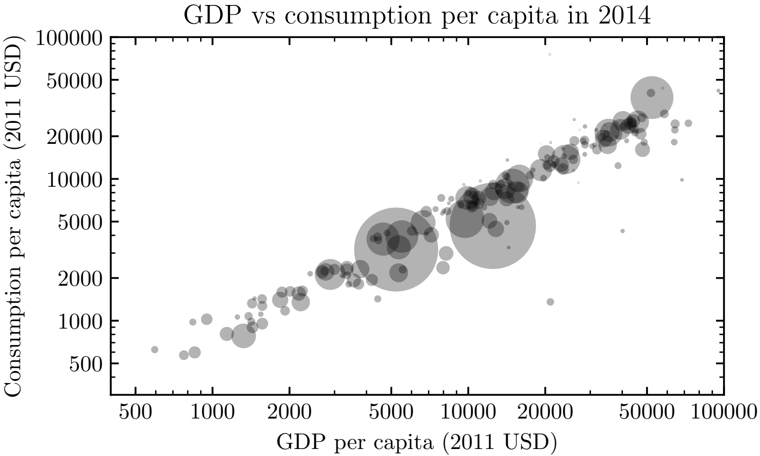
In case you need a review of basic concepts

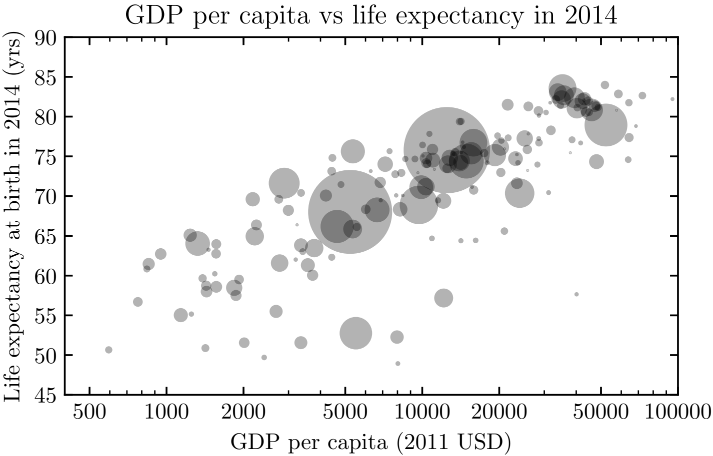
https://www.mruniversity.com/courses/principles-economics-macroeconomics

Chapters 1-3

GDP per capita and welfare: consumption

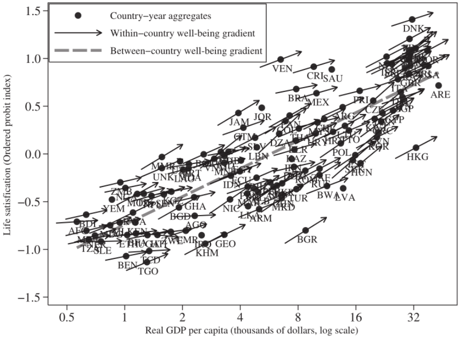


GDP per capita and welfare: life duration



GDP per capita and welfare: life satisfaction

Figure 11. Within-Country and Between-Country Estimates of the Life Satisfaction–Income Gradient: Gallup World Poll[a]



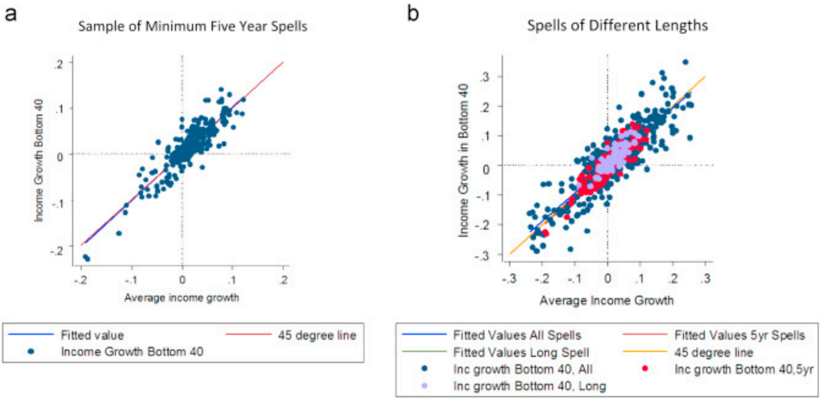
Source: Gallup World Poll, 2006; authors' regressions. Sources for GDP per capita are described in the text.

a. Each solid circle plots life satisfaction against GDP per capita for one of 131 developed and developing countries. The slope of the arrow represents the satisfaction-income gradient estimated for that country from a country-specific ordered probit of satisfaction on the log of annual real household income, controlling for gender, a quartic in age, and their interaction. Usable household income data were unavailable for eighteen countries. The dashed line represents the between-country satisfaction-income gradient estimated from an OLS regression of the satisfaction index on the logarithm of real GDP per capita. GDP per capita is at purchasing power parity in constant 2000 international dollars.

Stevenson and Wolfers (2013)

Economic Growth and Subjective Well-Being: Reassessing the Easterlin Paradox

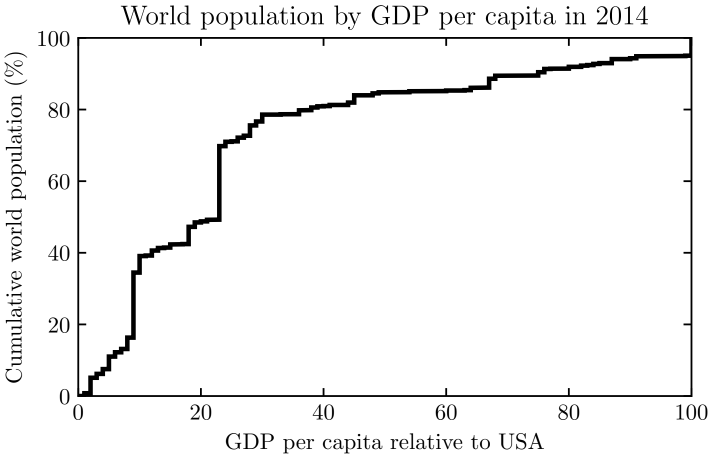
Growth in total income vs income of bottom 40%



Dollar, Kleineberg and Kraay (2016) *Growth still is good for the poor*

Growth fact 1

There is enormous variation in GDP per capita across economies.



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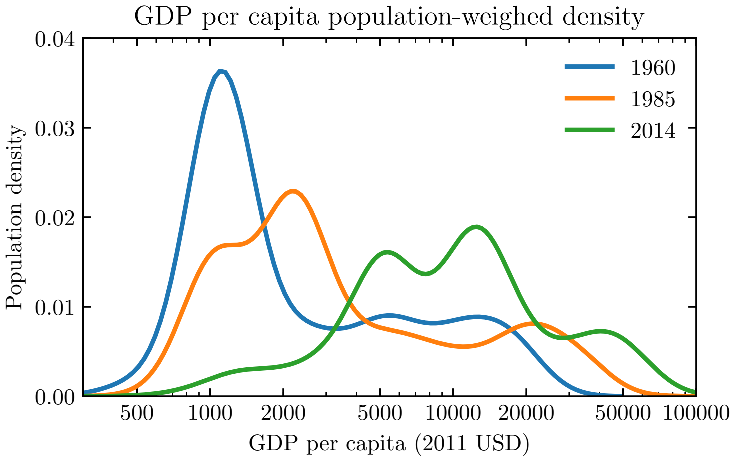


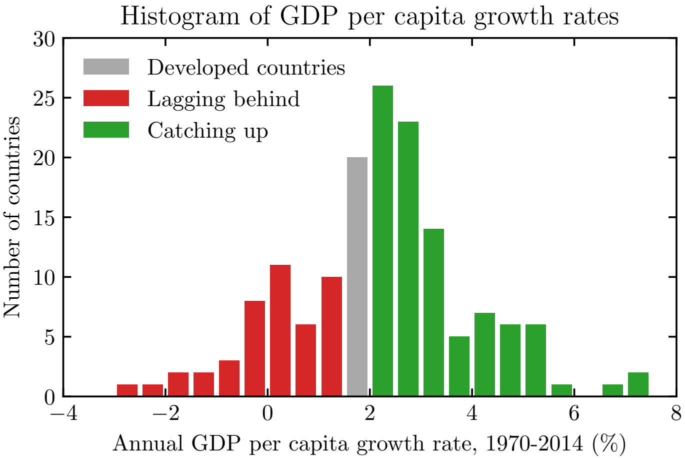
Growth fact 1

There is enormous variation in GDP per capita across economies. The poorest countries have per capita incomes that are less than 5 percent of per capita incomes in the richest countries.

- Income per capita (or GDP per capita) is not the sole measure of well-being, but it's a useful summary statistic.
- Income per capita ignores distribution of income within a country.
- Comparing income per capita across countries is not trivial:
  - You have to convert between currencies.
  - Countries have different relative prices for goods.
  - Large uncertainty in comparing income across countries and over time: Johnson et al. (2013) *Is newer better? Penn World Table Revisions and their impact on growth estimates.*

Growth fact 2

Rates of economic growth vary substantially across countries.



Growth fact 2

Rates of economic growth vary substantially across countries.

- We will try to distinguish whether these are long-term differences or just transitional differences.
- If they are long-term, then eventually some countries will be infinitely rich compared to others.
- We think most differences are transitional.

Small differences in rates of growth translate to big differences in incomes over time:

Rate of growth	Initial income	Income after ... years 25	50	70	100
1.0%	100	128	164	201	270
1.5%	100	145	211	284	443
2.0%	100	164	269	400	724
2.5%	100	185	344	563	1181
3.0%	100	209	438	792	1922

Rule of 70

A way to estimate the number of years it takes for a certain variable to double.

Find T for which $x_T = 2x_0$, assuming constant (annual) growth rate g

$$
\begin{aligned}
x_T &= x_0 \cdot (1+g)^T \\
2x_0 &= x_0 \cdot (1+g)^T \\
2 &= (1+g)^T \quad | \quad \ln \\
\ln 2 &= T \cdot \ln(1+g) \\
0.7 &\approx T \cdot g \\
T &\approx \frac{0.7}{g} = \frac{70}{100 \cdot g}
\end{aligned}
$$

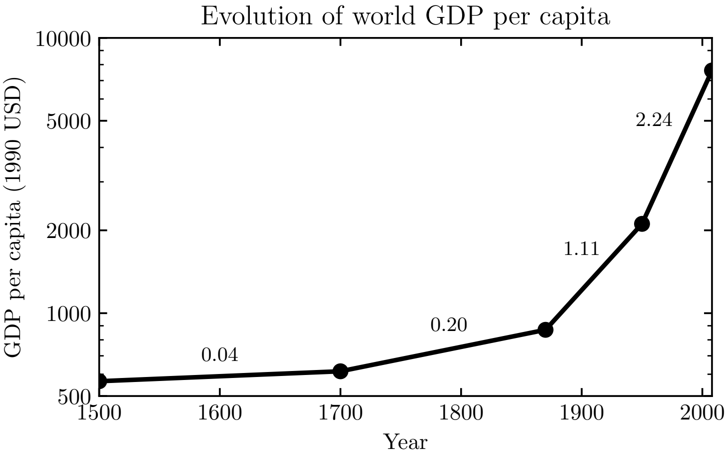
If g is expressed in percent, the number of years for a variable to double is approximately equal to 70 divided by g (expressed in percentage points).

There are both "growth miracles" and "growth disasters"

Country	GDP p.c. 2014	GDP p.w. 2014	Emp. rate 2014	GDP p.c. 1970	Growth 70-14 (%)	Yrs. to double
United States	52 292	112 517	0.46	23 608	1.8	38
United Kingdom	40 242	83 612	0.48	15 176	2.2	31
France	39 374	95 498	0.41	16 436	2.0	35
Japan	35 358	68 989	0.51	12 956	2.3	30
Singapore	72 583	117 472	0.62	5 814	5.9	12
Hong Kong	51 808	100 467	0.52	7 613	4.5	16
Taiwan	44 328	92 979	0.48	4 738	5.2	13
South Korea	35 104	67 247	0.52	2 100	6.6	10
Botswana	16 175	37 637	0.43	798	7.1	10
China	12 473	21 394	0.58	1 285	5.3	13
Indonesia	9 707	21 853	0.44	995	5.3	13
India	5 224	13 261	0.39	1 282	3.2	21
Zimbabwe	1 869	4 384	0.43	2 429	-0.6	-117
Madagascar	1 237	2 833	0.44	1 479	-0.4	-171
Dem. Rep. of Congo	1 217	3 757	0.32	2 536	-1.7	-42
Niger	852	2 397	0.36	1 395	-1.1	-62

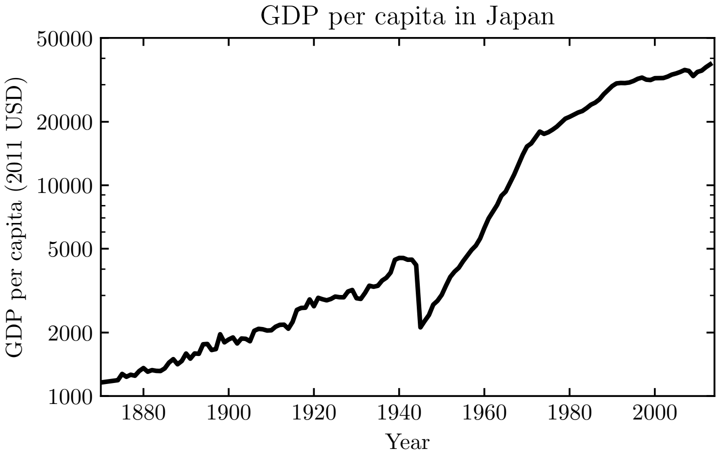
Growth fact 3

World growth rates have increased sharply in the twentieth century.



Growth fact 3

For individual countries, growth rates also change over time.



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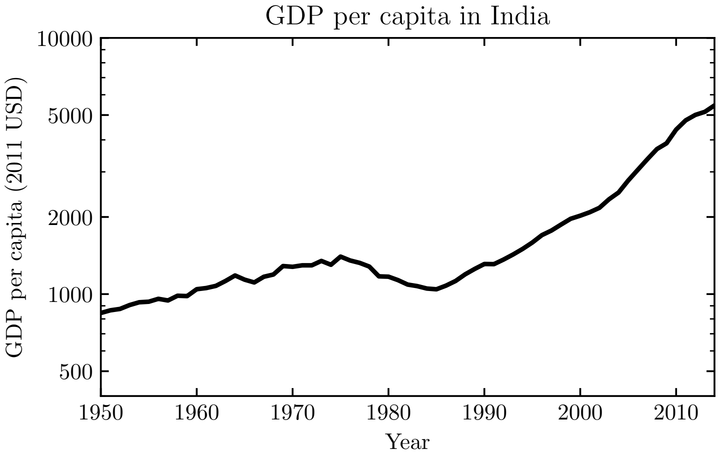


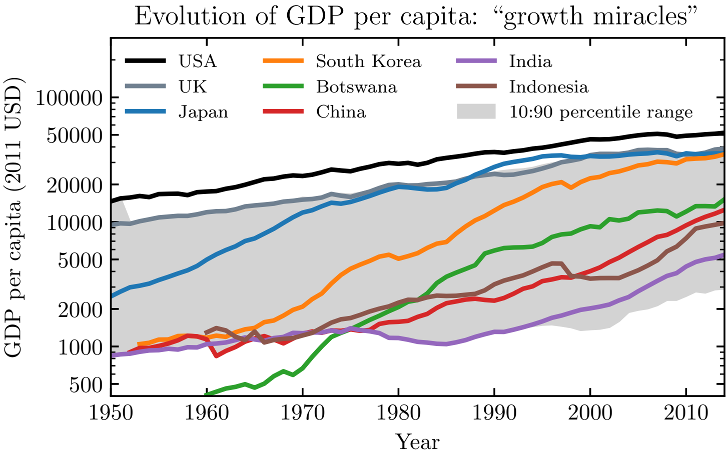
Growth fact 3

Growth rates are not generally constant over time.
For the world as a whole, growth rates were close to zero over most of history but have increased sharply in the twentieth century.
For individual countries, growth rates also change over time.

- The big changes in growth rates over history are from pre-Industrial Revolution (close to 0% growth) to modern times (roughly 1.85% growth per year for developed countries).
- The big changes in growth rates within countries tend to be as they transition from poor to rich (e.g. Japan), after which growth slows down.

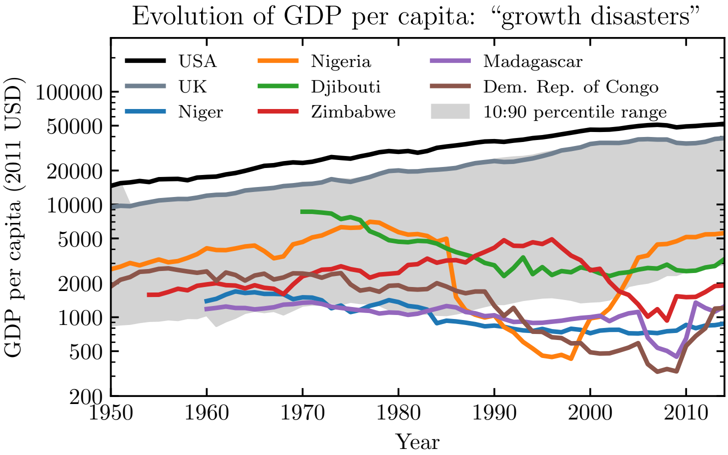
Growth fact 4

Countries can go from being "poor" to being "rich".



Growth fact 4

Countries can go from being “rich” to being “poor”.



Growth fact 4

A country's relative position in the world distribution of per capita incomes is not immutable. Countries can go from being "poor" to being "rich", and vice versa.

- The "growth miracles" in 1960 were very poor.
  Now they are catching up to the rich countries.
- The "growth disasters" were in 1960 richer than East Asia.
  Now they are well behind.

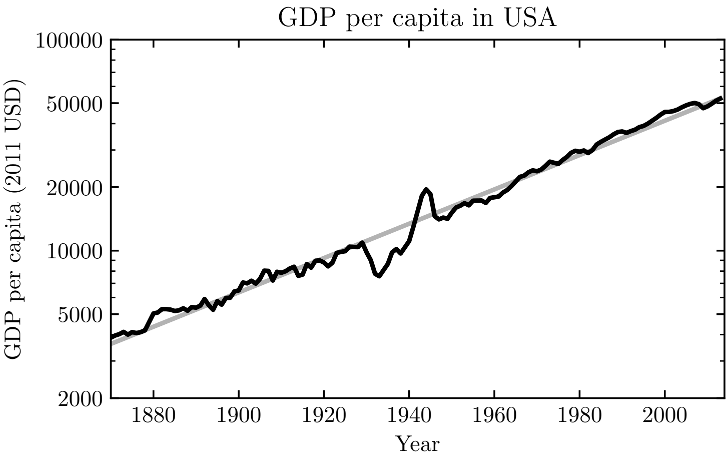
Kaldor’s stylized facts

In the USA (and other developed countries):

1. Per capita output grows over time,
   and its growth rate does not tend to diminish.
2. Physical capital per worker grows over time.
3. The rate of return to capital is not trending.
4. The ratio of physical capital to output is nearly constant.
5. The shares of labor and physical capital in national income
   are nearly constant.
6. Real wage grows over time.

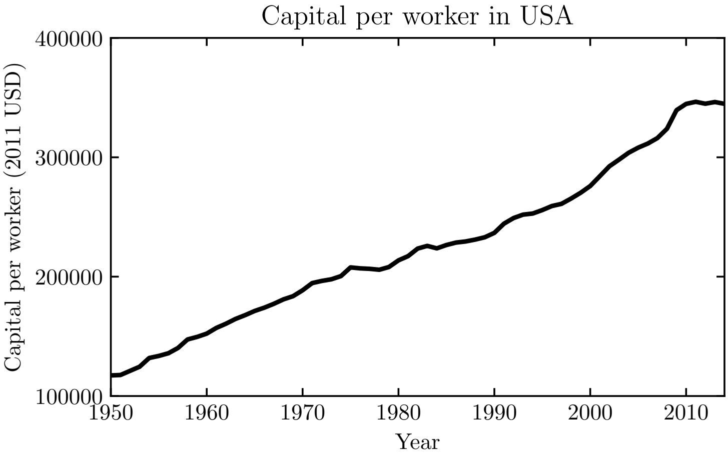
Kaldor's stylized fact 1

Per capita output grows over time,
and its growth rate does not tend to diminish.



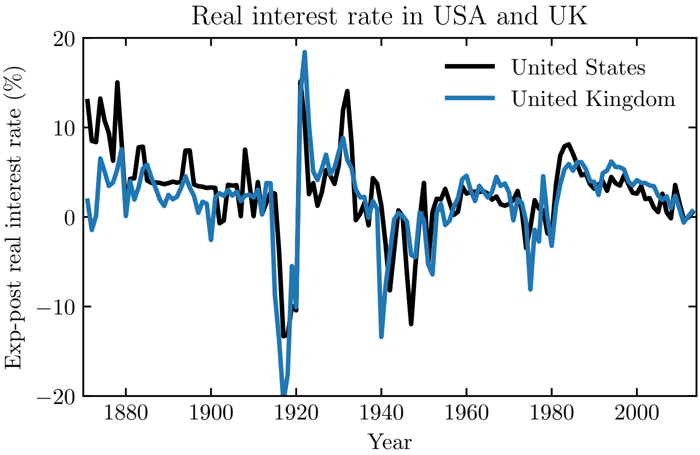
Kaldor's stylized fact 2

Physical capital per worker grows over time.



Kaldor's stylized fact 3

The rate of return to capital is not trending.



Kaldor’s stylized fact 3

The rate of return to capital is not trending.

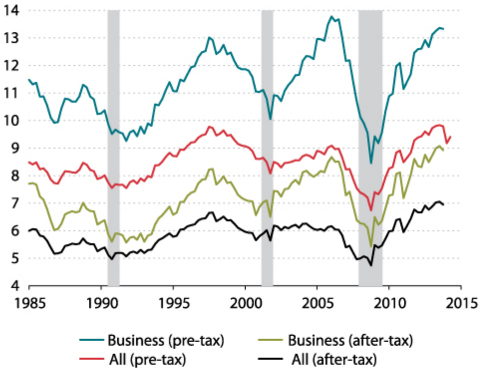
DeLong (2015)

Kaldor's stylized fact 3

The rate of return to capital is not trending.

Figure 2

Real Returns on Capital (percent)

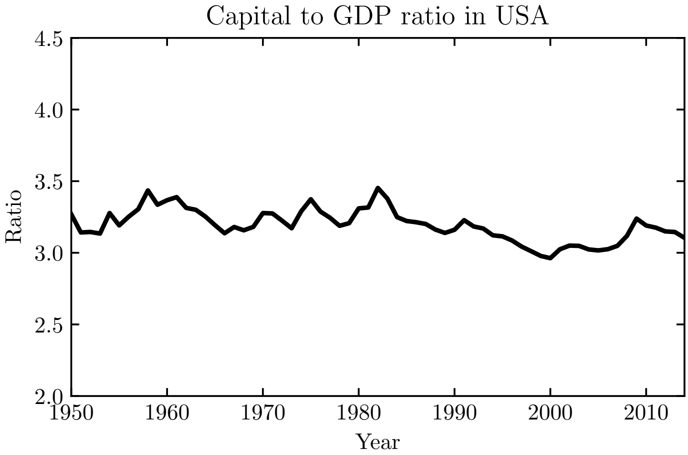


SOURCE: Authors' calculations; for details, see Gomme, Ravikumar, and Rupert (2011).

Gomme, Ravikumar and Rupert (2015) *Secular Stagnation and Returns on Capital*

Kaldor’s stylized fact 4

The ratio of physical capital to output is nearly constant.

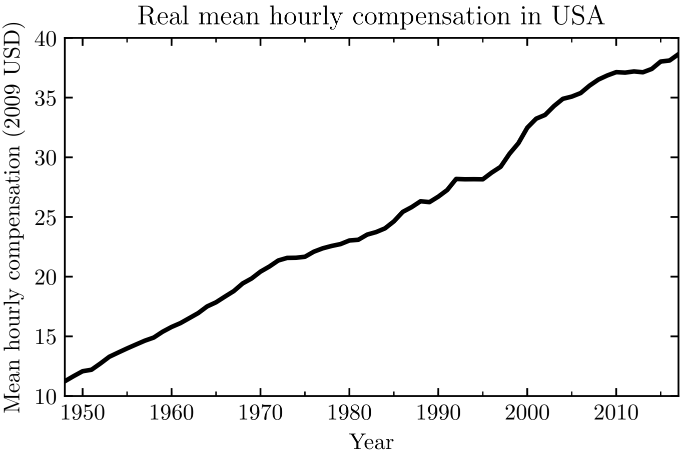


Kaldor's stylized fact 5

The shares of labor and physical capital in national income are nearly constant.

Kaldor's stylized fact 6

Real wage grows over time.



Explaining growth

We want to explain:

- Why some countries are poor and other rich?
- Why some countries that were previously poor become rich?
- Why not all poor countries catch up to rich countries?
- Why do rich countries still grow?

Solow-Swan model

Developed by Robert Solow (1956) and Trevor Swan (1956).

Growth in income per capita comes from two sources:

- Capital accumulation (endogenous).
- Improvements in technology (exogenous).

But capital accumulation alone cannot sustain growth in the absence of technology improvements.

Does not explain “deep” sources of economic growth.

Departure point for growth theory.

Simplifications and assumptions

- Closed economy.
- No government.
- Single, homogenous final good with its price normalized to 1 in each period (all variables are expressed in real terms).
- Two types of representative agents:
  - Firms.
  - Households.

Production

Real GDP is produced according to a neoclassical prod. function:

$$Y_t = F\left(K_t, A_t L_t\right)$$

where Y is real GDP, F is a neoclassical prod. function, K is capital stock, A is the technology level and L is the number of workers.

Technology grows at a rate g and increases productivity of labor (otherwise Kaldor's stylized facts would be violated):

$$A_{t+1} = \left(1 + g\right) A_t$$

Very often we use a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha}$$

Like other neoclassical prod. functions, it exhibits constant returns to scale – doubling inputs K and L doubles the amount produced:

$$\left(zK_t\right)^{\alpha} \left(A_t \cdot zL_t\right)^{1-\alpha} = z^{\alpha} z^{1-\alpha} K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha} = zY_t$$

Firms

Perfectly competitive firms maximize their profit:

$$\max_{K_t, L_t} \quad \Pi_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha} - r_t^k K_t - w_t L_t$$

where r^k denotes the rental rate on capital.

FOCs:

$$\begin{aligned} K_t \quad &: \quad \alpha K_t^{\alpha-1} \left(A_t L_t\right)^{1-\alpha} - r_t^k = 0 && \rightarrow \quad r_t^k = \alpha \frac{Y_t}{K_t} \\ L_t \quad &: \quad (1-\alpha) K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha} - w_t = 0 && \rightarrow \quad w_t = (1-\alpha) \frac{Y_t}{L_t} \end{aligned}$$

Total factor payments are equal to GDP:

$$r_t^k K_t + w_t L_t = \alpha \frac{Y_t}{K_t} K_t + (1-\alpha) \frac{Y_t}{N_t} L_t = \alpha Y_t + (1-\alpha) Y_t = Y_t$$

Factor shares

Calculate the fraction of GDP that is paid to each factor:

$$\frac{w_t L_t}{Y_t} = \frac{(1-\alpha)\,\frac{Y_t}{L_t} \cdot L_t}{Y_t} = (1-\alpha) \quad \text{and} \quad \frac{r_t^k K_t}{Y_t} = \frac{\alpha \frac{Y_t}{K_t} \cdot K_t}{Y_t} = \alpha$$

Cobb-Douglas function implies constant shares
of labor and physical capital in income.

Confronting with the US data, we can obtain $\alpha \approx \frac{1}{3}$ and $(1-\alpha) \approx \frac{2}{3}$.

Households

Own factors of production (capital and labor)
and earn income from renting them to firms.

Each households supplies one unit of labor: $L_t = N_t$
and population grows at a rate n:

$$N_{t+1} = (1 + n)\, N_t$$

Capital accumulates from investment I_t and depreciates at rate δ:

$$K_{t+1} = I_t + (1 - \delta)\, K_t$$

Income of households is consumed or saved (invested):

$$Y_t = w_t L_t + r_t^k K_t = C_t + S_t = C_t + I_t$$

Households **don't optimize**, save a constant fraction s of income:

$$I_t = sY_t \quad \text{and} \quad C_t = (1 - s)\, Y_t$$

GDP per worker

Usually we are most interested in GDP per worker (or per capita), y:

$$y_t \equiv \frac{Y_t}{L_t} = \frac{K_t^\alpha \left(A_t L_t\right)^{1-\alpha}}{L_t} = A_t \left(\frac{K_t}{A_t L_t}\right)^\alpha \equiv A_t \hat{k}_t^\alpha$$

where $\hat{k}$ is capital K divided per effective unit of labor (AL).

Clearly, GDP per worker increases due to improvements
in technology and due to capital accumulation.

The prod. function exhibits diminishing marginal returns to capital.
GDP p.w. increases with $\hat{k}$, but the size of the increase falls with $\hat{k}$.

It is also useful to define output per effective unit of labor $\hat{y}$:

$$\hat{y}_t = \frac{y_t}{A_t} = \hat{k}_t^\alpha$$

Capital accumulation

Capital accumulates according to:

$$K_{t+1} = sY_t + (1-\delta)\,K_t$$

And capital per effective labor according to:

$$\begin{aligned}
K_{t+1} &= sY_t + (1-\delta)\,K_t \quad \Big| \quad : A_t L_t \\
\frac{K_{t+1}}{A_{t+1}L_{t+1}}\frac{A_{t+1}}{A_t}\frac{L_{t+1}}{L_t} &= s\frac{Y_t}{A_t L_t} + (1-\delta)\,\frac{K_t}{A_t L_t} \\
\hat{k}_{t+1}\,(1+g)\,(1+n) &= s\hat{y}_t + (1-\delta)\,\hat{k}_t \\
\hat{k}_{t+1} &= \frac{s\hat{k}_t^{\alpha} + (1-\delta)\,\hat{k}_t}{(1+g)\,(1+n)}
\end{aligned}$$

The growth rate of capital per effective labor equals:

$$\begin{aligned}
\hat{k}_{t+1} - \hat{k}_t &= s\hat{k}_t^{\alpha} - \delta\hat{k}_t - (n+g+ng)\,\hat{k}_{t+1} \\
g_{\hat{k}} \equiv \Delta\hat{k}_{t+1}/\hat{k}_t &\approx s\hat{k}_t^{\alpha-1} - (\delta+n+g)
\end{aligned}$$

where the approximation assumes $ng \approx 0$, $n\Delta\hat{k}_{t+1} \approx 0$, $g\Delta\hat{k}_{t+1} \approx 0$.

Balanced growth path (steady state)

Variables per effective labor converge to their steady state values.
If $\hat{k}_{t+1} = \hat{k}_t = \hat{k}^*$ then:

$$\hat{k}^* (1 + n + g) = s(\hat{k}^*)^\alpha + (1 - \delta) \hat{k}^*$$

$$\hat{k}^* (\delta + n + g) = s(\hat{k}^*)^\alpha$$

$$\hat{k}^* = \left(\frac{s}{\delta + n + g} \right)^{\frac{1}{1-\alpha}}$$

$$\hat{y}^* = \left(\frac{s}{\delta + n + g} \right)^{\frac{\alpha}{1-\alpha}}$$

Along the balanced growth path (BGP) variables per worker grow together with increases in technology:

$$y_t^* = A_t \hat{y}^* \quad \rightarrow \quad g_y^* = \frac{\Delta y_{t+1}^*}{y_t^*} = \frac{\Delta A_{t+1}}{A_t} = g$$

And aggregate variables like aggregate capital and GDP grow at the sum of rates of increase in population and technology.

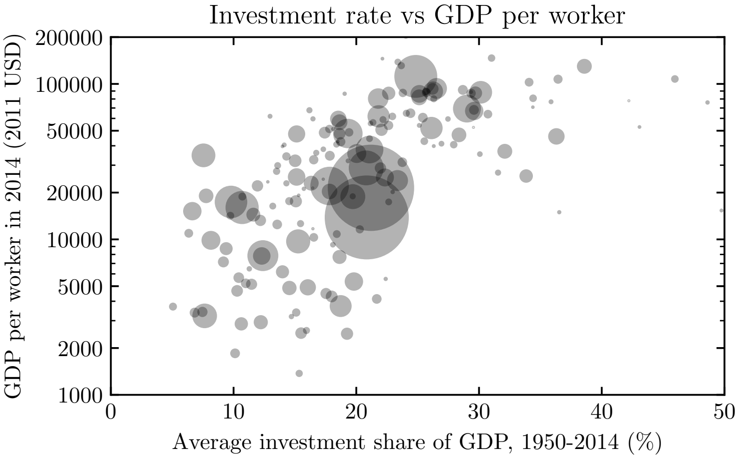
Comparative statics

Solow-Swan model predicts that the BGP level of GDP per worker:

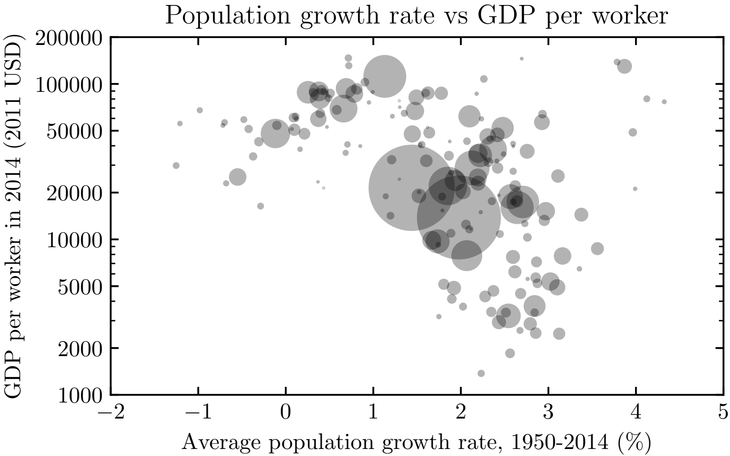
$$y_t^* = A_t \left(\frac{s}{\delta + n + g} \right)^{\frac{\alpha}{1-\alpha}}$$

is higher in countries with higher investment share of GDP s
and higher technology level A,
and lower in countries with higher population growth rate n.

Investment share of GDP s vs GDP per worker y



Population growth rate n vs GDP per worker y



Transitional dynamics

We are also interested in the determinants of growth rates in GDP per worker outside of the BGP.

Start with growth rates of GDP per effective labor:

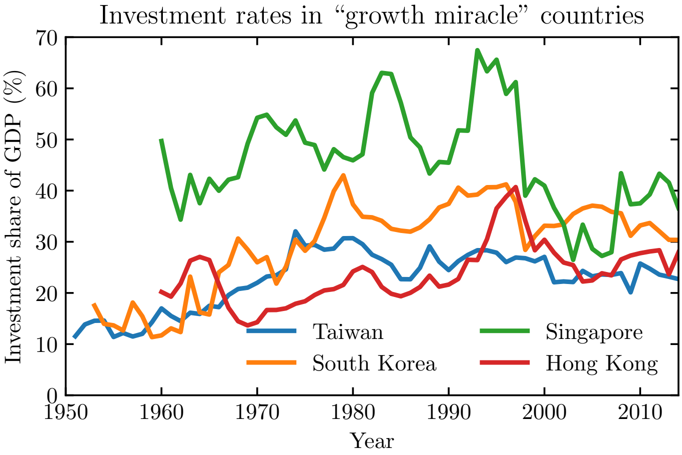
$$g_{\hat{y}} \approx \ln\left(\hat{y}_{t+1}/\hat{y}_t\right) = \ln(\hat{k}_{t+1}^{\alpha}/\hat{k}_t^{\alpha}) = \alpha \ln(\hat{k}_{t+1}/\hat{k}_t) \approx \alpha g_{\hat{k}}$$
$$g_{\hat{y}} \approx \alpha \left[s\hat{k}_t^{\alpha-1} - (\delta + n + g)\right]$$

To obtain growth rate of GDP per worker, add the growth rate of technology g:

$$\begin{aligned} g_y &= \alpha \left[s\hat{k}_t^{\alpha-1} - (\delta + n + g)\right] + g \\ &= \alpha \left[s\hat{k}_t^{\alpha-1} - (\delta + n)\right] + (1-\alpha)\, g \end{aligned}$$

An increase in s or a decrease in n **temporarily** increases the growth rate of GDP per worker. Note that even if higher g decreases $\hat{k}^*$, it increases the rate of growth of GDP per worker.

Investment share of GDP s in "growth miracle" countries



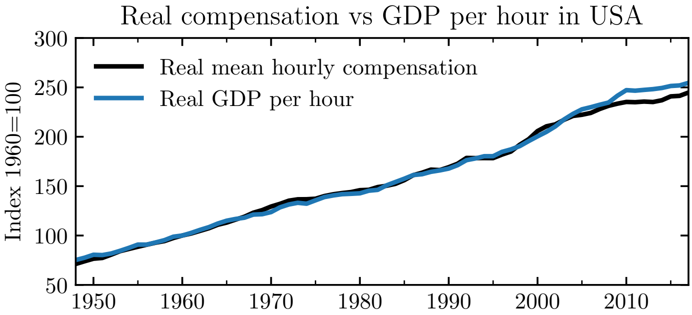
Factor payments once again

Using $\hat{k}^*$ as capital per effective labor along the BGP, let us revisit factor prices:

$$(r_t^k)^* = \alpha K_t^{\alpha-1} (A_t L_t)^{1-\alpha} = \alpha (\hat{k}^*)^{\alpha-1}$$

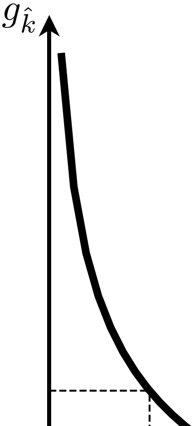
$$w_t^* = (1-\alpha) K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha} = (1-\alpha) A_t (\hat{k}^*)^{\alpha}$$

The model predicts that along the BGP the interest rates are constant while hourly wages grow at the same rate as GDP per hour:



Convergence

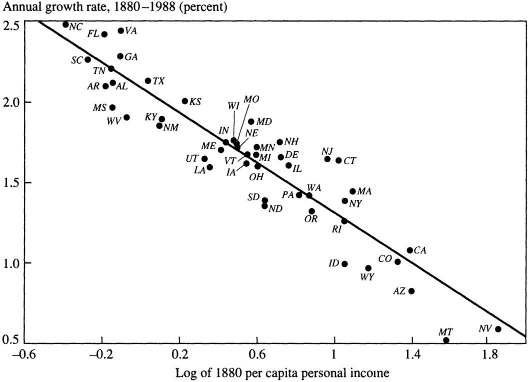
Solow-Swan model predicts that if countries have access to the same technology and share the same steady state, then ones that are poorer should grow faster:



Convergence: USA

We can observe convergence across individual states in USA:

Figure 1. Convergence of Personal Income across U.S. States: 1880 Income and Income Growth from 1880 to 1988

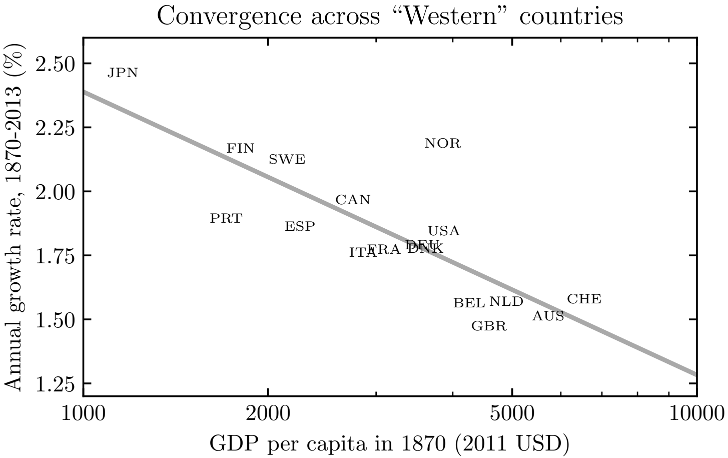


Sources: Bureau of Economic Analysis (1984), Easterlin (1960a, 1960b), and *Survey of Current Business*, various issues. The postal abbreviation for each state is used to plot the figure. Oklahoma, Alaska, and Hawaii are excluded from the analysis.

Barro and Sala-i-Martin (1991) *Convergence across States and Regions*

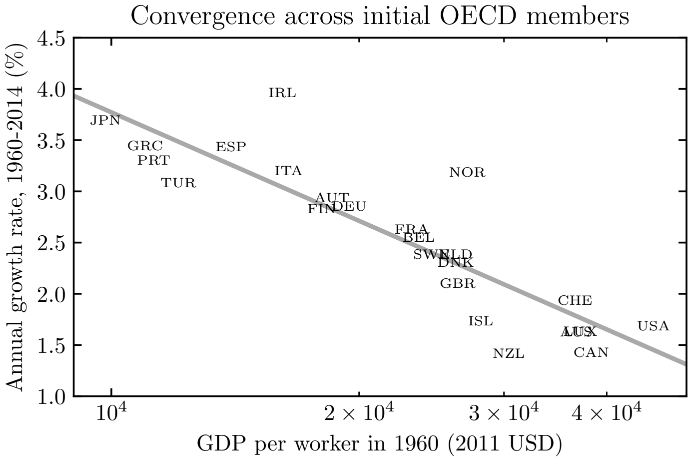
Convergence: “West”

We can observe convergence across “Western” countries (+ Japan):



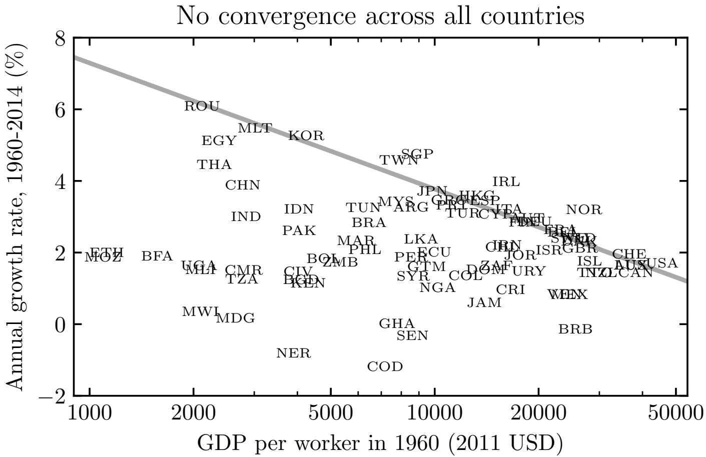
Convergence: OECD

We can observe convergence across initial OECD members:



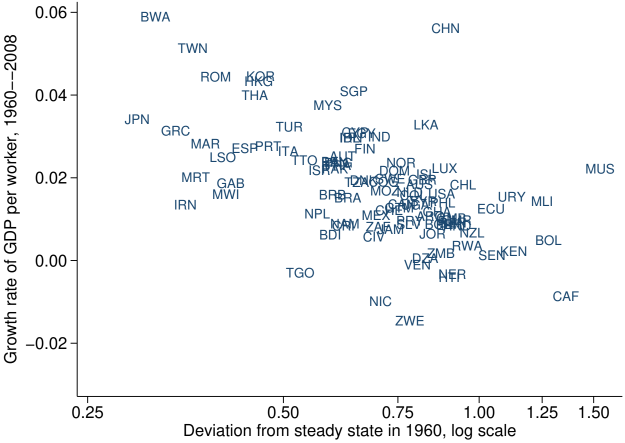
Convergence: conditional/club, but not absolute

In general it is not true that poorer countries grow faster:



Conditional convergence

But countries grow faster the further away they are from their own steady state:



Jones and Vollrath (2013) *Introduction to Economic Growth*

Speed of convergence

The model implies a relationship between the distance from steady state and the current rate of growth:

$$g_y \approx -\underbrace{(1-\alpha)(\delta+n+g)}_{\beta}(\ln y_t - \ln y_t^*)$$

Econometric studies both on individual countries and states within USA find that $\beta \approx 0.02$, meaning that it takes about 35 years to close half of the gap between the current income and the steady state.

Given sensible parameter values: $\alpha = 0.33$, $\delta = 0.05$, $n = 0.01$, $g = 0.02$, the model generates $\beta = 0.053$, implying that it would take about 13 years to close half of the gap, a very unrealistic number.

Adding human capital allows the model to assign lower weight to raw labor and be consistent with slow convergence.

Human capital augmented Solow model

The production function that accounts for human capital:

$$
\begin{aligned}
Y_t &= K_t^{\alpha}\left(A_t H_t\right)^{1-\alpha} \\
H_t &= h\left(u_t\right) L_t
\end{aligned}
$$

where u are average years of schooling. Benchmark empirical estimates on returns to schooling are expressed via the h function:

$$
\ln h(u) = \begin{cases} 0.134 \cdot u & \text{if } u \leq 4 \\ 0.134 \cdot 4 + 0.101 \cdot (u - 4) & \text{if } 4 < u \leq 8 \\ 0.134 \cdot 4 + 0.101 \cdot 4 + 0.068 \cdot (u - 8) & \text{if } u > 8 \end{cases}
$$

The estimates capture the empirical regularity that schooling boosts individuals' wages. Wages contain not only rewards to raw labor, but also to human capital. Empirical estimates of the income share of "broad capital" are consistent with convergence factor $\beta \approx 0.02$.

Human capital augmented Solow model

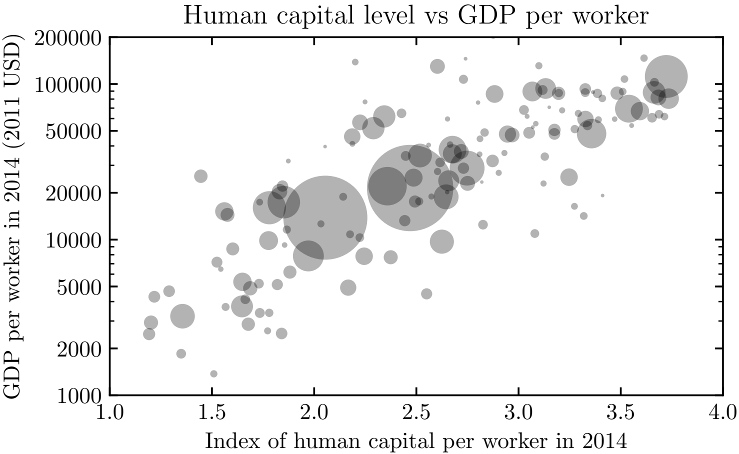
The production function that accounts for human capital:

$$\begin{aligned} Y_t &= K_t^{\alpha} \left(A_t H_t\right)^{1-\alpha} \\ H_t &= h\left(u_t\right) L_t \end{aligned}$$

Human capital generates level effects for GDP per worker along the BGP:

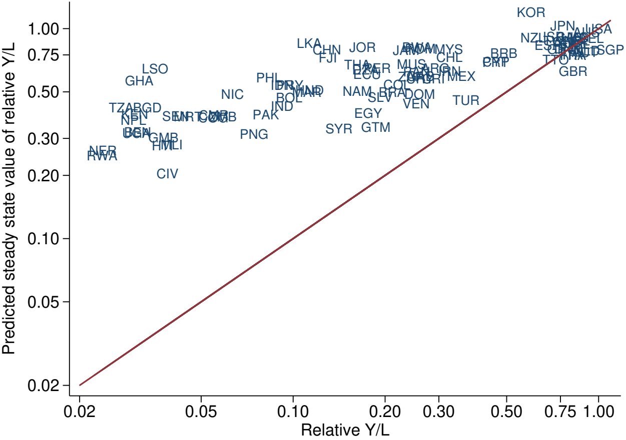
$$y_t = A_t h\left(u_t\right) \left(\frac{s}{\delta + n + g}\right)^{\frac{\alpha}{1-\alpha}}$$

Human capital per capita h vs real GDP per worker y



Fit of human capital-augmented Solow model

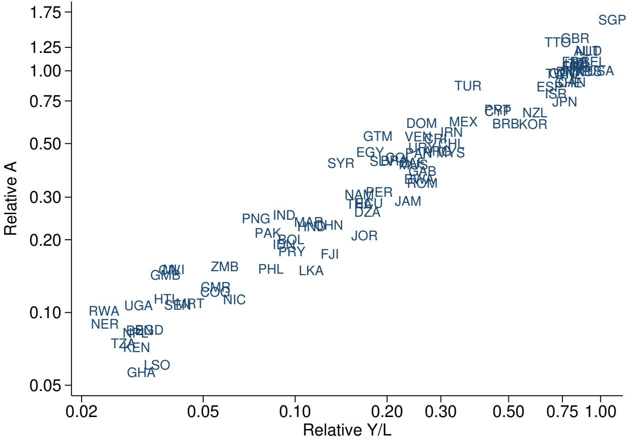
Suggests that poor countries “should” be richer:



Jones and Vollrath (2013) *Introduction to Economic Growth*

Solow residual: accounting for technology differences

There are also significant differences in technology across countries:



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Takeaway

- Long run growth stems from improvements in technology.
- Countries can achieve higher balanced growth paths if they accumulate more physical and human capital.
- Just as important as accumulation is technology adoption.
- Did not touch on "deep" causes of growth – we treated many choice variables as exogenous parameters:
  - Countries with low s may not protect private ownership properly or have underdeveloped financial system.
  - Countries with high n may have high mortality rates incentivizing families to have many children in hopes that at least some survive into adulthood to be able to support their (then old) parents.
  - Countries may have high u because they are already rich and children do not have to work there.
  - Groups of interest within a country may obstruct technology adoption if they have a monopoly over the old technology.