University of Warsaw
Faculty of Economic Sciences

Introduction to modern macroeconomics

Advanced Macroeconomics QF: Lecture 1

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Course organization

Website & contact information

- Website: http://coin.wne.uw.edu.pl/mbielecki/
  $\hookrightarrow$ Advanced Macroeconomics QF
  - Lecture slides and/or notes available on Monday prior to the lecture
- E-mail: mbielecki@wne.uw.edu.pl
- Office hours: Fridays, by appointment

Assessment

You will be graded on the basis of

- Final exam (70%), consisting of two parts
  - Open questions (30%): you will answer to 3 out of 4 questions
  - Problems (40%): you will solve 2 out of 3 problems
- Homeworks (30%)
  - There will be 5 problem sets and you will be given 2 weeks to hand in the solutions

Points from the final exam and homeworks add up

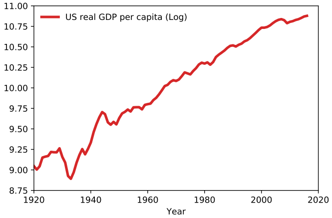
You need at least 50% to pass the course

Score	[0, 50)	[50, 60)	[60, 70)	[70, 80)	[80, 90)	[90, 100]
Grade	2	3	3.5	4	4.5	5

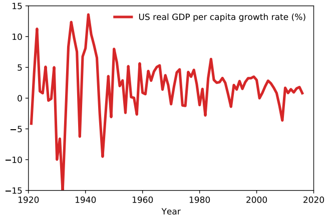
Topics of interest

We want to understand the mechanisms behind

Long-run growth



Business cycles



using the tools of modern macroeconomics

Course overview

- Microfoundations (3 lectures)
  - Introduction to modern macroeconomics
  - Consumption
  - Investment
- Economic growth (4 lectures)
  - Growth facts. Solow-Swan model
  - Ramsey-Cass-Koopmans model
  - Overlapping Generations model
  - Endogenous growth models
- Business cycles (6 lectures)
  - Business cycle facts. Real Business Cycles model
  - Models of unemployment
  - Incomplete markets models. Inequality
  - New Keynesian model
  - Monetary policy design
  - Financial frictions. Great Recession

Questions?

Modern macroeconomics

Modern macroeconomics[1]

- Why is it so different from your previous macro courses?
- Cornerstone of "modern" (post-1970s) macroeconomics

Macroeconomics is microeconomics

(at a high level of aggregation)

[1] This section heavily borrows from Lutz Hendricks.

An "old" macroeconomic model

- Consumption function: $C = C_0 + cY$
- Investment function: $I = I_0 - bi$
- Identity: $Y = C + I + G$
- IS curve:

$$(1 - c)\,Y = C_0 + I_0 + G - bi$$

- Real money demand: $L = L_0 + kY - hi$
- Real money supply: M/P
- LM curve:

$$M/P = L_0 + kY - hi$$

Shortcomings of the IS-LM model

- Government spending always raises output
  - Supply side constraints are missing
- Consuming more / saving less always raises output
  - The model lacks capital
- Behavior depends on parameters: c, b, k, h and C_0, I_0, L_0
  - Which parameters are stable?
  - Can policy affect these parameters?
- Expectations are not modeled

Modern macroeconomic models

- Behavior of agents is **derived** from the solutions to their optimization problems
  - Often involving time and uncertainty
- Agents have model-consistent **endogenous expectations**
- **Aggregate** outcomes result from **individual** decisions
- The economy is in **general equilibrium**
  - Which does not mean it is "at rest"
  - Nor that the outcomes are desirable

What do we gain from this approach?

- Consistency
  - Aggregate relationships satisfy all individual constraints
- Transparency
  - Assumptions about the fundamentals are clearly stated
- Non-arbitrary behavior
  - In old macro, results depend on the assumed behavior
  - In modern macro, behavior is derived
- Testability
  - Models can be tested against both macro and micro data
- Welfare analysis
  - It is possible to evaluate how a policy change affects the welfare (utility) of each agent

How to build a model in 4 simple steps

Step 1: Describe the economy

- List the agents (households, firms, governments)
- For each agent define
  - Demographics: population grows at rate n
  - Preferences: households maximize utility $u(c)$
  - Endowments: each household has one unit of time
  - Technologies: output is produced using $f(k)$
- Define the markets in which agents interact
  - Households work for firms (labor market)
  - Households purchase goods from firms (goods market)

How to build a model in 4 simple steps

Step 2: Solve each agent's problem

- Write down the maximization problem each agent solves
  - Households maximize utility, subject to the budget constraint
  - Firms maximize profits, subject to the production function
- Derive a set of equations that determine the agent's choice
  - Households' consumption and saving functions
  - Firms' demands for factors of production

How to build a model in 4 simple steps

Step 3: Market clearing

- For each market, calculate supply and demand by each agent
- Aggregate supply $=\sum$ individual supplies (same for demand)
- Aggregate supply $=$ Aggregate demand

Step 4: Define the equilibrium

- Collect all endogenous objects
  - Consumption, output, wage rate, ...
- Collect all equations
  - First order conditions, market clearing conditions
- You should have N equations to solve for N variables
  - Quantities and prices

Before we run, we walk

- We will not consider these general equilibrium models in the first couple of lectures
- First, let us focus on getting comfortable with Steps 1 and 2
- Very similar to the partial equilibrium approach in micro

Example 1: One period, two goods

Step 1: Describe the economy

- Agents and their demographics
  - A household who lives for one period
- Preferences
  - The household values consumption of two goods with preferences described by the utility function $U(c_1, c_2)$
- Endowments
  - The household receives endowments of two goods (y_1, y_2)
- Technology
  - There is no production, but goods can be traded
- Markets
  - There are competitive markets for the two goods
  - The prices of the two goods are p_1 and p_2

Step 2: Solve the agent's problem

- The household maximizes $U(c_1, c_2)$
  subject to the budget constraint
- The household takes as given the following state variables
  - Market prices for the two goods, p_1 and p_2
  - Endowments y_1 and y_2
- The choice variables are c_1 and c_2
- We can normalize the price of good 1 to unity: $p_1 = 1$
  and call the relative price $p \equiv p_2/p_1$

Households' utility maximization problem (UMP)

- Budget constraint:
  Value of consumption $\leq$ Value of endowments

- The household solves the problem

$$\begin{aligned} \max \quad & U(c_1, c_2) \\ \text{subject to} \quad & c_1 + pc_2 \leq y_1 + py_2 \end{aligned}$$

- A solution to the problem is the pair (c_1, c_2) conditional on the relative price p and endowments (y_1, y_2)
  - Can I replace the symbol "$\leq$" with "$=$" ?
  - Ideas on how to solve this problem?

Method of Lagrange multipliers

- Many good approaches to solving such simple problems
- Fewer good approaches when problems get more complex
- One tool to rule them all – Lagrange function (Lagrangian)

Solving the household's problem

- Set up the **Lagrangian**

$$\mathcal{L} = U(c_1, c_2) + \lambda [y_1 + py_2 - c_1 + pc_2]$$

- Derive the **first order conditions** (FOCs)

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{\partial U(c_1, c_2)}{\partial c_1} - \lambda = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = \frac{\partial U(c_1, c_2)}{\partial c_2} - \lambda p = 0 \quad (2)$$

  The **Lagrange multiplier** λ has a useful interpretation

  - It is the marginal utility of relaxing the constraint a bit
  - In this example λ is the marginal utility of wealth

- The solution is then a vector (c_1, c_2, λ) that satisfies
  - FOCs (1), (2) and the budget constraint $c_1 + pc_2 = y_1 + py_2$

Simplify the conditions

- It is convenient to substitute out the Lagrange multiplier λ

- The ratio of the FOCs implies

$$\frac{\partial U\left(c_1, c_2\right)/\partial c_2}{\partial U\left(c_1, c_2\right)/\partial c_1} \equiv \frac{U_2}{U_1} = p \tag{3}$$

- This is the familiar tangency condition
  - marginal rate of substitution equals relative price

- Now the solution is a pair (c_1, c_2) that satisfies (3) and the budget constraint $c_1 + pc_2 = y_1 + py_2$

Let's make our example more concrete

- Assume logarithmic utility

$$U(c_1, c_2) = \ln c_1 + \beta \ln c_2$$

- The FOCs are

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} - \lambda = 0 \quad \rightarrow \quad \lambda = \frac{1}{c_1}$$
$$\frac{\partial \mathcal{L}}{\partial c_2} = \beta \frac{1}{c_2} - \lambda p = 0 \quad \rightarrow \quad \lambda p = \beta \frac{1}{c_2}$$

- Ratio of FOCs

$$p = \beta \frac{c_1}{c_2} \quad \rightarrow \quad c_2 = \frac{\beta}{p} c_1$$

Let's make our example more concrete

- Ratio of FOCs

$$c_2 = \frac{\beta}{p} c_1$$

- Plug into the budget constraint

$$c_1 + pc_2 = y_1 + py_2$$
$$c_1 + \beta c_1 = y_1 + py_2$$

- Optimal consumption levels

$$c_1 = \frac{y_1 + py_2}{1 + \beta} \quad \text{and} \quad c_2 = \frac{\beta}{p} \frac{y_1 + py_2}{1 + \beta}$$

Example 2: Two periods, one good

Step 1: Describe the economy

- Agents and their demographics
  - A household who lives for two periods
- Preferences
  - The household values consumption in two **time periods** with preferences described by the utility function $U(c_1, c_2)$
- Endowments
  - The household receives **income** in two time periods (y_1, y_2)
- Technology
  - There is no production, but the agent can **save or borrow**
- Markets
  - There is a competitive **financial market**: one unit of first period good saved delivers $(1 + r)$ units of second period good
  - First period good costs 1, second period good costs $1/(1 + r)$

Step 2: Solve the agent's problem

- The household maximizes

$$U(c_1, c_2) = \ln c_1 + \beta \ln c_2$$

  subject to the budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

- The household takes the following state variables as given
  - Interest rate r
  - Incomes y_1 and y_2
- The choice variables are c_1 and c_2

Solving the household's problem

- Set up the Lagrangian

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left[y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right]$$

- Derive the first order conditions (FOCs)

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial c_2} = \beta \frac{1}{c_2} - \lambda \frac{1}{1+r} = 0$$

- Simplify

$$\lambda = \frac{1}{c_1} \quad \text{and} \quad \lambda = \beta \left(1+r\right) \frac{1}{c_2}$$

- Obtain the **intertemporal** condition / **Euler equation**

$$\frac{1}{c_1} = \beta \left(1+r\right) \frac{1}{c_2} \quad \Longleftrightarrow \quad c_2 = \beta \left(1+r\right) c_1$$

Solving the household's problem

- Obtain the **intertemporal** condition / **Euler equation**

$$c_2 = \beta\,(1+r)\,c_1$$

- Combine it with the budget constraint

$$\begin{aligned} c_1 + \frac{c_2}{1+r} &= y_1 + \frac{y_2}{1+r} \\ c_1 + \frac{\beta\,(1+r)\,c_1}{1+r} &= y_1 + \frac{y_2}{1+r} \\ c_1 + \beta c_1 &= y_1 + \frac{y_2}{1+r} \end{aligned}$$

- Optimal consumption levels

$$\begin{aligned} c_1 &= \frac{1}{1+\beta}\left[y_1 + \frac{y_2}{1+r}\right] \\ c_2 &= \frac{\beta}{1+\beta}\left[(1+r)\,y_1 + y_2\right] \end{aligned}$$