Marcin Bielecki, Advanced Macroeconomics QF, Fall 2018
Homework 4 – deadline: 23rd January, 11.30 AM

Problem 1

Consider an economy where final goods are produced according to the following production function:

$$Y_t = L^{1-\alpha} \sum_{i=1}^{M_t} x_{it}^{\alpha}$$

where L is the constant labor force, M_t is the number of invented types of intermediate goods and x_{it} denotes usage of intermediate good type i in the final goods production. The inventor of type i holds a perpetual patent that gives exclusive, monopolistic rights to produce this type of an intermediate good. Assume that the government taxes the monopolists and each of them pays tax T per period. The rest of the problem structure is the same as in Lecture 8 Notes, section 1.

(a) Solve the profit maximization problem of the final goods producer to obtain the (inverse) demand function for intermediate goods.

(b) Solve the profit maximization problem of the intermediate goods producers (monopolists). Find the optimal price, quantity produced and maximal after-tax profit per period.

(c) Assume that inventing a new type of an intermediate good costs $1/\eta$ units of the final good. Equalize the cost of invention with the discounted after-tax value of profit flows of a new monopolist.

(d) Transform the expression from (c) to obtain the real interest rate. Use the Euler equation: $g = (r - \rho) / \sigma$ to obtain the equilibrium growth rate of the economy.

(e) Discuss how the growth rate of the economy depends on the level of taxation T. Should the government aim to reduce the after-tax profits of the monopolists to 0?

Problem 2

Consider the following model. Households maximize expected utility subject to the standard budget constraint:

$$\begin{aligned} \max \quad & U_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t \right] \\ \text{subject to} \quad & c_t + k_{t+1} = w_t h_t + (1 + r_t) k_t + div_t \\ & h_t = 1 \end{aligned}$$

Firms maximize profits subject to the Cobb-Douglas production function:

$$\begin{aligned} \max \quad & div_t = y_t - w_t h_t - (r_t + \delta) k_t \\ \text{subject to} \quad & y_t = z_t k_t^{\alpha} h_t^{1-\alpha} \end{aligned}$$

where $\delta = 1$ (capital deprecates fully). The technology constant z evolves according to the process:

$$z_t = (1 - \rho_z) + \rho_z z_{t-1} + \varepsilon_{z,t}$$

(a) Derive the first order conditions of the households.

(b) Derive the first order conditions of the firm.

(c) Find the steady state of the system.

(d) Assuming that household behavior can be expressed as $c_t = (1 - s) y_t$ where s is a constant, find the value of s as a function of model parameters.

(e) Find the expression for k_{t+1} as a function of variables at time t.

Problem 3

In Lecture 10 Slides we considered a model where permanent changes to marginal productivity of labor reduced the unemployment rate. This would imply that with trend productivity growth unemployment would disappear over time. Let's modify our model so that it is consistent with stationary unemployment in face of trend productivity growth. Suppose that the flow cost of a vacancy κ and the imputed value of free time b are functions of the wage rate w (instead being exogenous). In particular, assume that $\kappa_t = \kappa_0 w_t$ and $b_t = b_0 w_t$.

(a) Determine the formula for job creation and wage setting along the balanced growth path (steady state).

(b) How do θ and wages along the balanced growth path react to productivity changes?

(c) Does a continuous growth of productivity lead to a decrease in the long run unemployment rate?