Marcin Bielecki, Advanced Macroeconomics QF, Fall 2018
Homework 3 – deadline: 19th December, 11.30 AM

Problem 1

Suppose you have a two-period overlapping generations (OLG) model. Each generation of agents has the same number of agents, N. There is no production; each agent receives an endowment in each period. However, agents are able to buy government bonds when they are young and redeem them when they are old. The interest rate is constant over time. Each young agent born at time t maximizes:

$$\begin{aligned} \max \quad & U_t^y = \ln c_t^y + \beta \ln c_{t+1}^o \\ \text{subject to} \quad & c_t^y + b_{t+1} = y^y - \tau_t^y \\ & c_{t+1}^o = y^o - \tau_{t+1}^o + (1+r)\, b_{t+1} \end{aligned}$$

where y^y and y^o denote, respectively, endowments of young and old, with $y^y - \tau_t^y > y^o - \tau_t^o$ and it is assumed that $\beta = 1/(1+r)$.

Each old agent alive at t (born at time $t-1$) maximizes:

$$\begin{aligned} \max \quad & U_t^o = \ln c_t^o \\ \text{subject to} \quad & c_t^o = y^o - \tau_t^o + (1+r)\, b_t \end{aligned}$$

Suppose the government reduces the taxes of period t old by issuing bonds to the period t young and buys back the bonds in $t+1$ by raising the taxes on the period $t+1$ old. In short:

$$\begin{aligned} d\tau_t^o &= -db_{t+1} = -T \\ d\tau_{t+1}^o &= -(1+r)\, d\tau_t^o = (1+r)\, T \end{aligned}$$

where T is a positive value denoting the size of the tax change.

(a) What would be the value of aggregate period t consumption $C_t = Nc_t^y + Nc_t^o$ if the tax change was not implemented?

(b) What happens to aggregate period t consumption when taxes are changed at the start of period t, i.e., what is $dC_t/d\tau_t^o$?

(c) Given (a) and (b), does the Ricardian equivalence hold? Why or why not?

Problem 2

Consider a Ramsey-Cass-Koopmans economy where for simplicity we assume $g = 0$ and $A = 1$. The representative households solve the following utility maximization problem:

$$\begin{aligned} \max \quad & U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \\ \text{subject to} \quad & (1+n)\, a_{t+1} = (1+r_t)\, a_t + w_t - c_t + v_t \end{aligned}$$

where v is the lump-sum transfer from the government to households. Discount factor β can be expressed as $1/(1+\rho)$ where $\rho > 0$ denotes households' discount rate.

The representative firm solves the following profit maximization problem:

$$\begin{aligned} \max \quad & \pi = (1-\tau^y)\, Y_t - (r_t + \delta)\, K_t - w_t L_t \\ \text{subject to} \quad & Y_t = K_t^\alpha L_t^{1-\alpha} \end{aligned}$$

where τ^y is the firm revenue tax.

(a) Derive the first order conditions of the household.

(b) Recast the problem of the firm in per worker terms. Derive the first order conditions of the firm.

(c) Write down the government budget constraint. Using the assumptions of closed economy and balanced government budget, find the conditions for general equilibrium in this economy.

(d) Find the steady state level of capital per worker k^* and consumption per worker c^* in this economy. Discuss how they depend on the tax rate τ^y.

Problem 3

Consider a perfectly competitive economy where individual price taking firms face the following production function:

$$Y_{it} = A_t K_{it}^{\alpha} L_{it}^{1-\alpha}$$

Assume that publicly available technology depends on the average level of capital per worker:

$$A_t = \left(\frac{\sum_i K_{it}}{\sum_i L_{it}} \right)^{\gamma} = k_t^{\gamma}$$

where γ represents a learning-by-doing externality.

The aggregate final goods production is a sum of individual firms' outputs:

$$Y_t = \sum_i Y_{it}$$

For simplicity population growth is assumed to be 0. Consumers maximize their lifetime utility function:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

subject to the budget constraint:

$$a_{t+1} = w_t + (1 + r_t)\, a_t - c_t$$

(a) Find the first order conditions characterizing the optimal choice of the consumer.

(b) Find the first order conditions characterizing the optimal behavior of the firm assuming that there is a constant rate of capital depreciation δ.

(c) Describe the general equilibrium in this economy using (a) and (b).

(d) Draw a phase diagram in the (k, c) space; will the long run equilibrium in this economy be stable if $\alpha + \gamma < 1$? What about the case of $\alpha + \gamma = 1$?

(e) Assuming that the initial level of capital in this economy is below its steady-state value describe the behavior of k, c, y and the growth rate of per capita income over time in the two above mentioned cases.