Marcin Bielecki, Advanced Macroeconomics QF, Fall 2018
Homework 2 – deadline: 21st November, 11.30 AM

Problem 1

Consider the following problem of a manager maximizing the value of the firm:

$$\max_{L_t, I_t^n, K_{t+1}} \quad \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \left[(1-\tau) \left(K_{t+i}^{\alpha} L_{t+i}^{1-\alpha} - w_{t+i} L_{t+i} - \delta K_{t+i} - I_{t+i}^n \right) - \frac{\chi}{2} \frac{\left(I_{t+i}^n \right)^2}{K_{t+i}} \right]$$
$$\text{subject to} \quad K_{t+1} = I_t^n + K_t$$

where τ is a tax levied on firm's profits, K is firm's capital stock, L is the number of firm's employees, $\alpha \in (0,1)$ is output elasticity w.r.t. capital, I^n is net investment and $\delta \in (0,1)$ stands for capital depreciation. Parameter χ describes the magnitude of capital installation costs. Note that the tax code does not treat installation costs as tax deductible.

(a) Write down the problem in the Lagrangian form and derive the first order conditions.

(b) Find the steady state level of q (the Lagrange multiplier). Is it equal to 1?

(c) Find the desired level of firm's capital stock per employee, $k \equiv K/L$, treating interest rate r as given.

(d) How does the desired level of firm's capital stock per employee change when taxes increase? Provide intuition for this result.

(e) How does the desired level of firm's capital stock per employee change when capital installation costs increase? Provide intuition for this result.

Problem 2

Let's examine the role of taxes in the Solow-Swan model. Imagine that the behavior of an economy may be summarized by the following three equations:

$$K_{t+1} = I_t - \delta K_t$$
$$I_t = s(1-\tau) Y_t$$
$$Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}$$

Assume that population grows at rate n and technology at rate g, so that $L_{t+1}/L_t = N_{t+1}/N_t = 1+n$ and $A_{t+1}/A_t = 1+g$, respectively. Income in this economy is taxed with rate τ and the tax revenues are used for government consumption which is useless from the point of view of households.

(a) Transform the three equations into per effective labor form, i.e. divide them by $A_t L_t$. Make use of notational convention $\hat{x}_t \equiv X_t / (A_t L_t)$.

(b) Find the balanced growth path level of capital per effective labor $\hat{k}^*$ in this economy.

(c) Discuss the effects of changes in parameters δ, n, g, s, τ on the economy's balanced growth path level of capital per effective labor $\hat{k}^*$.

(d) Discuss the effects of changes in parameters δ, n, g, s, τ on the economy's balanced growth path level of consumption per effective labor $\hat{c}^*$.

(e) Households care about the level of consumption per capita, i.e. c_t. This variable grows at rate g once the economy reaches its balanced growth path. Discuss whether low g or high g is better from the point of view of households.

Problem 3

Robert Solow in his 1956 article "A Contribution to the Theory of Economic Growth" considered the behavior of economy when output was produced according to other than Cobb-Douglas production functions. One of them was the following Constant Elasticity of Substitution (CES) function:

$$Y_t = \left[aK_t^{\frac{b-1}{b}} + (1-a)\, L_t^{\frac{b-1}{b}} \right]^{\frac{b}{b-1}}$$

where $a \in (0,1)$ and $b > 0$ and for simplicity the technology growth rate is set to 0. The CES function reduces to the Cobb-Douglas function in the limit of $b \to 1$.

Saving and investment behaviour of the economy are described respectively as:

$$\begin{aligned} i_t &= s \cdot y_t \\ (1+n)\, k_{t+1} &= i_t + (1-\delta)\, k_t \end{aligned}$$

where lower case letters i_t, y_t, k_t denote per worker quantities, n denotes population growth, and δ denotes the depreciation rate.

(a) Find the intensive form of the production function $y_t = f(k_t)$ (output per worker).

(b) Find the steady state value for k_t.

(c) Show how an increase in the saving rate affects the steady state level of capital per worker. You may use a graph to show that.

(d) Show how an increase in the population growth rate affects the steady state level of capital per worker. You may use a graph to show that.

(e) Derive the expression for the capital share of income ($r^k k/y$) in the steady state.