University of Warsaw
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Monopolistic competition. Price stickiness New Keynesian model. Monetary policy

Advanced Macroeconomics IE: Lecture 19

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Monopolistic competition: introduction

- Most (if not all) sectors of the economy are not perfectly competitive
- There is a significant markup on prices, averaging around 33%
- Monopolistic competition allows us to introduce a new shock
- Is a stepping stone for nominal frictions models

Empirical evidence on markups

Christopoulou and Vermeulen (2008) Markups in the Euro Area and the US over the period 1981–2004: a comparison of 50 sectors

Table 1. Weighted average markup, 1981-2004

Country	Manufacturing & Construction		Market Services		All (Manufacturing, Construction & Market Services)	
Germany	1.16	(0.01)*	1.54	(0.03)*	1.33	(0.01)*
France	1.15	(0.01)*	1.26	(0.02)*	1.21	(0.01)*
Italy	1.23	(0.01)*	1.87	(0.02)*	1.61	(0.01)*
Spain	1.18	(0.00)*	1.37	(0.01)*	1.26	(0.01)*
Netherlands	1.13	(0.01)*	1.31	(0.02)*	1.22	(0.01)*
Belgium	1.14	(0.00)*	1.29	(0.01)*	1.22	(0.01)*
Austria	1.20	(0.02)*	1.45	(0.03)*	1.31	(0.02)*
Finland	1.22	(0.01)*	1.39	(0.02)*	1.28	(0.01)*
Euro Area	1.18	(0.01)*	1.56	(0.01)*	1.37	(0.01)*
USA	1.28	(0.02)*	1.36	(0.03)*	1.32	(0.02)*

Monopolistic competition: setup

- Two sectors of producers – final and intermediate goods
- Final goods sector is perfectly competitive
- Intermediate goods sector is monopolistically competitive and produces differentiated goods
- There is a degree of market power captured by $\mu \geq 1$
- If $\mu = 1$ then we are in perfect competition
- Higher μ indicates higher monopoly power
- Final goods production function expressed as Dixit-Stiglitz aggregator

$$y_t = \left(\sum_i y_t\left(i\right)^{1/\mu} \right)^{\mu}$$

$$y_t = \left(\int_0^1 y_t\left(i\right)^{1/\mu} \mathrm{d}i \right)^{\mu}$$

- For small μ goods are close substitutes
- For large μ goods are complementary

Final goods producing firm I

Profit maximization problem

$$\begin{aligned} \max \quad & P_t y_t - \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i \\ \text{subject to} \quad & y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu} \end{aligned}$$

Lagrangian

$$\mathcal{L} = P_t y_t - \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i + \lambda_t \left[\left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu} - y_t \right]$$

FOCs

$$\begin{aligned} y_t \quad &: \quad P_t - \lambda_t = 0 \\ y_t(i) \quad &: \quad -P_t(i) + \lambda_t \left[\mu \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu - 1} \cdot \frac{1}{\mu} y_t(i)^{\frac{1}{\mu} - 1} \right] = 0 \end{aligned}$$

Final goods producing firm II

Result

$$P_t(i) = P_t \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i\right)^{\mu-1} y_t(i)^{\frac{1-\mu}{\mu}} \quad \Big| \quad (\cdot)^{\frac{\mu}{\mu-1}}$$

$$P_t(i)^{\frac{\mu}{\mu-1}} = P_t^{\frac{\mu}{\mu-1}} \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i\right)^{\mu} y_t(i)^{-1}$$

$$y_t(i) = \left(\frac{P_t}{P_t(i)}\right)^{\frac{\mu}{\mu-1}} y_t$$

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t$$

Aggregate price index derivation

$$P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i\right)^{1-\mu}$$

Intermediate goods producing firm (simplified) I

For now let's consider production function linear in hours

$$y_t(i) = z_t h_t(i)$$

Cost minimization problem

$$\begin{aligned} \min \quad & tc_t(i) = w_t h_t(i) \\ \text{subject to} \quad & y_t(i) = z_t h_t(i) \end{aligned}$$

Lagrangian

$$\mathcal{L} = -w_t h_t(i) + mc_t(i)\left(z_t h_t(i) - y_t(i)\right)$$

FOC

$$h_t(i) \quad : \quad -w_t + mc_t(i)\, z_t = 0$$

Marginal cost is identical across firms

$$mc_t(i) = mc_t = \frac{w_t}{z_t}$$

Intermediate goods producing firm (simplified) II

Profit maximization problem

$$\max \quad \frac{P_t(i)}{P_t} y_t(i) - mc_t y_t(i)$$

$$\text{subject to} \quad y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t$$

Rewrite

$$\max \quad P_t(i)^{1+\frac{\mu}{1-\mu}} P_t^{1+\frac{\mu}{\mu-1}} y_t - mc_t P_t(i)^{\frac{\mu}{1-\mu}} P_t^{\frac{\mu}{\mu-1}} y_t$$

FOC

$$\left(\frac{1}{1-\mu}\right) P_t(i)^{\frac{\mu}{1-\mu}} P_t^{1+\frac{\mu}{\mu-1}} y_t - mc_t \left(\frac{\mu}{1-\mu}\right) P_t(i)^{\frac{\mu}{1-\mu}-1} P_t^{\frac{\mu}{\mu-1}} y_t = 0$$

$$P_t(i) = \mu \cdot mc_t \cdot P_t$$

Identical marginal costs $\rightarrow$ identical prices
(and we normalize $P = 1$)

$$1 = \mu mc_t \quad \rightarrow \quad mc_t = \frac{1}{\mu} \quad \text{and} \quad w_t = \frac{z_t}{\mu}$$

Intermediate goods producing firm

In the case of production function with capital

$$y_t(i) = z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha}$$

we get

$$mc_t = \frac{1}{\mu}$$

and

$$w_t = \frac{(1-\alpha)}{\mu} z_t k_t(i)^{\alpha} h_t(i)^{-\alpha}$$
$$r_t = \frac{\alpha}{\mu} z_t k_t(i)^{\alpha-1} h_t(i)^{1-\alpha} - \delta$$

- The rest of the model is unchanged relative to the basic RBC model
- We will introduce shocks to vary market power parameter over time

Full set of equilibrium conditions

System of 9 equations and 9 unknowns: $\{c, h, y, r, w, k, i, z, \mu\}$

$$
\begin{aligned}
\text{Euler equation} &: \quad 1 = \beta E_t \left[\frac{c_t}{c_{t+1}} (1 + r_{t+1}) \right] && (1) \\
\text{Cons.-hours choice} &: \quad h_t = 1 - \phi \frac{c_t}{w_t} && (2) \\
\text{Prod. function} &: \quad y_t = z_t k_t^{\alpha} h_t^{1-\alpha} && (3) \\
\text{Real interest rate} &: \quad r_t = \frac{\alpha}{\mu_t} z_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta && (4) \\
\text{Real hourly wage} &: \quad w_t = \frac{(1-\alpha)}{\mu_t} z_t k_t^{\alpha} h_t^{-\alpha} && (5) \\
\text{Investment} &: \quad i_t = k_{t+1} - (1-\delta) k_t && (6) \\
\text{Output accounting} &: \quad y_t = c_t + i_t && (7) \\
\text{TFP process} &: \quad \ln z_t = \rho \ln z_{t-1} + \varepsilon_t && (8) \\
\text{Markup process} &: \quad \ln \mu_t = (1 - \rho_\mu) \ln \mu + \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu,t} && (9)
\end{aligned}
$$

Steady state

- The only thing to be careful about is the monopoly wedge
- Other than that steady state is identical to the basic RBC case

$$(8) \qquad z = 1$$

$$(9) \qquad \mu = \mu$$

$$(1) \qquad r = 1/\beta - 1$$

$$(4) \qquad \frac{k}{h} = \left(\frac{\alpha/\mu}{r+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$(3) \qquad \frac{y}{h} = \left(\frac{k}{h}\right)^{\alpha}$$

$$(5) \qquad w = \frac{(1-\alpha)}{\mu}\frac{y}{h}$$

$$(6) \qquad \frac{i}{h} = \delta\frac{k}{h}$$

$$(7) \qquad \frac{c}{h} = \frac{y}{h} - \frac{i}{h}$$

$$(2) \qquad h = \left(1 + \frac{\phi}{w}\frac{c}{h}\right)^{-1}$$

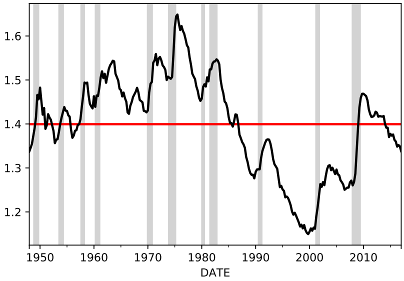
Parameters

From the consumption-labor choice and wage equations we get

$$\ln(1-h_t) = \ln\phi + \ln c_t - (-\ln\mu_t + \ln(1-\alpha) + \ln y_t - \ln h_t)$$

$$\ln\mu_t = -\ln\phi + \ln(1-\alpha) + \ln(1-h_t) - \ln h_t + \ln y_t - \ln c_t$$

All variables on the RHS are observable. The result is plotted below



Regression on the above markup implies $\rho_\mu = 0.99$ and $\varepsilon_\mu = 0.011$

Model comparison

	Rel. S. D.			Corr. w. y			Autocorr.		
	Data	RBC	MC	Data	RBC	MC	Data	RBC	MC
y	1.00	1.00	1.00	1.00	1.00	1.00	0.85	0.72	0.73
c	0.53	0.38	0.38	0.78	0.94	0.63	0.82	0.78	0.82
i	2.75	3.11	4.99	0.76	0.99	0.96	0.87	0.71	0.71
h	1.00	0.44	1.09	0.80	0.98	0.85	0.91	0.71	0.72
w	0.55	0.58	0.71	0.08	0.99	0.99	0.68	0.74	0.76
$\frac{y}{h}$	0.60	0.58	0.59	0.44	0.99	0.13	0.71	0.74	0.74
μ	1.08	–	0.88	-0.48	–	-0.72	0.83	–	0.72

Monopolistic competition – summary

- Introduction of second shock reduces model's reliance on TFP
- It improves hours volatility by a lot
- Markup shock does not have a good economic interpretation
- May be a result of many factors unrelated to monopoly power $\rightarrow$ increases sharply in recessions
- Chari, Kehoe, and McGrattan (2007) perform "business cycle accounting", where they identify "wedges" (residuals) from the first order conditions of a very basic RBC model
- We have just obtained the measure for the labor wedge

Aggregate price index derivation

Perfect competition in the final goods sector implies

$$P_t y_t = \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i$$

$$P_t y_t = \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t \mathrm{d}i$$

$$P_t y_t = P_t^{-\frac{\mu}{1-\mu}} y_t \cdot \int_0^1 P_t(i)^{1+\frac{\mu}{1-\mu}}\, \mathrm{d}i$$

$$P_t^{1+\frac{\mu}{1-\mu}} = \int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i$$

$$P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i\right)^{1-\mu}$$

back

Marginal cost for production function with capital I

Cost minimization problem

$$\begin{aligned} \min \quad & tc_t(i) = w_t h_t(i) + r_t^k k_t(i) \\ \text{subject to} \quad & y_t(i) = z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} \end{aligned}$$

Lagrangian

$$\mathcal{L} = -\left(w_t h_t(i) + r_t^k k_t(i)\right) + mc_t(i)\left(z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} - y_t(i)\right)$$

FOCs

$$\begin{aligned} h_t(i) \quad &: \quad w_t = mc_t(i)(1-\alpha) z_t k_t(i)^{\alpha} h_t(i)^{-\alpha} \\ k_t(i) \quad &: \quad r_t^k = mc_t(i)\,\alpha z_t k_t(i)^{\alpha-1} h_t(i)^{1-\alpha} \end{aligned}$$

Divide

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{k_t(i)}{h_t(i)} \quad \rightarrow \quad \frac{k_t(i)}{h_t(i)} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \quad \rightarrow \quad h_t(i) = \frac{1-\alpha}{\alpha}\frac{r_t^k}{w_t}k_t(i)$$

All firms have identical k/h ratio

Marginal cost for production function with capital II

Production function

$$y_t(i) = z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} = z_t k_t(i)^{\alpha} \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} k_t(i) \right)^{1-\alpha}$$

$$= z_t k_t(i) \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} \right)^{1-\alpha} \quad \rightarrow \quad k_t(i) = \frac{y_t(i)}{z_t} \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} \right)^{\alpha-1}$$

Total cost

$$tc_t(i) = w_t h_t(i) + r_t^k k_t(i) = \frac{\alpha}{1-\alpha} r_t^k k_t(i) + r_t^k k_t(i)$$

$$= \left(\frac{1-\alpha}{\alpha} + 1 \right) r_t^k k_t(i) = \frac{1}{\alpha} r_t^k k_t(i)$$

$$= \frac{1}{\alpha} r_t^k \frac{y_t(i)}{z_t} \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} \right)^{\alpha-1} = \frac{y_t(i)}{z_t} \frac{\left(r_t^k\right)^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}$$

Marginal cost is identical across firms back

$$mc_t(i) = \frac{\partial tc_t(i)}{\partial y_t(i)} = \frac{1}{z_t} \frac{\left(r_t^k\right)^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} = mc_t$$

Flexible vs sticky prices[1]

- Central assumption of the (new) classical economics
- Prices (of goods and factor services) are fully flexible
    - An increase in money supply increases prices 1:1 immediately
    - Money is (super)neutral, monetary policy has no power
      $\rightarrow$ classical dichotomy
    - In previous models we have abstracted
      from money and nominal variables
- (New) Keynesian economics
    - Prices are sticky (inertial), do not adjust instantly
    - Classical dichotomy no longer holds
      $\rightarrow$ nominal variables affect real
    - Scope for monetary policy
    - Additional propagation channels for other shocks

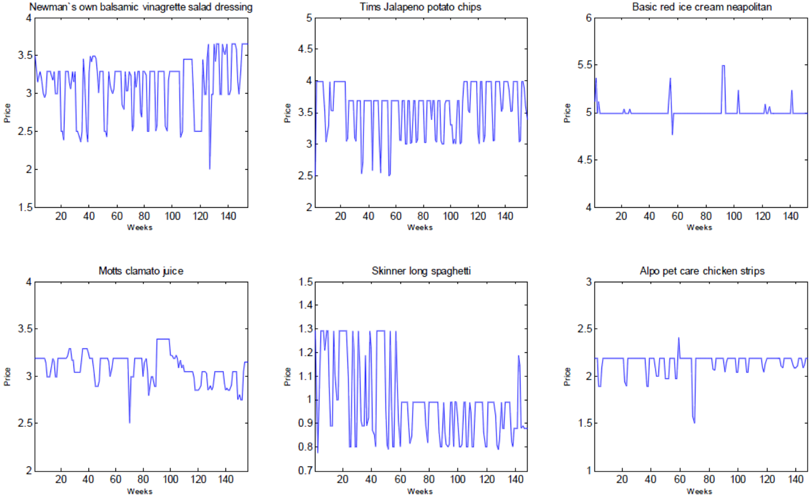
[1] The following slides were adapted from Michał Brzoza-Brzezina's lectures

Sticky prices: empirical evidence

- Price duration
  - US: average time between price changes is 2-4 quarters
    Blinder et al. (1998), Bils and Klenow (2004),
    Klenow and Kryvstov (2008), Nakamura and Steinsson (2008)
  - Euro area: average time between price changes is 4-5 quarters
    Dhyne et al. (2005), Altissimo et al. (2006)
  - Poland: average time between price changes is 4 quarters
    Macias and Makarski (2013)
- The higher inflation, the more frequently price changes occur
- Cross-industry heterogeneity
  - Prices of tradables less sticky than those of nontradables
  - Retail prices usually more sticky than producer prices

Example retail prices behavior

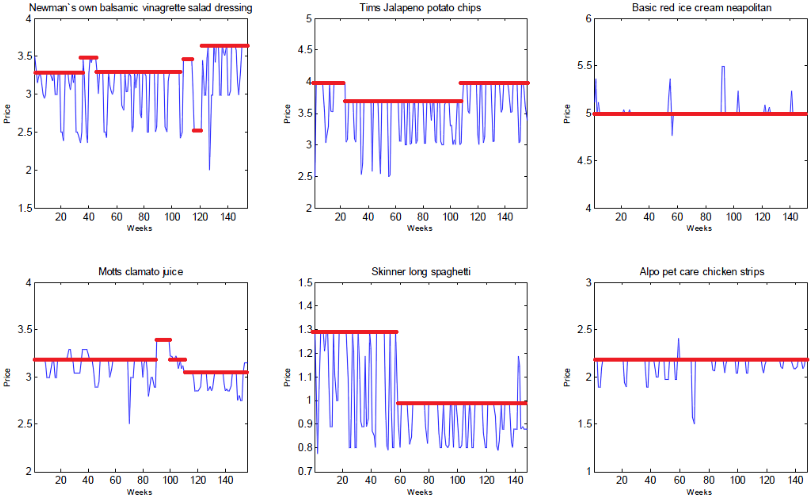
Raw retail scanner data



http://jpkoning.blogspot.com/2015/10/are-prices-getting-less-sticky.html

Example retail prices behavior

After “controlling” for short-lived sales prices – reference prices



Theories on price stickiness

- Lucas (1972) – imperfect information
  - When faced with a higher nominal demand for product a firm does not know whether real demand or price level went up
  - If it's real demand firm should increase output
  - If it's inflation firm should increase prices
  - Low inflation environment – rational to leave prices unchanged
  - Extensions: sticky information – Mankiw and Reis (2007); rational inattention – Sims (2003), Maćkowiak and Wiederholt (2009)
- Behavioral – psychological pricing, judging quality by price
- Costs of changing prices (explicit or implicit)
  - Menu costs – Sheshinski and Weiss (1977), Akerlof and Yellen (1985), Mankiw (1985)
  - Explicit contracts which are costly to renegotiate
  - Long-term relationships with customers $\rightarrow$ price changes less frequent in sectors with more monopoly power
- "Good" causes of price stickiness $\rightarrow$ in a stable economic environment agents trust in price stability

Price stickiness depends on sector

Altissimo, Ehrmann and Smets (2006)

Inflation persistence and price-setting behaviour in the euro area

Table 4.1 Frequency of consumer price changes by product type, in %

Country	Unprocessed food	Processed food	Energy (oil products)	Non-energy industrial goods	Services	Total, country weights	Total, Euro area weights
Belgium	31.5	19.1	81.6	5.9	3.0	17.6	15.6
Germany	25.2	8.9	91.4	5.4	4.3	13.5	15.0
Spain	50.9	17.7	n.a.	6.1	4.6	13.3	11.5
France	24.7	20.3	76.9	18.0	7.4	20.9	20.4
Italy	19.3	9.4	61.6	5.8	4.6	10.0	12.0
Luxembourg	54.6	10.5	73.9	14.5	4.8	23.0	19.2
The Netherlands	30.8	17.3	72.6	14.2	7.9	16.2	19.0
Austria	37.5	15.5	72.3	8.4	7.1	15.4	17.1
Portugal	55.3	24.5	15.9	14.3	13.6	21.1	18.7
Finland	52.7	12.8	89.3	18.1	11.6	20.3	-
Euro Area	28.3	13.7	78.0	9.2	5.6	15.1	15.8

Source: Dhyne et al. (2005). Figures presented in this table are computed on the basis of the 50 product sample, with the only exception of Finland for which figures based on the entire CPI are presented. The total with country weights is calculated using country-specific weights for each item, the total with euro area weights using common euro area weights for each sub-index. No figures are provided for Finland because of a lack of comparability of the sample of products used in this country.

Survey assessment of price stickiness theories

Altissimo, Ehrmann and Smets (2006)

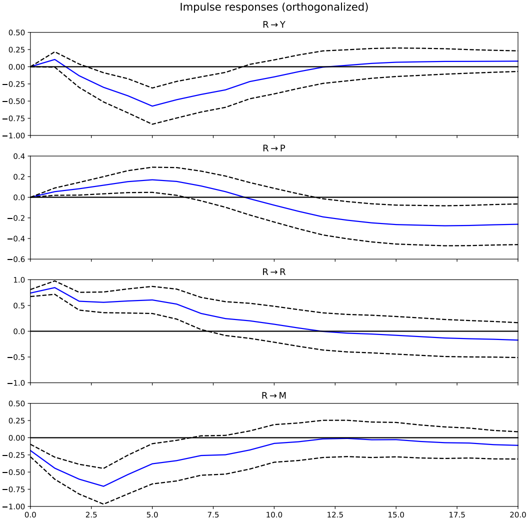
Inflation persistence and price-setting behaviour in the euro area

Table 4.6 Ranking of theories explaining price stickiness

	Belgium	Germany	Spain	France	Italy	Luxem-bourg	Nether-lands	Austria	Portugal	Euro Area
Implicit contracts	2.5		2.6	2.2		2.7	2.7	3.0	3.1	2.7
Explicit contracts	2.4	2.4	2.3	2.7	2.6	2.8	2.5	3.0	2.6	2.6
Cost-based pricing	2.4			2.5		2.7		2.6	2.7	2.6
Co-ordination failure	2.2	2.2	2.4	3.0	2.6	2.1	2.2	2.3	2.8	2.4
Judging quality by price	1.9		1.8			2.2	2.4	1.9	2.3	2.1
Temporary shocks	1.8	1.9	1.8	2.1	2.0	1.7	2.4	1.5	2.5	2.0
Change non-price factors	1.7		1.3			1.9	1.9	1.7		1.7
Menu costs	1.5	1.4	1.4	1.4	1.6	1.8	1.7	1.5	1.9	1.6
Costly information	1.6		1.3			1.8		1.6	1.7	1.6
Pricing thresholds	1.7		1.5	1.6	1.4	1.8	1.8	1.3	1.8	1.6

Source: Fabiani et al. (2005). Euro area figures are unweighted averages of country scores.

Effects of price stickiness – influence of nominal variables



Results from a 4-variable 6-lag vector autoregression

New Keynesian model – introduction

- New Keynesian model is an RBC model with
  - Monopolistic competition
  - Sticky prices
  - Monetary policy authority
- Model price stickiness via Calvo (1983) assumption
  - A firm can change its price only if it receives a signal
  - Firm does not receive the signal with probability θ
  - Expected (average) price duration is $\frac{1}{1-\theta}$

Households – problem

For simplicity consider a model without physical capital

$$\begin{aligned} \max \quad & E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \phi \frac{h_t^{1+\eta}}{1+\eta} \right) \right] \\ \text{subject to} \quad & P_t c_t + B_t = W_t h_t + R_{t-1} B_{t-1} + P_t div_t \end{aligned}$$

where nominal bonds B yield the gross nominal interest rate R

Rewrite budget constraint in real terms

$$\begin{aligned} c_t + \frac{B_t}{P_t} &= \frac{W_t}{P_t} h_t + R_{t-1} \frac{P_{t-1}}{P_t} \frac{B_{t-1}}{P_{t-1}} + div_t \\ c_t + b_t &= w_t h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + div_t \end{aligned}$$

where $\Pi_t = P_t / P_{t-1}$ is the gross inflation rate

Households – solution

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_0 \left[\begin{array}{c} \frac{c_t^{1-\sigma}}{1-\sigma} - \phi \frac{h_t^{1+\eta}}{1+\eta} \\ +\lambda_t \left(w_t h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + div_t - c_t - b_t \right) \end{array} \right]$$

FOCs

$$\begin{aligned} c_t &: \quad c_t^{-\sigma} - \lambda_t = 0 \\ h_t &: \quad -\phi h_t^{\eta} + \lambda_t w_t = 0 \\ b_t &: \quad -\lambda_t + \beta E_t \left[\lambda_{t+1} \left(R_t / \Pi_{t+1} \right) \right] = 0 \end{aligned}$$

Resulting

$$\begin{aligned} \text{Intratemporal choice } (c+h) &: \quad c_t^{-\sigma} w_t = \phi h_t^{\eta} \\ \text{Intertemporal choice } (c+b) &: \quad c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} \left(R_t / \Pi_{t+1} \right) \right] \end{aligned}$$

Final goods producing firm I

Profit maximization problem

$$\begin{aligned} \max \quad & P_t y_t - \int_0^1 P_t\left(i\right) y_t\left(i\right) \mathrm{d}i \\ \text{subject to} \quad & y_t = \left(\int_0^1 y_t\left(i\right)^{\frac{1}{\mu}} \mathrm{d}i\right)^{\mu} \end{aligned}$$

Lagrangian

$$\mathcal{L} = P_t y_t - \int_0^1 P_t\left(i\right) y_t\left(i\right) \mathrm{d}i + \lambda_t \left[\left(\int_0^1 y_t\left(i\right)^{\frac{1}{\mu}} \mathrm{d}i\right)^{\mu} - y_t\right]$$

FOCs

$$\begin{aligned} y_t \quad &: \quad P_t - \lambda_t = 0 \\ y_t\left(i\right) \quad &: \quad -P_t\left(i\right) + \lambda_t \left[\mu \left(\int_0^1 y_t\left(i\right)^{\frac{1}{\mu}} \mathrm{d}i\right)^{\mu-1} \cdot \frac{1}{\mu} y_t\left(i\right)^{\frac{1}{\mu}-1}\right] = 0 \end{aligned}$$

Final goods producing firm II

Result

$$P_t(i) = P_t\left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i\right)^{\mu-1} y_t(i)^{\frac{1-\mu}{\mu}} \quad \Big| \quad (\cdot)^{\frac{\mu}{\mu-1}}$$

$$P_t(i)^{\frac{\mu}{\mu-1}} = P_t^{\frac{\mu}{\mu-1}}\left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i\right)^{\mu} y_t(i)^{-1}$$

$$y_t(i) = \left(\frac{P_t}{P_t(i)}\right)^{\frac{\mu}{\mu-1}} y_t$$

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t$$

Aggregate price index derivation

$$P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i\right)^{1-\mu}$$

Intermediate goods producing firm I

Production function is linear in hours

$$y_t\left(i\right) = z_t h_t\left(i\right)$$

Cost minimization problem

$$\begin{aligned} \min \quad & tc_t\left(i\right) = w_t h_t\left(i\right) \\ \text{subject to} \quad & y_t\left(i\right) = z_t h_t\left(i\right) \end{aligned}$$

Lagrangian

$$\mathcal{L} = -w_t h_t\left(i\right) + mc_t\left(i\right)\left(z_t h_t\left(i\right) - y_t\left(i\right)\right)$$

FOC

$$w_t = mc_t\left(i\right) z_t$$

Marginal cost is identical across firms

$$mc_t\left(i\right) = mc_t = \frac{w_t}{z_t}$$

Intermediate goods producing firm II

Profit maximization problem (where $\Lambda_{0,t} = \lambda_t / \lambda_0$)

$$\max \quad E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_{0,t} \left(\frac{\tilde{P}_0(i)}{P_t} y_t(i) - mc_t y_t(i) \right) \right]$$

$$\text{subject to} \quad y_t(i) = \left(\frac{\tilde{P}_0(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} y_t$$

Define $\tilde{p}_0(i) = \tilde{P}_0(i) / P_0$ and $\Pi_{0,t} = P_t / P_0 = \Pi_1 \cdot \ldots \cdot \Pi_t$. Then

$$\frac{\tilde{P}_0(i)}{P_t} = \frac{\tilde{P}_0(i)}{P_0} \frac{P_0}{P_t} = \tilde{p}_0(i) \frac{1}{\Pi_{0,t}} = \frac{\tilde{p}_0(i)}{\Pi_{0,t}}$$

Rewrite

$$\max \qquad E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_{0,t} \left(\left(\frac{\tilde{p}_0(i)}{\Pi_{0,t}} \right)^{1+\frac{\mu}{1-\mu}} y_t - mc_t \left(\frac{\tilde{p}_0(i)}{\Pi_{0,t}} \right)^{\frac{\mu}{1-\mu}} y_t \right) \right]$$

Intermediate goods producing firm III

$$\max \quad E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{\lambda_t}{\lambda_0} \left(\tilde{p}_0(i)^{1+\frac{\mu}{1-\mu}} \Pi_{0,t}^{\frac{\mu}{\mu-1}-1} y_t - mc_t \tilde{p}_0(i)^{\frac{\mu}{1-\mu}} \Pi_{0,t}^{\frac{\mu}{\mu-1}} y_t \right) \right]$$

FOC

$$E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{\lambda_t}{\lambda_0} \left(\frac{1}{1-\mu} \right) \tilde{p}_0(i)^{\frac{\mu}{1-\mu}} \Pi_{0,t}^{\frac{\mu}{\mu-1}-1} y_t \right] =$$

$$= E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{\lambda_t}{\lambda_0} mc_t \left(\frac{\mu}{1-\mu} \right) \tilde{p}_0(i)^{\frac{\mu}{1-\mu}-1} \Pi_{0,t}^{\frac{\mu}{\mu-1}} y_t \right]$$

Optimal relative price

$$\tilde{p}_0(i) = \mu \cdot \frac{E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \lambda_t mc_t \Pi_{0,t}^{\frac{\mu}{\mu-1}} y_t \right]}{E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \lambda_t \Pi_{0,t}^{\frac{\mu}{\mu-1}-1} y_t \right]}$$

Intermediate goods producing firm IV

Optimal relative price is the same across all firms resetting prices

$$\tilde{p}_t = \mu \cdot \frac{E_t\left[\sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+j} mc_{t+j} \Pi_{t,t+j}^{\frac{\mu}{\mu-1}} y_{t+j}\right]}{E_t\left[\sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+j} \Pi_{t,t+j}^{\frac{1}{\mu-1}} y_{t+j}\right]}$$

This expression has a convenient recursive representation derivation

$$\tilde{p}_t = \mu \frac{Num_t}{Den_t}$$

$$Num_t = \lambda_t mc_t y_t + \beta\theta E_t\left[\Pi_{t+1}^{\frac{\mu}{\mu-1}} Num_{t+1}\right]$$

$$Den_t = \lambda_t y_t + \beta\theta E_t\left[\Pi_{t+1}^{\frac{1}{\mu-1}} Den_{t+1}\right]$$

If prices are not sticky ($\theta = 0$) then

$$\tilde{p}_t = \mu \cdot mc_t$$

NK collapses to RBC with monopolistic competition

Inflation dynamics

Recall the formula for aggregate price index

$$P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}} \, \mathrm{d}i \right)^{1-\mu}$$

$$P_t^{\frac{1}{1-\mu}} = \int_0^\theta P_{t-1}(i)^{\frac{1}{1-\mu}} \, \mathrm{d}i + \int_\theta^1 \tilde{P}_t^{\frac{1}{1-\mu}} \, \mathrm{d}i$$

$$P_t^{\frac{1}{1-\mu}} = \theta P_{t-1}^{\frac{1}{1-\mu}} + (1-\theta) \tilde{P}_t^{\frac{1}{1-\mu}} \quad \Big| \quad : P_{t-1}^{\frac{1}{1-\mu}}$$

$$\left(\frac{P_t}{P_{t-1}} \right)^{\frac{1}{1-\mu}} = \theta \left(\frac{P_{t-1}}{P_{t-1}} \right)^{\frac{1}{1-\mu}} + (1-\theta) \left(\frac{\tilde{P}_t}{P_t} \frac{P_t}{P_{t-1}} \right)^{\frac{1}{1-\mu}}$$

$$\Pi_t^{\frac{1}{1-\mu}} = \theta + (1-\theta) \left(\tilde{p}_t \Pi_t \right)^{\frac{1}{1-\mu}}$$

$$\Pi_t = \left[\theta / \left(1 - (1-\theta) \tilde{p}_t^{\frac{1}{1-\mu}} \right) \right]^{1-\mu}$$

Market clearing

Factor markets clear

$$h_t = \int_0^1 h_t\left(i\right) \mathrm{d}i$$

Intermediate goods markets are in equilibrium

$$z_t h_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t$$

$$\int_0^1 z_t h_t\left(i\right) \mathrm{d}i = \int_0^1 \left(\frac{P_t\left(i\right)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t \mathrm{d}i$$

$$z_t h_t = y_t \int_0^1 \left(\frac{P_t\left(i\right)}{P_t}\right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$$

$$z_t h_t = y_t \Delta_t$$

where price dispersion Δ creates inefficiency

$$y_t = \frac{z_t h_t}{\Delta_t}$$

Output accounting

Dividends

$$div_t = y_t - w_t h_t$$

Budget constraint

$$c_t + b_t = w_t h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + div_t$$

In equilibrium representative agent holds 0 bonds ($b_t = b_{t-1} = 0$)

$$\begin{aligned} c_t &= w_t h_t + y_t - w_t h_t \\ c_t &= y_t \end{aligned}$$

Price dispersion: source of inefficiency

Define

$$\Delta_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$$

Dynamics

$$\Delta_t = \int_0^\theta \left(\frac{P_{t-1}(i)}{P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{\frac{\mu}{1-\mu}} \mathrm{d}i + \int_\theta^1 \left(\frac{\tilde{P}_t}{P_t} \right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$$

$$\Delta_t = \theta \Delta_{t-1} \Pi_t^{\frac{\mu}{\mu-1}} + (1-\theta)\, \tilde{p}_t^{\frac{\mu}{1-\mu}}$$

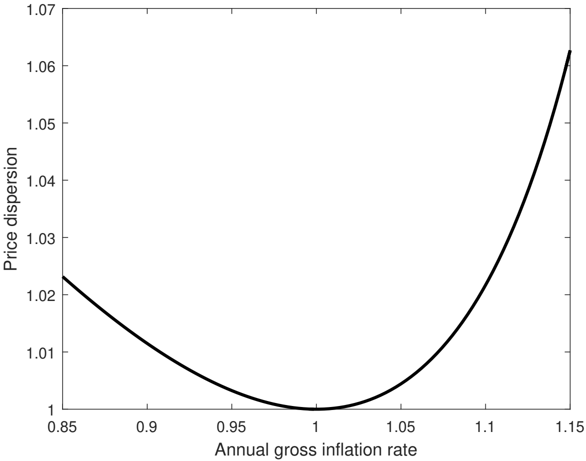
One can show that $\Delta_t \geq 1$ and in consequence

$$y_t \leq z_t h_t$$

Costs of non-zero inflation

Price dispersion as a function of steady state gross annual inflation

Parameters used: $\mu = 1.33$, $\theta = 0.75$



Costs of non-zero inflation

- Inflation is more harmful than deflation
- Costs of inflation are convex
  - An annual inflation of 2% causes about 0.05% loss in GDP
  - An annual inflation of 5% causes about 0.4% loss in GDP
  - An annual inflation of 10% causes about 2% loss in GDP
  - An annual inflation of 15% causes about 6% loss in GDP
  - An annual inflation of 20% causes about 15% loss in GDP
- For high levels of inflation the model breaks down
  $\rightarrow$ not suitable for analysing hyperinflations
- Even before that firms would change prices more often
  $\rightarrow$ Calvo pricing is a modeling shortcut, not microfounded
- Despite efficiency losses from price dispersion,
  higher inflation target lowers probability of hitting ZLB
- Before the crisis the consensus for inflation target was 2%
- After the crisis: Blanchard, Ball and others propose 4%

Equilibrium conditions

Euler equation	:	$1 = \beta E_t \left(c_t / c_{t+1} \right)^{\sigma} \left(R_t / \Pi_{t+1} \right)$
Consumption-hours	:	$w_t = \phi h_t^{\eta} c_t^{\sigma}$
Real wages	:	$w_t = mc_t z_t h_t$
Production function	:	$y_t = z_t h_t / \Delta_t$
Price dispersion	:	$\Delta_t = \theta \Delta_{t-1} \Pi_t^{\frac{\mu}{\mu-1}} + (1-\theta) \tilde{p}_t^{\frac{\mu}{1-\mu}}$
Inflation dynamics	:	$\Pi_t = [1/\theta - (1/\theta - 1) \tilde{p}_t^{\frac{1}{1-\mu}}]^{\mu-1}$
Optimal reset price	:	$\tilde{p}_t = \mu \cdot (Num_t / Den_t)$
Numerator	:	$Num_t = c_t^{-\sigma} mc_t y_t + \beta\theta E_t \Pi_{t+1}^{\frac{\mu}{\mu-1}} Num_{t+1}$
Denominator	:	$Den_t = c_t^{-\sigma} y_t + \beta\theta E_t \Pi_{t+1}^{\frac{1}{\mu-1}} Den_{t+1}$
Output accounting	:	$y_t = c_t$
TFP AR(1) process	:	$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t}$

Monetary policy

- There are 11 equations but 12 variables!
  $\{y, c, h, w, z, mc, R, \Pi, \Delta, \tilde{p}, Num, Den\}$
- Need another equation to close the model
- Model is closed by adding a monetary policy rule
- Often the Taylor rule is used

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_i} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_x}\right)^{1-\gamma_i} \varepsilon_{R,t}$$

- Model stable only if Taylor principle fulfilled:[2] $\gamma_\pi > 1$
- The model can then be reduced to just three equations: for output (gap), inflation and nominal interest rate (three-equation New Keynesian model)

[2] More precisely, $\kappa\left(\gamma_\pi - 1\right) + \left(1 - \beta\right)\gamma_x > 0$

Three-equation New Keynesian model

- Full derivation here
- Denote output gap x_t as the log-difference between the actual output y_t and counterfactual output under flexible prices y_t^f
- New Keynesian IS curve (from Euler equation)

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) + \varepsilon_t^{IS}$$

- New Keynesian Phillips curve (from inflation dynamics)

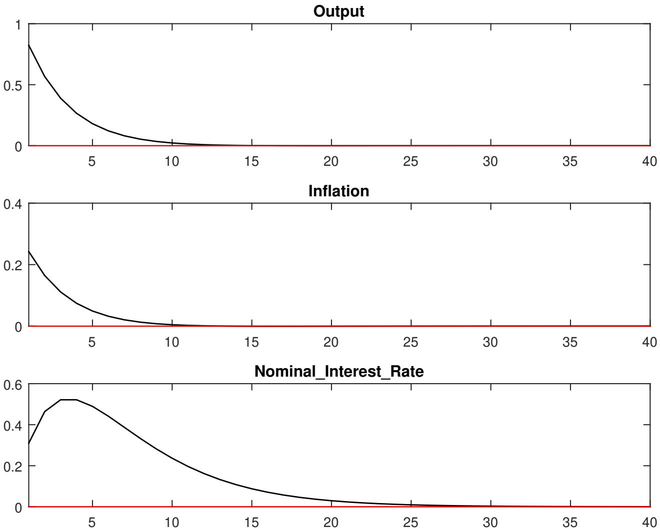
$$\pi_t = \underbrace{\frac{(1-\beta\theta)(1-\theta)}{\theta}(\sigma + \eta)}_{\kappa} x_t + \beta E_t \pi_{t+1} + \varepsilon_t^{PC}$$

- Taylor rule

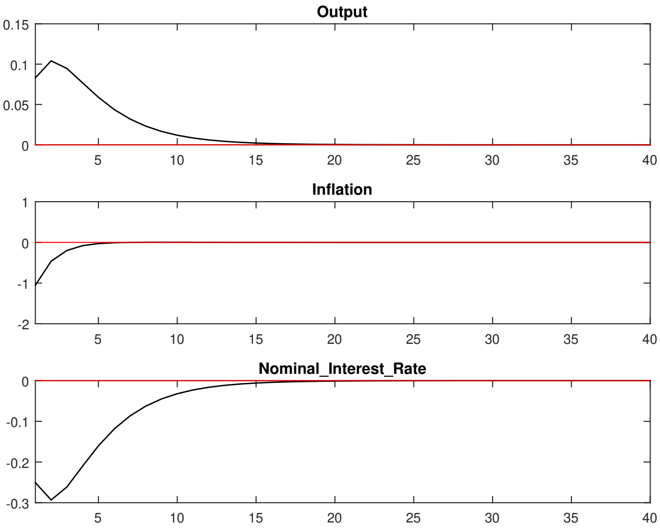
$$i_t = \gamma_i i_{t-1} + (1-\gamma_i)\left(\gamma_\pi \pi_t + \gamma_x x_t\right) + \varepsilon_t^{TR}$$

- Shocks: demand ε_t^{IS}, cost-push ε_t^{PC} ($\neq$ TFP shock), monetary ε_t^{TR} (unexpected deviation from monetary rule)

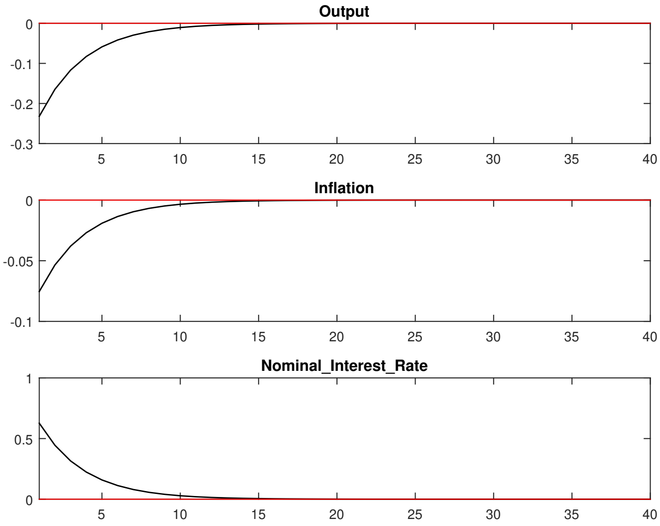
Positive demand shock



Positive cost-push shock



Negative monetary shock



Important properties[3]

- Short-run non-neutrality of monetary policy
- The (extended) NK model can generate impulse responses consistent with empirical studies
- In reaction to demand shocks output and inflation move in the same direction
- In reaction to supply shocks output and inflation move in opposite directions

[3] The following slides were adapted from Michał Brzoza-Brzezina's lecture

Role of expectations

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \varepsilon_t^{PC}$$

- Current inflation is affected by inflation expectations
- Modern monetary policy: management of expectations
- Woodford (2005, p. 3):
  For not only do expectations about policy matter, but,
  at least under current conditions, very little *else* matters

Rules (commitment) vs. discretion debate

- Old debate
  - should monetary policy be bound by rules
    or should it be free to do whatever it wants every period?
  - Kydland and Prescott (1977) and Barro and Gordon (1983)
    show that central bank pursuing an overly ambitious
    output goal will end up with inflation bias
  - agents know that the central bank prefers
    high output (positive gap) and adjust expectations
  - as a result inflation is higher, but output gap is 0!
  - thus CB should credibly commit to keeping output at potential
- Today
  - we do not think of central banks as trying
    to keep permanently positive output gaps
  - but Clarida, Gali and Gertler (1999) show that
    even without such targets, commitment can be good

Optimal monetary policy I

- Price dispersion is lowest when all prices are equal
- This happens with zero inflation
- If sticky prices are the only distortion then optimal monetary policy in the short run is to stabilize inflation perfectly
- In the simple NK model stabilizing inflation at 0 also stabilizes (welfare-relevant) output gap (if $\varepsilon_t^{PC} = 0$)
  $\rightarrow$ Blanchard and Gali (2007): "divine coincidence"
- Attention: there may be other distortions, e.g. sticky wages
- Then optimal policy becomes more complicated (e.g. it may also have to stabilize wages)

Optimal monetary policy II

- Under richer models optimal policy has to solve trade-offs
- Rotemberg and Woodford (1998): when real imperfections are present, the second order approximation to social welfare is

$$W_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda x_t^2 \right) \right]$$

- Trade-off between between stabilizing inflation and output gap
- Consistent with behavior of central banks, who aim to stabilize both inflation and output gaps
- Question arises whether policy should be conducted discretionary or under commitment

Optimal policy under discretion

- Under optimal discretionary policy (ODP) the central bank is not able to influence expectations about future policy
- Optimizing boils down to solving static problems

$$\begin{aligned} \min \quad & \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) \\ \text{subject to} \quad & \pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \varepsilon_t^{PC} \end{aligned}$$

- Note that expectation terms are taken as given, since the CB is assumed not to influence them
- Solution: $\pi_t = -\frac{\lambda}{\kappa} x_t$
- This is called targeting rule (in contrast to instrument rules)
- After an inflationary shock the CB allows the output gap to become negative

Optimal policy under commitment I

- Under (credible) commitment the CB is able to influence expectations about future policy
- The problem is now dynamic

$$\min \quad \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda x_t^2\right)$$

$$\text{subject to} \quad \pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \varepsilon_t^{PC}$$

- Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_0 \left[\frac{1}{2} \left(\pi_t^2 + \lambda x_t^2\right) + \mu_t \left(\kappa x_t + \beta \pi_{t+1} + \varepsilon_t^{PC} - \pi_t\right) \right]$$

- FOCs

$$x_{t+1} \quad : \quad \beta^{t+1} \left[\lambda x_{t+1} + \mu_{t+1} \kappa\right] = 0 \qquad \rightarrow \qquad \mu_t = -\frac{\lambda}{\kappa} x_t$$

$$\pi_{t+1} \quad : \quad \beta^{t+1} \left[\pi_{t+1} - \mu_{t+1}\right] + \beta^t \left[\mu_t \beta\right] = 0 \qquad \rightarrow \qquad \pi_t = \mu_t - \mu_{t-1}$$

Optimal policy under commitment II

- For $t = 0$ we get ($\mu_{-1} = 0$)

$$\pi_0 = \mu_0 = -\frac{\lambda}{\kappa} x_0$$

- Same as under discretion
- For $t \geq 1$

$$\pi_t = \mu_t - \mu_{t-1} = -\frac{\lambda}{\kappa} \left(x_t - x_{t-1} \right)$$

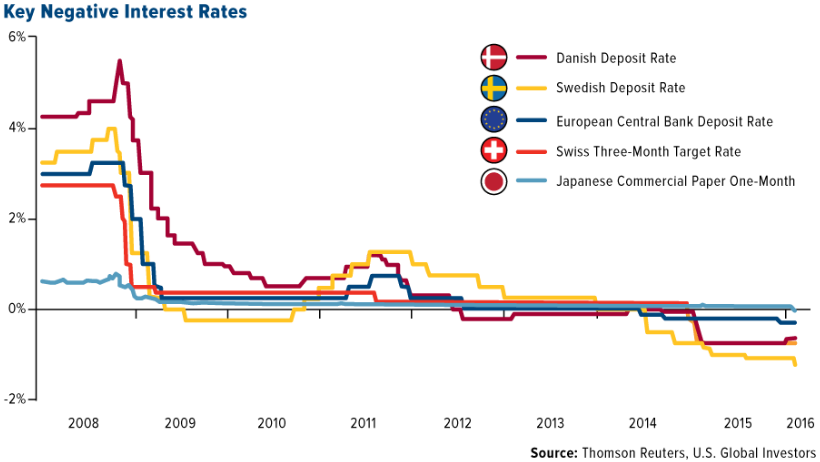
- Different than in period $t = 0$
- Takes past developments into account
- Optimal commitment policy (OCP) means doing something today and promising to do something different from tomorrow on
- But tomorrow will be today tomorrow
  $\rightarrow$ **time inconsistency**

Optimal policy under commitment III

- OCP is time inconsistent – solutions?
    1. Appoint very credible central bankers
    2. Act in “timeless perspective”: pretend that OCP has been applied long ago and use the formula for $t \geq 1$ from the beginning
- What is better: OCP or ODP?
- Neither invokes an inflation bias
- ODP generates a stabilization bias $\rightarrow$ economy is more volatile
- The superiority of commitment calls for a credible, long-term arrangement for the central bank

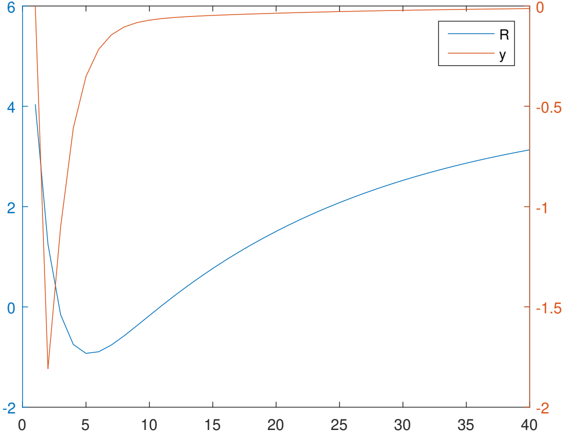
Zero Lower Bound (liquidity trap)

Literally zero?



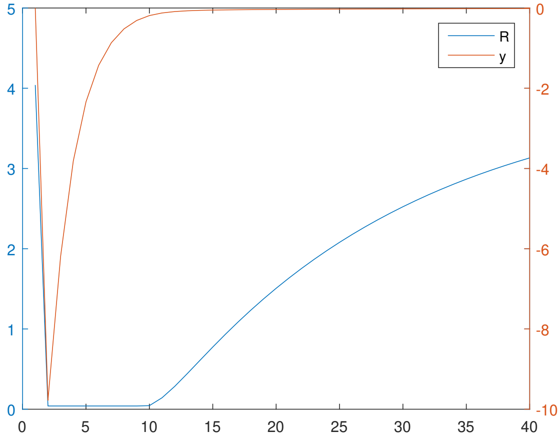
Zero Lower Bound (liquidity trap)

Nonbinding



Zero Lower Bound (liquidity trap)

Binding



Aggregate price index derivation

Perfect competition in the final goods sector implies

$$P_t y_t = \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i$$

$$P_t y_t = \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} y_t \mathrm{d}i$$

$$P_t y_t = P_t^{-\frac{\mu}{1-\mu}} y_t \cdot \int_0^1 P_t(i)^{1+\frac{\mu}{1-\mu}}\, \mathrm{d}i$$

$$P_t^{1+\frac{\mu}{1-\mu}} = \int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i$$

$$P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i \right)^{1-\mu}$$

back

Optimal reset price – numerator

$$
\begin{aligned}
Num_t &= \\
&= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+j} mc_{t+j} \Pi_{t,t+j}^{\frac{\mu}{\mu-1}} y_{t+j} \\
&= \lambda_t mc_t y_t + E_t \sum_{j=1}^{\infty} (\beta\theta)^j \lambda_{t+j} mc_{t+j} \Pi_{t,t+j}^{\frac{\mu}{\mu-1}} y_{t+j} \\
&= \lambda_t mc_t y_t + E_t \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+1+j} mc_{t+1+j} \Pi_{t,t+1+j}^{\frac{\mu}{\mu-1}} y_{t+1+j} \\
&= \lambda_t mc_t y_t + E_t \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+1+j} mc_{t+1+j} \left(\Pi_{t,t+1} \cdot \Pi_{t+1,t+1+j}\right)^{\frac{\mu}{\mu-1}} y_{t+1+j} \\
&= \lambda_t mc_t y_t + E_t \left[\Pi_{t,t+1}^{\frac{\mu}{\mu-1}} \cdot \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+1+j} mc_{t+1+j} \Pi_{t+1,t+1+j}^{\frac{\mu}{\mu-1}} y_{t+1+j}\right] \\
&= \lambda_t mc_t y_t + E_t \left[\Pi_{t,t+1}^{\frac{\mu}{\mu-1}} \cdot Num_{t+1}\right]
\end{aligned}
$$

Optimal reset price – denominator

$$
\begin{aligned}
Den_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+j} \Pi_{t,t+j}^{\frac{1}{\mu-1}} y_{t+j} \\
&= \lambda_t y_t + E_t \sum_{j=1}^{\infty} (\beta\theta)^j \lambda_{t+j} \Pi_{t,t+j}^{\frac{1}{\mu-1}} y_{t+j} \\
&= \lambda_t y_t + E_t \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+1+j} \Pi_{t,t+1+j}^{\frac{1}{\mu-1}} y_{t+1+j} \\
&= \lambda_t y_t + E_t \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+1+j} \left(\Pi_{t,t+1} \cdot \Pi_{t+1,t+1+j}\right)^{\frac{1}{\mu-1}} y_{t+1+j} \\
&= \lambda_t y_t + E_t \left[\Pi_{t,t+1}^{\frac{1}{\mu-1}} \cdot \sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+1+j} \Pi_{t+1,t+1+j}^{\frac{1}{\mu-1}} y_{t+1+j}\right] \\
&= \lambda_t y_t + E_t \left[\Pi_{t,t+1}^{\frac{1}{\mu-1}} \cdot Den_{t+1}\right]
\end{aligned}
$$

back