University of Warsaw
Faculty of Economic Sciences

Models of unemployment

Advanced Macroeconomics IE: Lecture 17

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RBC model vs data comparison

		Std. Dev.		Corr. w. y		Autocorr.	
		Data	Model	Data	Model	Data	Model
Output	y	1.63	1.63	1.00	1.00	0.85	0.72
Consumption	c	0.87	0.63	0.77	0.94	0.83	0.79
Investment	i	4.51	5.07	0.76	0.99	0.87	0.71
Capital	k	0.59	0.45	0.40	0.09	0.95	0.96
Hours	h	**1.91**	**0.71**	0.88	0.98	0.91	0.71
Wage	w	0.97	0.95	**0.11**	**0.99**	0.68	0.75
TFP	z	0.84	1.15	**0.53**	**1.00**	0.73	0.72
Productivity	$\frac{y}{h}$	1.06	0.95	**0.41**	**0.99**	0.71	0.75

RBC model vs data comparison

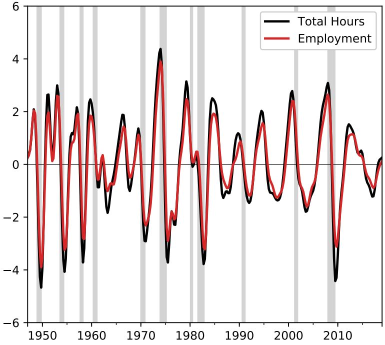
- Model performance is quite good
  – it was a big surprise in the 1980s!

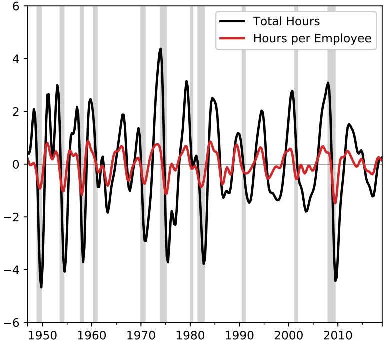
- There are some problems with it though
  - In the data, hours are slightly more volatile than output
  - In the model, hours are less than half as volatile as output
  - In the data, real wage can be either pro- or countercyclical
  - In the model, real wage is strongly procyclical
  - In the data TFP and productivity are mildly correlated with output
  - In the model both are 1:1 correlated with output

- Those results suggest that
  - Need some room for nominal variables
  - More shocks than just TFP are needed
  - **We need to focus more on labor market**
    – should improve behavior of hours and real wage

Indivisible labor: introduction

Most of the variation in hours worked is on the *extensive* margin (employment-unemployment) rather than on the *intensive* margin (hours worked by individual employees)





Indivisible labor: introduction

Most of the variation in hours worked is on the *extensive* margin (employment-unemployment) rather than on the *intensive* margin (hours worked by individual employees)

$$H_t = L_t h_t$$

$$\text{Var}(\log H) = \text{Var}(\log L) + \text{Var}(\log h) + 2 \cdot \text{Cov}(\log L, \log h)$$

Variance-covariance matrix of Hodrick-Prescott deviations

	H	L	h
Total hours H	**3.55**		
Employment L		**2.48**	**0.41**
Hours per employee h		**0.41**	**0.25**

About 70% of variance of total hours worked is accounted for by variance of employment level and only 7% is accounted for by variance of hours worked by individual employees (the rest is accounted for by covariance)

Indivisible labor: setup

- "Realistic" hours worked variation
  results from a two-step process:
  - Decision between working and not working
  - Conditional on working, how much to work
- For simplicity we will focus on the first step only
- Hansen (1985, JME) and Rogerson (1988, JME)
  invented a clever technical solution
- In the RBC model households choose how much to work
- Here they will choose the probability p of working $\bar{h}$ hours:
  - All workers are identical and can work for either 0 hours
    or a fixed number of hours $\bar{h}$
  - Each worker is a part of big family and consumes
    the same amount regardless of working or not
  - All workers will choose the same probability of working p

Households' problem

Consider first a single-period problem:

$$\max \quad U = \ln c + E\left[\phi \ln\left(1-h\right)|p\right]$$

Expand the expected term:

$$E\left[\phi \ln\left(1-h\right)|p\right] = p\phi \ln(1-\bar{h}) + \left(1-p\right)\phi \log\left(1-0\right) = p\phi \ln(1-\bar{h})$$

Since all workers choose the same p, the average number of hours per worker household h is equal to probability p times working hours per employed $\bar{h}$:

$$h = p\bar{h} \quad \rightarrow \quad p = h/\bar{h}$$

Going back to the expected term:

$$E\left[\phi \ln\left(1-h\right)|p\right] = p\phi \ln(1-\bar{h}) = h\frac{\phi \ln(1-\bar{h})}{\bar{h}} \equiv -Bh$$

where $B = -\phi \ln(1-\bar{h})/\bar{h} > 0$. Utility becomes linear in h!

Households' solution I

Households solve the expected utility maximization problem:

$$\begin{aligned} \max \quad & U_t = E_t \left[\sum_{i=0}^{\infty} \beta^i \left(\ln c_{t+i} - B h_{t+i} \right) \right] \\ \text{subject to} \quad & a_{t+1} + c_t = (1 + r_t)\, a_t + w_t h_t + d_t \end{aligned}$$

Lagrangian:

$$\mathcal{L} = \sum_{i=0}^{\infty} \beta^i E_t \left[\begin{array}{c} \ln c_{t+i} - B h_{t+i} \\ + \lambda_{t+i} \left[(1 + r_{t+i})\, a_{t+i} + w_{t+i} h_{t+i} + d_t - a_{t+1+i} - c_{t+i} \right] \end{array} \right]$$

First order conditions:

$$\begin{aligned} c_t &: \quad \frac{1}{c_t} - \lambda_t = 0 && \rightarrow \quad \lambda_t = \frac{1}{c_t} \\ h_t &: \quad -B + \lambda_t w_t = 0 && \rightarrow \quad \lambda_t = \frac{B}{w_t} \\ a_{t+1} &: \quad -\lambda_t + \beta E_t \left[\lambda_{t+1} (1 + r_{t+1}) \right] = 0 && \rightarrow \quad \lambda_t = \beta E_t \left[\lambda_{t+1} (1 + r_{t+1}) \right] \end{aligned}$$

Households' solution II

First order conditions:

$$\begin{aligned} c_t &: \quad \lambda_t = \frac{1}{c_t} \\ h_t &: \quad \lambda_t = \frac{B}{w_t} \\ a_{t+1} &: \quad \lambda_t = \beta E_t \left[\lambda_{t+1} \left(1 + r_{t+1} \right) \right] \end{aligned}$$

Resulting in:

$$\begin{aligned} \text{Intertemporal condition } (c+a) &: \quad \frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(1 + r_{t+1} \right) \right] \\ \text{Intratemporal condition } (c+h) &: \quad c_t = B w_t \end{aligned}$$

Full set of equilibrium conditions

System of 8 equations and 8 unknowns: $\{c, h, y, r, w, k, i, z\}$

$$
\begin{aligned}
\text{Euler equation} &: \quad 1/c_t = \beta E_t \left[(1/c_{t+1}) (1 + r_{t+1}) \right] \\
\text{Consumption-hours choice} &: \quad c_t = B w_t \\
\text{Production function} &: \quad y_t = z_t k_t^{\alpha} h_t^{1-\alpha} \\
\text{Real interest rate} &: \quad r_t = \alpha z_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta \\
\text{Real hourly wage} &: \quad w_t = (1-\alpha) y_t / h_t \\
\text{Investment} &: \quad i_t = k_{t+1} - (1-\delta) k_t \\
\text{Output accounting} &: \quad y_t = c_t + i_t \\
\text{TFP AR(1) process} &: \quad z_t = (1-\rho_z) + \rho_z z_{t-1} + \varepsilon_t
\end{aligned}
$$

Steady state – closed form solution

Start with the Euler equation:

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1}) \right] \quad \rightarrow \quad 1 = \beta (1 + r) \quad \rightarrow \quad r = \frac{1}{\beta} - 1$$

From the interest rate equation obtain the k/h ratio:

$$r = \alpha k^{\alpha - 1} h^{1-\alpha} - \delta \quad \rightarrow \quad \left(\frac{k}{h} \right)^{\alpha - 1} = \frac{r + \delta}{\alpha} \quad \rightarrow \quad \frac{k}{h} = \left(\frac{\alpha}{r + \delta} \right)^{\frac{1}{1-\alpha}}$$

From the production function obtain the y/h ratio and then wage:

$$y = k^{\alpha} h^{1-\alpha} \quad \rightarrow \quad \frac{y}{h} = \left(\frac{k}{h} \right)^{\alpha} \quad \text{and} \quad w = (1 - \alpha) \frac{y}{h}$$

From investment and output accounting eqns. obtain the c/h ratio:

$$i = \delta k \quad \rightarrow \quad y = c + \delta k \quad \rightarrow \quad \frac{c}{h} = \frac{y}{h} - \delta \frac{k}{h}$$

Get c from the consumption-hours choice. Then obtain h:

$$c = Bw \quad \rightarrow \quad h = \frac{c}{c/h}$$

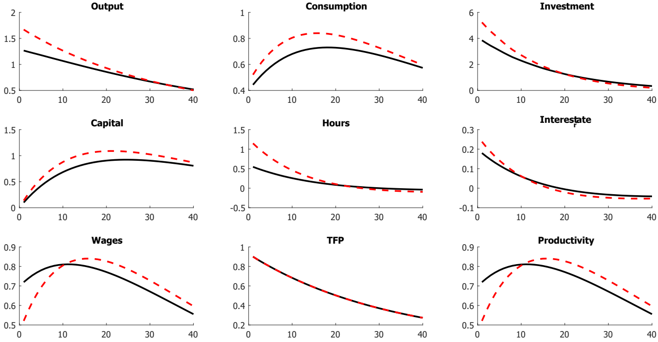
Parameters

- To best compare our two models,
  we need them to generate identical steady states
- We replace parameter ϕ with parameter B
- We choose the value for B so that it matches $h = 1/3$
- For this model $B = 2.63$

Model comparison: impulse response functions

RBC model IRF: black solid lines

Indivisible labor IRF: red dashed lines

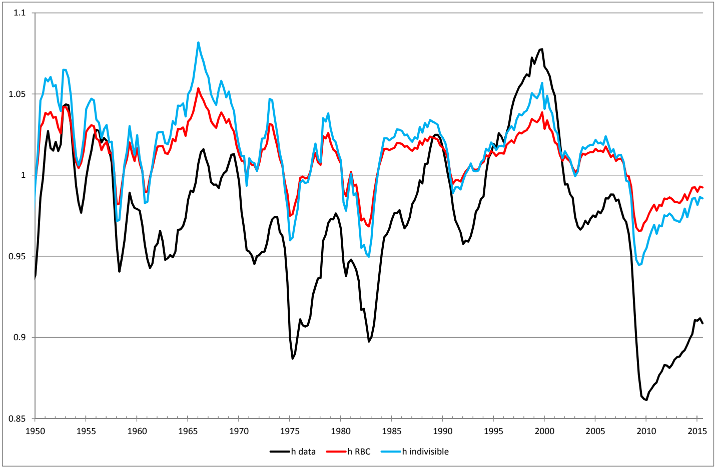


Percentage deviations from steady state (percentage points for r)

Model comparison: moments

	Std. Dev.			Corr. w. y			Autocorr.		
	Data	RBC	Ind	Data	RBC	Ind	Data	RBC	Ind
y	1.63	1.63	1.63	1.00	1.00	1.00	0.85	0.72	0.72
c	0.87	0.63	0.57	0.77	0.94	0.92	0.83	0.79	0.80
i	4.51	5.07	5.28	0.76	0.99	0.99	0.87	0.71	0.71
k	0.59	0.45	0.46	0.40	0.09	0.08	0.95	0.96	0.96
h	1.91	0.71	1.13	0.88	0.98	0.98	0.91	0.71	0.71
w	0.97	0.95	0.57	0.11	0.99	0.92	0.68	0.75	0.80
z	0.84	1.15	0.88	0.53	1.00	1.00	0.73	0.72	0.72
$\frac{y}{h}$	1.06	0.95	0.57	0.41	0.99	0.92	0.71	0.75	0.80

Model comparison: model-generated hours worked



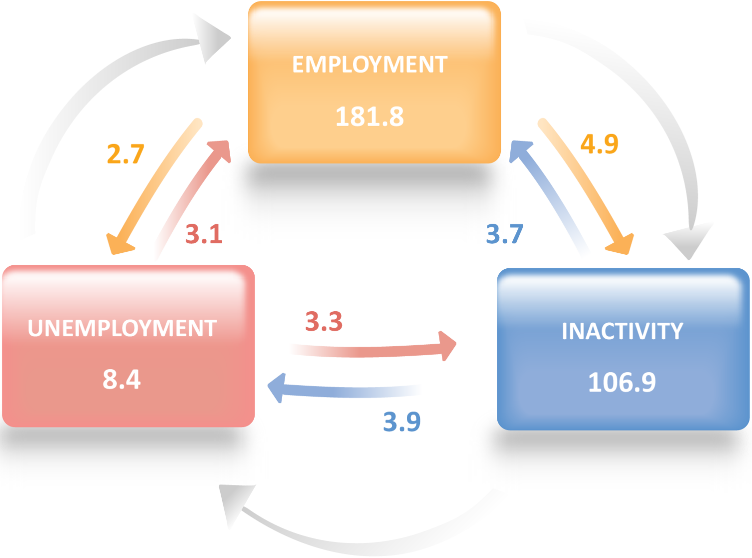
Indivisible labor: summary

- Model enhances hours volatility (but it’s still too low)
- Improves correlation of wages and productivity with output
- Slightly decreases empirical match in other dimensions
- Technical advantage: requires smaller TFP shocks
- Philosophical advantage: more “realistic” labor market

Search and matching: introduction

- Labor markets are in a state of constant flux
- At the same time there are job-seeking workers and worker-seeking firms
- Labor markets are decentralized and active search is needed
- Search friction leads to unemployment even in the steady state
- Peter Diamond, Dale Mortensen and Christopher Pissarides were awarded the Nobel Prize in 2010 for developing this model

Labor market status and flows: EU 2018Q4

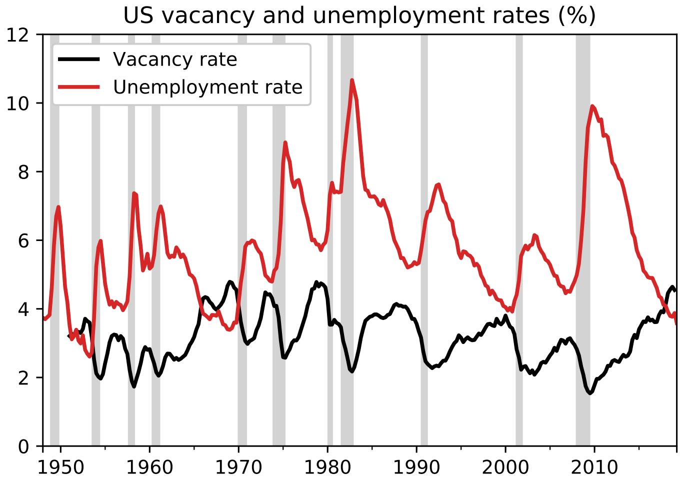


Labor market status change probabilities: EU 2018Q4

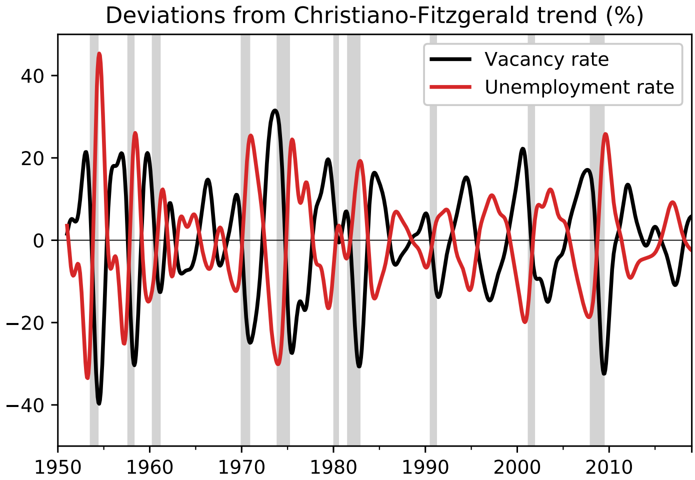
	EMPLOYMENT 2018Q4	UNEMPLOYMENT 2018Q4	INACTIVITY 2018Q4
EMPLOYMENT 2018Q3	96.0 %	1.4 %	2.6 %
UNEMPLOYMENT 2018Q3	20.9 %	57.0 %	22.1%
INACTIVITY 2018Q3	3.2 %	3.4 %	93.3 %

Source: Eurostat

Unemployment and vacancy rates: USA 1948-2019



Labor market fluctuations: USA 1950-2019



Matching function

- Firms create open job positions (openings, vacancies)
- Workers search for jobs
- Both jobs and workers are heterogeneous
  $\hookrightarrow$ not every possible match is attractive
- Matching function captures this feature
- New matches M are a function of the pool of unemployed U and pool of vacancies V:

$$M_t = M\left(V_t, U_t\right) = \chi V_t^{\eta} U_t^{1-\eta}$$

where $\chi > 0$ and $\eta \in (0, 1)$

Job finding and job filling probabilities

- Unemployed workers are interested in job finding probability p:

$$p_t = \frac{M_t}{U_t} = \chi \left(\frac{V_t}{U_t} \right)^{\eta} = \chi \theta_t^{\eta} = q_t \theta_t$$

  where $\theta = V/U$ is called labor market tightness

- Firms with vacancies care about job filling probability q:

$$q_t = \frac{M_t}{V_t} = \chi \left(\frac{V_t}{U_t} \right)^{\eta - 1} = \chi \theta_t^{\eta - 1} = \frac{p_t}{\theta_t}$$

- Dual externality from congestion:
  - High number of unemployed decreases p and increases q
  - High number of vacancies increases p and decreases q

Employment dynamics

- Ignoring labor market inactivity, employment rate n and unemployment rate u sum to unity:

$$n_t + u_t = 1 \quad \rightarrow \quad n_t = 1 - u_t$$

- Existing matches are destroyed with exogenous probability s
- New matches increase next period employment:

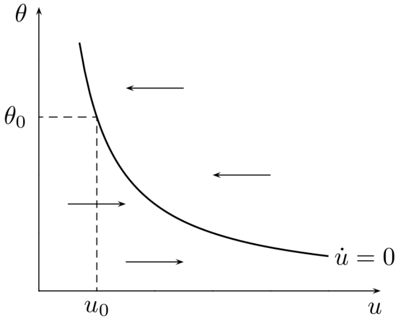
$$n_t = n_{t-1} - sn_{t-1} + m_{t-1}$$
$$u_t = u_{t-1} + sn_{t-1} - m_{t-1}$$

- We can find the steady state unemployment rate as a function of separation and job finding probabilities:

$$u = u + s\left(1 - u\right) - p\left(\theta\right)u$$
$$u = \frac{s}{s + p\left(\theta\right)}$$

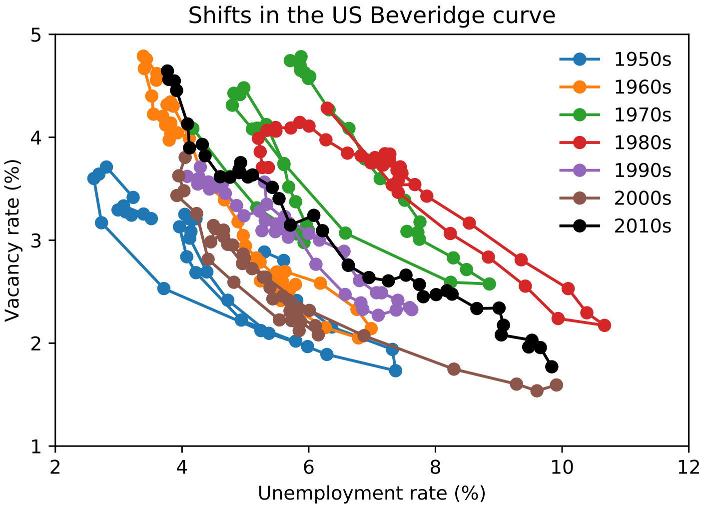
- This generates a Beveridge curve: a negative relationship between the unemployment and vacancy rates

Beveridge curve: theory



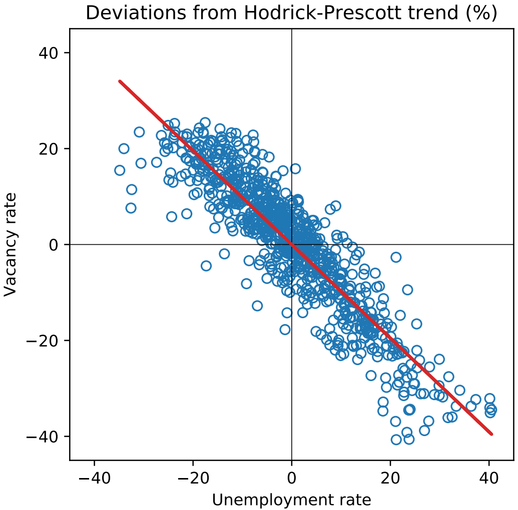
Graph by Leszek Wincenciak

Beveridge curve: data

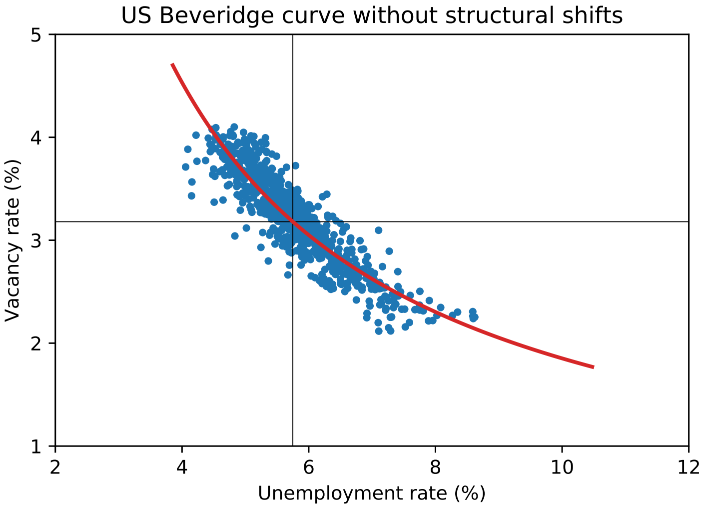


Beveridge curve: data

Detrending with Hodrick-Prescott filter takes out structural shifts



Beveridge curve: “estimation”



Firm side

- Assume firms and workers discount future with β
- Period net gain from a filled job equals marginal product of employee *mpn* less wage *w*
- Existing matches are destroyed with probability *s*:

$$\mathcal{J}_t = (mpn_t - w_t) + \beta E_t \left[(1 - s) \mathcal{J}_{t+1} + s \mathcal{V}_{t+1} \right]$$

- Period net loss from open vacancy is its cost κ
- With probability *q* the vacancy will be filled:

$$\mathcal{V}_t = -\kappa + \beta E_t \left[q_t \mathcal{J}_{t+1} + (1 - q_t) \mathcal{V}_{t+1} \right]$$

- Free entry in vacancies ensures that always $\mathcal{V} = 0$
- In the steady state ($r = 1/\beta - 1$):

$$mpn - w = (r + s) \kappa / q(\theta)$$

Worker side

- Period net gain from employment equals wage w
- Existing matches are destroyed with probability s:

$$\mathcal{E}_t = w_t + \beta E_t \left[(1 - s) \, \mathcal{E}_{t+1} + s \mathcal{U}_{t+1} \right]$$

- Period net gain from unemployment equals benefits (and possibly utility from leisure) b
- With probability p an unemployed person finds a job:

$$\mathcal{U}_t = b + \beta E_t \left[p_t \mathcal{E}_{t+1} + (1 - p_t) \, \mathcal{U}_{t+1} \right]$$

Wage setting

- The negotiated wage can be anywhere between the gain from unemployment b and the marginal product of employee mpn plus match gain $\kappa\theta$
- Nash bargaining allows to model the outcome of negotiations
- Let $\gamma \in [0, 1]$ denote the relative bargaining power of firms
- The negotiated wage is the solution of the problem:

$$\max_{w_t} \quad [\mathcal{J}_t(w_t)]^{\gamma} [\mathcal{E}_t(w_t) - \mathcal{U}_t]^{1-\gamma}$$

- Solving the problem results in: derivation

$$w_t = \gamma b + (1-\gamma)(mpn_t + \kappa\theta_t)$$

- Intuitively: $w \to b$ if $\gamma \to 1$ and $w \to mpn + \kappa\theta$ if $\gamma \to 0$

Full set of equilibrium conditions

System of 9 equations and 9 unknowns: $\{w, mpn, \theta, \mathcal{J}, q, u, n, m, v\}$

$$
\begin{aligned}
w_t &= \gamma b + (1-\gamma)\left(mpn_t + \kappa\theta_t\right) \\
\mathcal{J}_t &= (mpn_t - w_t) + (1-s)\cdot \beta E_t\left[\mathcal{J}_{t+1}\right] \\
\kappa &= q_t \cdot \beta E_t\left[\mathcal{J}_{t+1}\right] \\
u_t &= 1 - n_t \\
n_t &= (1-s)\, n_{t-1} + m_{t-1} \\
q_t &= \chi \theta_t^{\eta-1} \\
\theta_t &= v_t / u_t \\
m_t &= \chi v_t^{\eta} u_t^{1-\eta} \\
mpn_t &= (1-\rho) + \rho \cdot mpn_{t-1} + \varepsilon_t
\end{aligned}
$$

Steady state: key equations

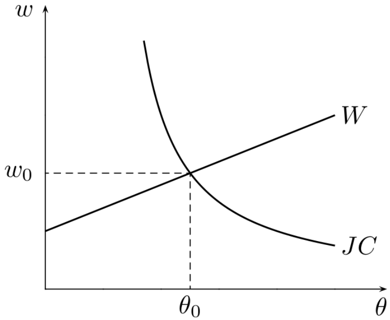
In the steady state the model is fully summarized by the following three key equations:

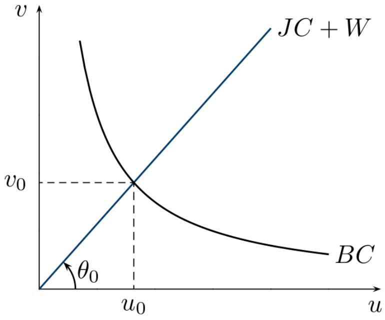
$$\text{Beveridge curve (BC)} \quad : \quad u = \frac{s}{s + p(\theta)}$$

$$\text{Job (vacancy) creation (JC)} \quad : \quad w = mpn - (r + s)\frac{\kappa}{q(\theta)}$$

$$\text{Wage setting (W)} \quad : \quad w = \gamma b + (1 - \gamma)(mpn + \kappa\theta)$$

Steady state: graphical solution



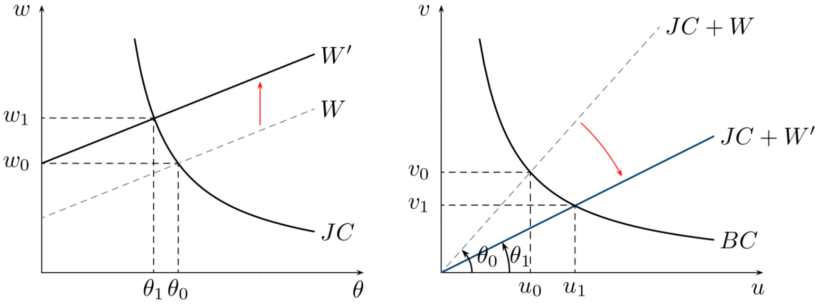


Graph by Leszek Wincenciak

Comparative statics I

Effects of an increase in unemployment benefits ($b \uparrow$) or in workers' bargaining power ($\gamma \downarrow$):

- Increase in real wage *w*
- Decrease in labor market tightness θ
- Decrease in vacancy rate *v*
- Increase in unemployment rate *u*

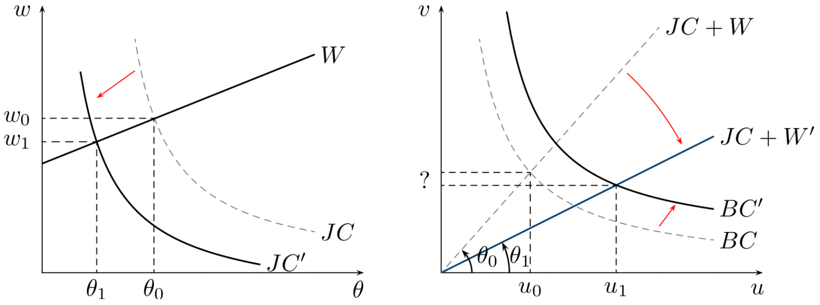


Graph by Leszek Wincenciak

Comparative statics II

Effects of an increase in separation rate ($s \uparrow$)
or a decrease in matching efficiency ($\chi \downarrow$):

- Decrease in real wage w
- Decrease in labor market tightness θ
- Ambiguous effect on vacancy rate v
- Increase in unemployment rate u

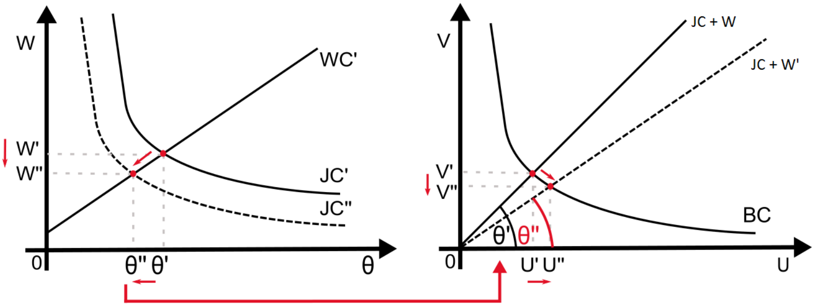


Graph by Leszek Wincenciak

Comparative statics III

Effects of an increase in interest rate ($r \uparrow$)
or an increase in impatience ($\rho \uparrow \quad \rightarrow \quad \beta \downarrow$):

- Decrease in real wage w
- Decrease in labor market tightness θ
- Decrease in vacancy rate v
- Increase in unemployment rate u

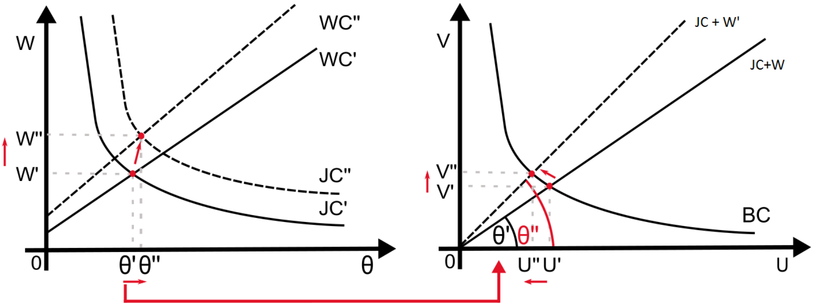


Graph by Matthias Hertweck

Comparative statics IV

Effects of an increase in labor productivity ($mpn \uparrow$):

- Increase in real wage w
- Increase in labor market tightness θ
- Increase in vacancy rate v
- Decrease in unemployment rate u



Graph by Matthias Hertweck

Transitional dynamics

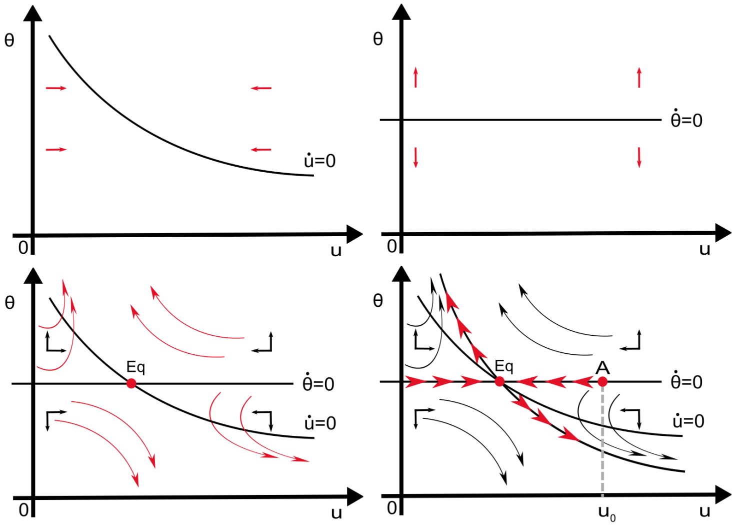
Reduced form of the model (with *mpn* treated as exogenous):

$$\dot{u} = s\,(1-u) - \chi\theta^{\eta}\cdot u$$

$$\dot{\theta} = \frac{\theta}{1-\eta}\left[(r+s) - \gamma\,(mpn-b)\,\frac{\chi\theta^{\eta-1}}{\kappa} + (1-\gamma)\,\chi\theta^{\eta}\right]$$

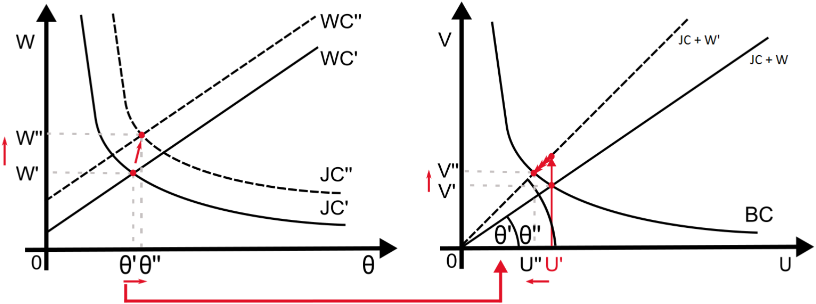
The dynamic equation for θ is independent of u:
$\dot{\theta} = 0$ is a flat line in the (u,θ) space

Transitional dynamics: phase diagram



Graph by Matthias Hertweck

Transitional dynamics: positive productivity shock



Graph by Matthias Hertweck

Parameters

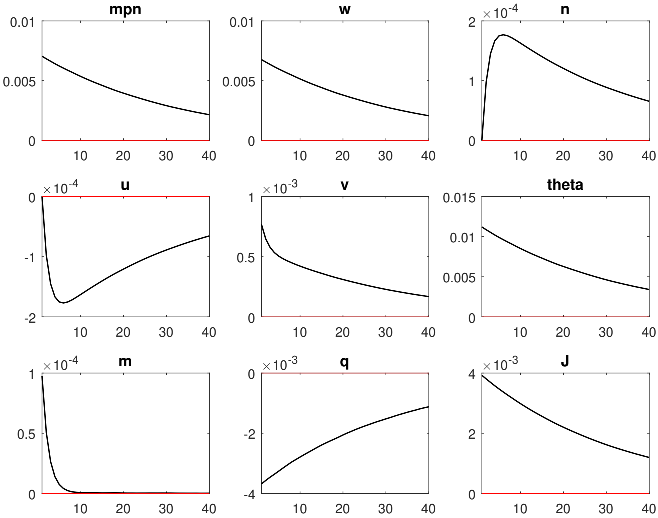
Values come from Shimer (2005, AER)

	Description	Value
χ	matching efficiency	0.45
η	matching elasticity of v	0.28
s	separation probability	0.033
β	discount factor	0.99
mpn	steady state marginal product	1
κ	vacancy cost	0.21
b	unemployment benefit	0.4
γ	firm bargaining power	0.28

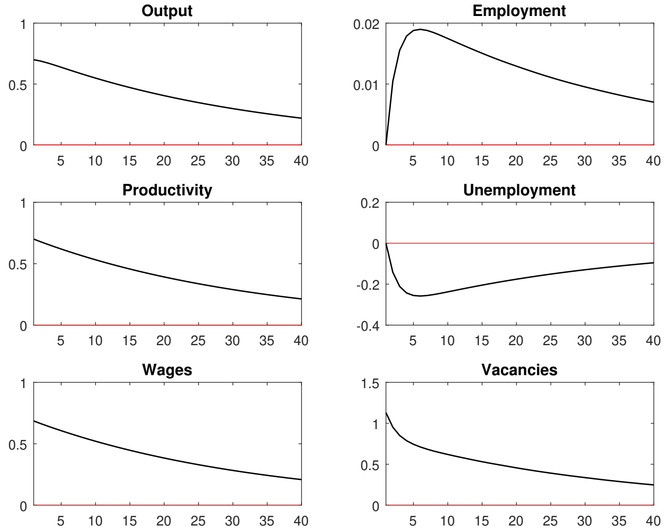
Implied steady state values

	Description	Value
u	unemployment rate	0.0687
v	vacancy rate	0.0674
m	new matches	0.031
θ	tightness	0.98
p	job finding probability	0.448
q	job filling probability	0.456
w	wage	0.98

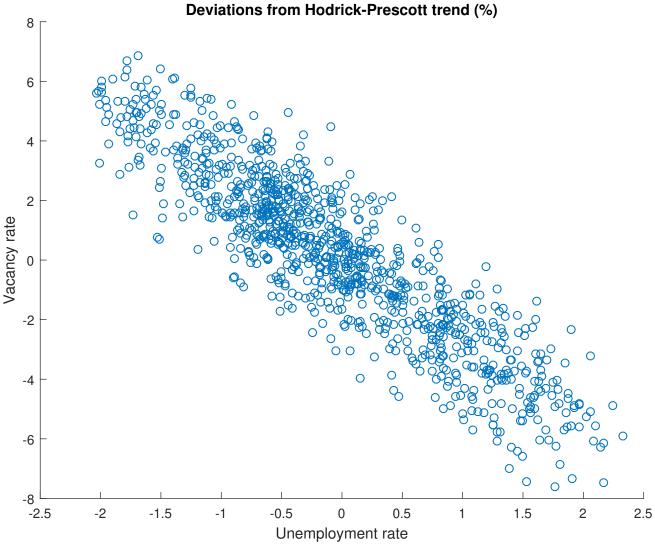
Impulse response functions I



Impulse response functions II



Model generated Beveridge curve



Summary

- We have a “realistic” model of the labor market
- Able to match both steady state (average) and some cyclical properties of the labor market
- Replicates the negative slope of the Beveridge curve
- Not enough variation in employment
- Beveridge curve too steep
- Too much variation in wages

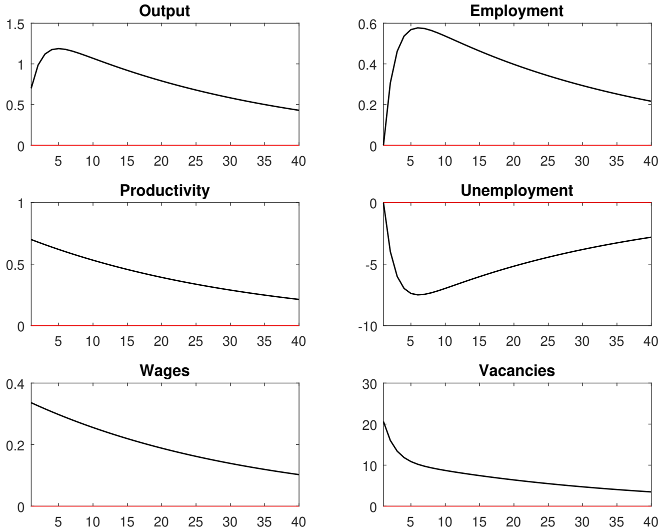
Alternative parametrization

Values come from Hagedorn & Manovskii (2008, AER)

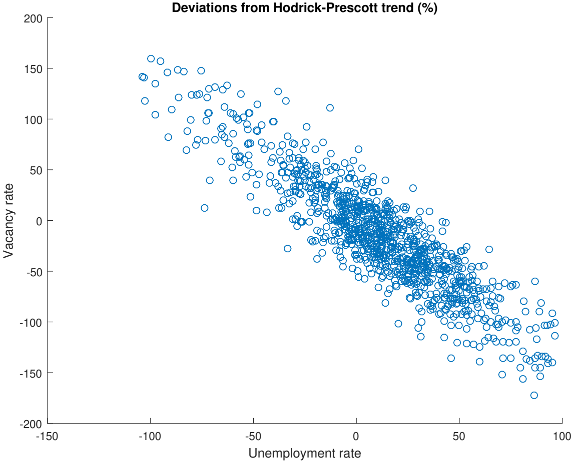
	Description	Value
η	matching elasticity of v	0.45
b	unemployment benefit	0.965
γ	firm bargaining power	0.928

- Firms have very strong bargaining position
- But unemployment gain includes leisure utility
- Steady state unchanged

Hagedorn & Manovskii: Impulse response functions



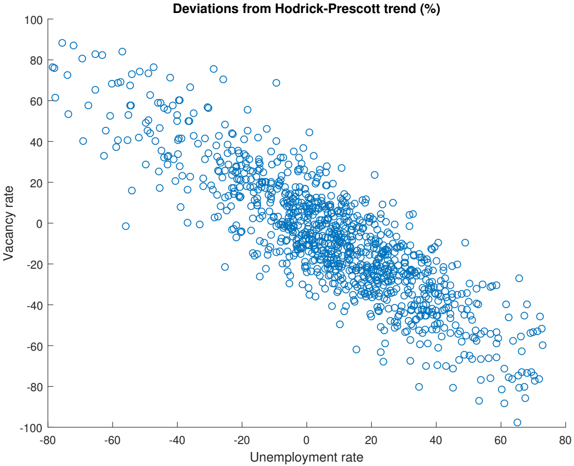
Hagedorn & Manovskii: Beveridge curve



Mortensen & Nagypal: Beveridge curve

Mortensen & Nagypal (2007) set $\eta = 0.54$

Model BC replicates slope of the data BC:



Summary

- Alternative parametrizations yield better results
- Both unemployment and employment become more volatile
- Volatility of wages is diminished
- Key problem for the search and matching model identified: period-by-period Nash bargaining
- Further extensions make alternative assumptions about the wage setting process

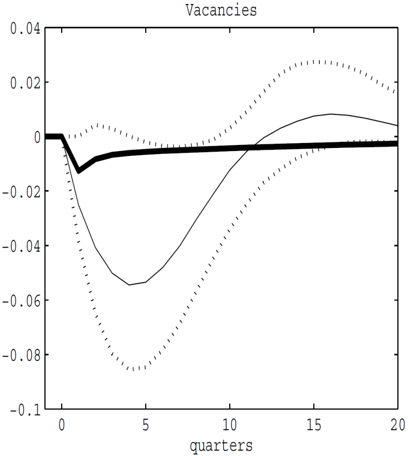
Integration with the RBC framework

- Very easy
- Get *mpn* from the usual firm problem
- Adjust β for $\beta \frac{\lambda_{t+1}}{\lambda_t}$ in the firm's valuation since the latter is the correct stochastic discounting factor
- Solve for labor market variables
- Get back to the RBC part
- Include vacancy costs in the national accounting equation:

$$y_t = c_t + i_t + \kappa v_t$$

Observation of Fujita

Fujita (2004): model IRF for vacancies is counterfactual



Alternative hiring cost function

- We have assumed linear vacancy costs:

$$w_t = \gamma b + (1-\gamma)\left(mpn_t + \kappa\theta_t\right)$$
$$\frac{\kappa}{q_t} = \beta E_t \left[mpn_{t+1} - w_{t+1} + (1-s)\frac{\kappa}{q_{t+1}} \right]$$

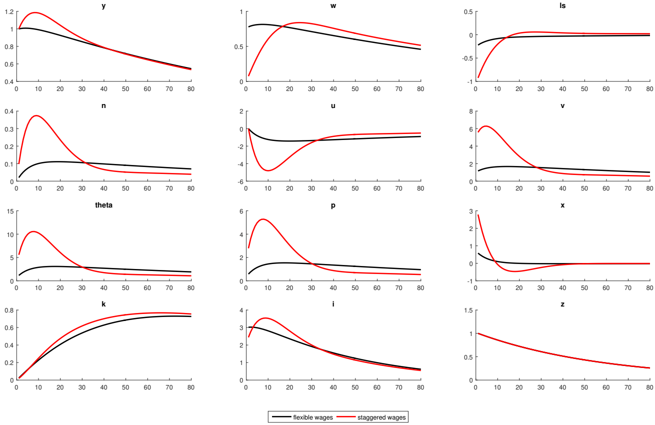
- Gertler & Trigari (2009, JPE) assume convex costs:

$$x_t \equiv \frac{m_t}{n_t}$$
$$w_t = \gamma b + (1-\gamma)\left(mpn_t + \frac{\kappa}{2}x_t^2 + p_t\kappa x_t\right)$$
$$\kappa x_t = \beta E_t \left[mpn_{t+1} - w_{t+1} + (1-s)\kappa x_{t+1} + \frac{\kappa}{2}x_t^2 \right]$$

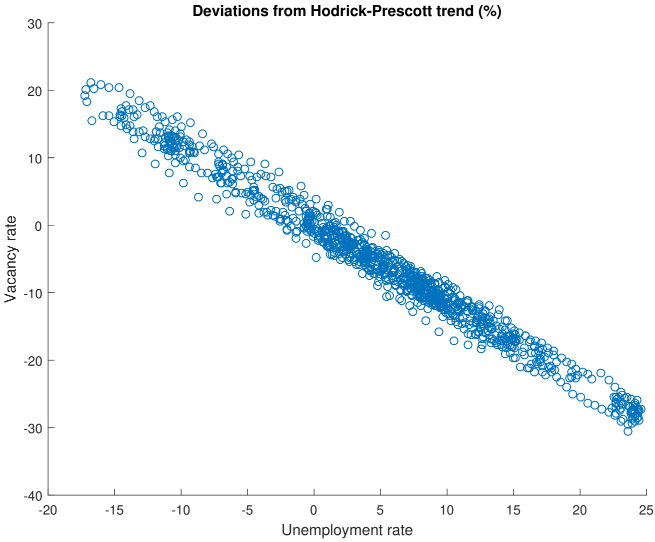
- They also consider multi-period wage contracts: within each period only a fraction of wage contracts are renegotiated

Gertler & Trigari: Impulse response functions

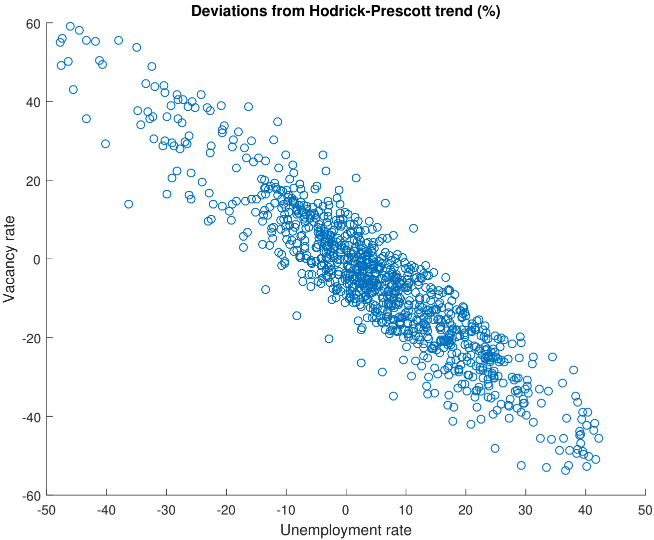
Monthly period frequency



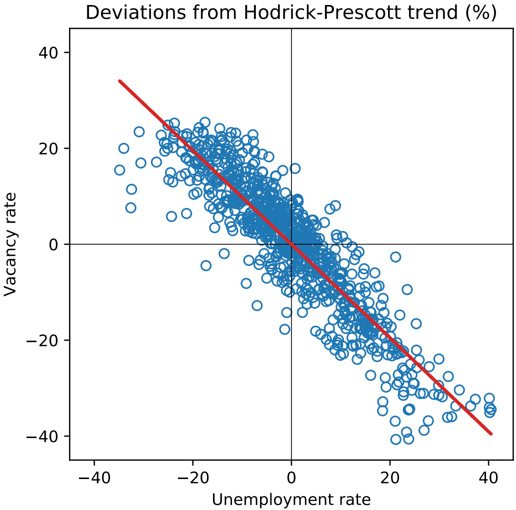
Gertler & Trigari: Beveridge curve (flexible wages)



Gertler & Trigari: Beveridge curve (staggered wages)



Beveridge curve: data



Gertler & Trigari: business cycle statistics

	y	w	ls	n	u	v	θ	a	i	c
	A. U.S. Economy, 1964:1–2005:1									
Relative standard deviation	1.00	.52	.51	.60	5.15	6.30	11.28	.61	2.71	.41
Autocorrelation	.87	.91	.73	.94	.91	.91	.91	.79	.85	.87
Correlation with y	1.00	.56	−.20	.78	−.86	.91	.90	.71	.94	.81
	B. Model Economy, $\lambda = 0$ (Flexible Wages)									
Relative standard deviation	1.00	.87	.09	.10	1.24	1.58	2.72	.93	3.11	.37
Autocorrelation	.81	.81	.58	.92	.92	.86	.90	.78	.80	.85
Correlation with y	1.00	1.00	−.54	.59	−.59	.98	.92	1.00	.99	.93
	C. Model Economy, $\lambda = 8/9$ (3 Quarters)									
Relative standard deviation	1.00	.56	.57	.35	4.44	5.81	9.84	.71	3.18	.35
Autocorrelation	.84	.95	.65	.90	.90	.82	.88	.76	.86	.86
Correlation with y	1.00	.66	−.56	.77	−.77	.91	.94	.97	.99	.90
	D. Model Economy, $\lambda = 11/12$ (4 Quarters)									
Relative standard deviation	1.00	.48	.58	.44	5.68	7.28	12.52	.64	3.18	.34
Autocorrelation	.85	.96	.68	.91	.91	.86	.90	.74	.88	.86
Correlation with y	1.00	.55	−.59	.78	−.78	.93	.95	.95	.99	.90

Summary

- After adding multi-period contracts, Gertler & Trigari obtain a very good empirical match of the RBC model with search & matching features
- This is one of the best matches for single-shock models
- Key to the success was:
  - Convex vacancy posting
  - Staggered (multi-period) wage contracts

Possible further extensions

- Endogenous (non-constant) separation rate
- On-the-job search
- Hours per worker adjustments

Derivation of the wage setting equation I

The negotiated wage is the solution of the problem:

$$\max_{w_t} \quad \left[\mathcal{J}_t\left(w_t\right)\right]^{\gamma}\left[\mathcal{E}_t\left(w_t\right)-\mathcal{U}_t\right]^{1-\gamma}$$

Derivatives of $\mathcal{J}_t$ and $\mathcal{E}_t$ with respect to wage w_t:

$$\mathcal{J}_t = mpn_t - w_t + (1-s)\cdot\beta E_t\left[\mathcal{J}_{t+1}\right] \quad \rightarrow \quad \frac{\partial \mathcal{J}_t}{\partial w_t} = -1$$

$$\mathcal{E}_t = w_t + \beta E_t\left[(1-s)\,\mathcal{E}_{t+1} + s\mathcal{U}_{t+1}\right] \quad \rightarrow \quad \frac{\partial \mathcal{E}_t}{\partial w_t} = 1$$

First order condition:

$$\gamma \mathcal{J}_t^{\gamma-1}\cdot\frac{\partial \mathcal{J}_t}{\partial w_t}\cdot\left(\mathcal{E}_t-\mathcal{U}_t\right)^{1-\gamma} + \mathcal{J}_t^{\gamma}\cdot(1-\gamma)\left(\mathcal{E}_t-\mathcal{U}_t\right)^{-\gamma}\cdot\frac{\partial \mathcal{E}_t}{\partial w_t} = 0$$

$$\gamma\left(\mathcal{E}_t-\mathcal{U}_t\right) = (1-\gamma)\,\mathcal{J}_t$$

Derivation of the wage setting equation II

Plug in expressions for $\mathcal{E}_t$, $\mathcal{U}_t$ and $\mathcal{J}_t$:

$$\gamma \left\{ (w_t - b) + \beta \left(1 - s - p_t\right) E_t \left[\mathcal{E}_{t+1} - \mathcal{U}_{t+1}\right] \right\} \\ = (1-\gamma) \left\{ (mpn_t - w_t) + \beta E_t \left[(1-s)\,\mathcal{J}_{t+1}\right] \right\}$$

$$w_t - \gamma b + (1 - s - p_t)\,\beta E_t \left[\gamma \left(\mathcal{E}_{t+1} - \mathcal{U}_{t+1}\right)\right] \\ = (1-\gamma)\,mpn_t + (1-s)\,\beta E_t \left[(1-\gamma)\,\mathcal{J}_{t+1}\right]$$

$$w_t - \gamma b + (1 - s - p_t)\,\beta E_t \left[(1-\gamma)\,\mathcal{J}_{t+1}\right] \\ = (1-\gamma)\,mpn_t + (1-s)\,\beta E_t \left[(1-\gamma)\,\mathcal{J}_{t+1}\right]$$

Derivation of the wage setting equation III

$$
\begin{aligned}
w_t &= \gamma b + (1-\gamma)\left\{mpn_t + p_t \beta E_t\left[\mathcal{J}_{t+1}\right]\right\} \\
\kappa / q_t &= \beta E_t\left[\mathcal{J}_{t+1}\right] \\
w_t &= \gamma b + (1-\gamma)\left(mpn_t + p_t \kappa / q_t\right) \\
w_t &= \gamma b + (1-\gamma)\left(mpn_t + \kappa \theta_t\right)
\end{aligned}
$$

◂ back