University of Warsaw
Faculty of Economic Sciences

Models of inequality

Advanced Macroeconomics IE: Lecture 13

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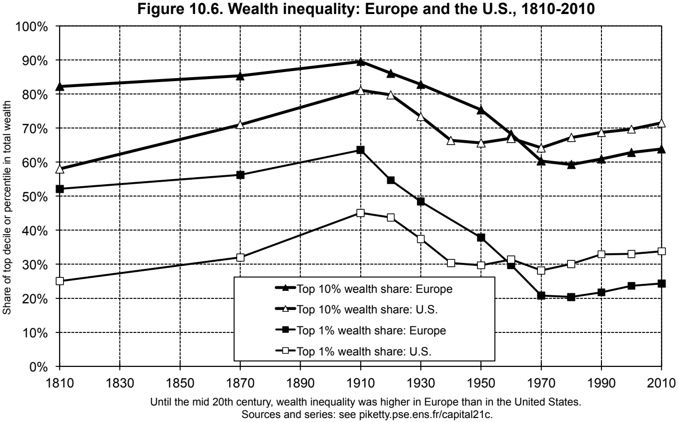
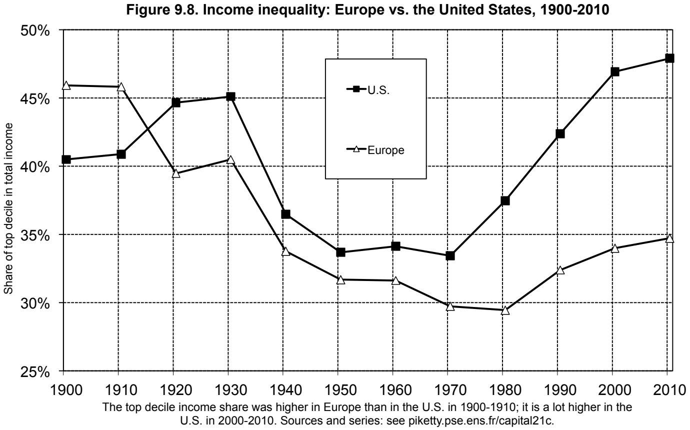
University of Warsaw

Evolution of top wealth



Piketty (2014) *Capital in the Twenty-First Century*

Evolution of top incomes



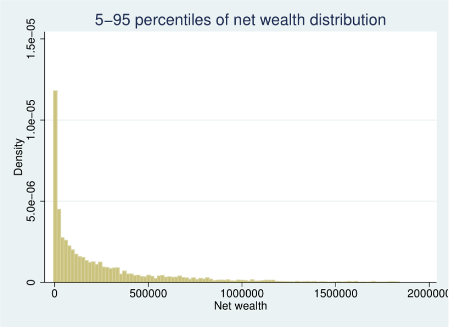
Piketty (2014) Capital in the Twenty-First Century

Lecture plan

1. Model of (top) wealth inequality
   based on Jones (2015)
2. Simple model of precautionary savings
   and the role of borrowing constraints
3. Quantitative models of income and wealth inequality
   based on De Nardi (2015)

(Top) wealth inequality – Jones (2015)

US wealth distribution

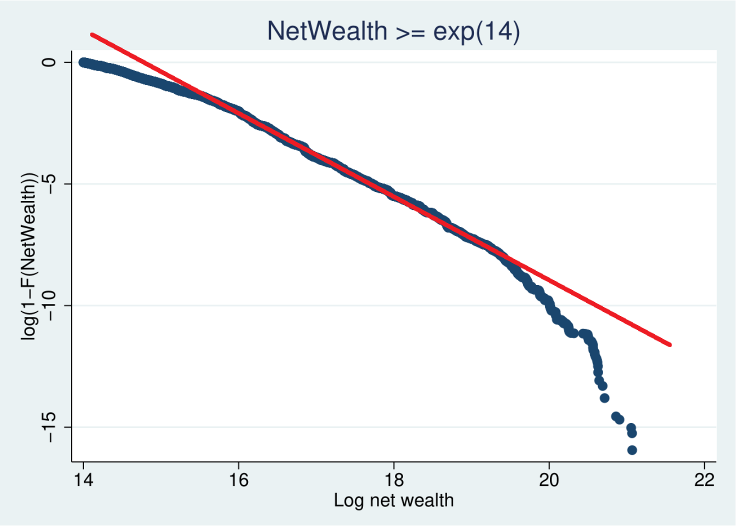


Ben Moll's lecture

Features of US Wealth Distribution:

- right skewness
- heavy upper tail
- Pareto distribution

US wealth distribution



Ben Moll's lecture

Properties of Pareto distribution

- When a variable (e.g. wealth) is Pareto distributed, it satisfies:

$$\Pr\left[\text{wealth} > a\right] = \left(a/a_{min}\right)^{-1/\eta}$$

  which means the fraction of people with wealth greater than some cutoff is proportional to the cutoff raised to some power

- Under Pareto distribution, computation of “top shares” is easy. The fraction of wealth going to the top p percentiles is given by:

$$\left(100/p\right)^{\eta-1}$$

- The higher η is, the more unequal the distribution

- For $\eta = 0.5$ the top 1% wealth share is $100^{-0.5} = 10\%$
  For $\eta = 0.75$ the top 1% wealth share is $100^{-0.25} \approx 32\%$

- Piketty (2014): in the US the top 1% wealth share $\approx 33\%$, in UK and France between 25% and 30%

Core intuition

- Assume (for now) that the size of population does not change
- Suppose households face a constant probability of death d
- Then the probability that an individual is of at least age x is:

$$\Pr\left[\text{age} > x\right] = e^{-dx} \approx \left(1 - d\right)^x$$

- Assume (for now) that everyone had the same initial wealth = 1
- Let the wealth of households increase with age at rate μ:

$$a\left(x\right) = e^{\mu x} \approx \left(1 + \mu\right)^x \quad \rightarrow \quad x\left(a\right) = \left(1/\mu\right) \cdot \ln a$$

- Then we can easily map the probability of holding at least some amount of wealth to the probability of being old enough:

$$\Pr\left[\text{wealth} > a\right] = \Pr\left[\text{age} > x\left(a\right)\right] = \exp\left(-\left(d/\mu\right) \cdot \ln a\right) = a^{-d/\mu}$$

- Wealth is Pareto distributed with $\eta = \mu/d$

Demographics

- Maintain the assumption of constant death probability
- Allow population size to change over time
- Denote with B_t the number of people born in period t
- Define a (crude) birth rate b and assume it's constant:

$$b = B_t / N_t \quad \rightarrow \quad B_t = bN_t$$

- Population growth rate n is the difference between crude birth and death rates:

$$1 + n = \frac{N_{t+1}}{N_t} = \frac{N_t (1 - d) + B_t}{N_t} = 1 - d + b$$
$$n = b - d \quad \rightarrow \quad b = n + d$$

- Share of people born in t relative to population in period t:

$$B_t / N_t = b$$

Age distribution

- Share of people born in t relative to population in period $t+1$:

$$\frac{B_t\left(1-d\right)}{N_{t+1}} = \frac{B_t\left(1-d\right)}{N_t\left(1+n\right)} = b\frac{1-d}{1+n}$$

- Share of people born in t relative to population in period $t+2$:

$$\frac{B_t\left(1-d\right)^2}{N_{t+2}} = \frac{B_t\left(1-d\right)^2}{N_t\left(1+n\right)^2} = b\left(\frac{1-d}{1+n}\right)^2$$

- Share of people aged x in the population is given by:

$$sh\left(x\right) = b\left(\frac{1-d}{1+n}\right)^x \approx b\left(1-d-n\right)^x = b\left(1-b\right)^x \approx be^{-bx}$$

- Probability that a person is at least of age x:

$$\Pr\left[\text{age} > x\right] = \int_x^\infty be^{-bs}\,ds = e^{-bx}$$

Households' choice

- Households solve the following utility maximization problem:

$$\max \quad U = \sum_{t=0}^{\infty} \left[\beta\left(1-d\right)\right]^{t} \ln c_t$$

$$\text{subject to} \quad a_{t+1} = \left(1+r-\tau\right) a_t - c_t$$

where households do not receive any labor income
and τ is a proportional tax on wealth

- Euler equation: $$c_{t+1} = \beta\left(1-d\right)\left(1+r-\tau\right) c_t$$

- Guess-and-verify that households consume a fixed fraction α of their wealth ($c_t = \alpha a_t$, value of $\alpha \approx \rho + d$ is of no importance):

$$\begin{aligned} \alpha a_{t+1} &= \beta\left(1-d\right)\left(1+r-\tau\right) \alpha a_t \\ \alpha\left[\left(1+r-\tau\right) a_t - \alpha a_t\right] &= \beta\left(1-d\right)\left(1+r-\tau\right) \alpha a_t \\ \left(1+r-\tau\right) - \alpha &= \beta\left(1-d\right)\left(1+r-\tau\right) \end{aligned}$$

Wealth dynamics

- Budget constraint then determines the dynamics of wealth:

$$a_{t+1} = (1 + r - \tau - \alpha)\, a_t$$
$$a_t = (1 + r - \tau - \alpha)^t\, a_0$$

- Let $a_t(x)$ denote the wealth of a person aged x at time period t:

$$a_t(x) = (1 + r - \tau - \alpha)^x\, a_{t-x}(0)$$

- Assume that newly born agents inherit wealth of the deceased:

$$a_t(0) = \frac{dK_t}{B_t} = \frac{dK_t}{bN_t} = \frac{d}{b} k_t$$

- Assume the BGP economy with exogenous tech. progress:

$$k_t = (1 + g)^t\, k_0$$

- Wealth inherited by newborns in period $t - x$:

$$a_{t-x}(0) = \frac{d}{b} k_{t-x} = \frac{d}{b} (1 + g)^{-x}\, k_t$$

Wealth distribution

- Wealth of people aged x at time period t:

$$a_t(x) = (1 + r - \tau - \alpha)^x \cdot \frac{d}{b}(1+g)^{-x} k_t \approx \frac{d}{b} k_t \cdot e^{(r-g-\tau-\alpha)x}$$

- Age x needed to accumulate wealth a:

$$x(a_t) = \frac{1}{r-g-\tau-\alpha} \cdot \ln\left(\frac{a_t}{(d/b)\,k_t}\right)$$

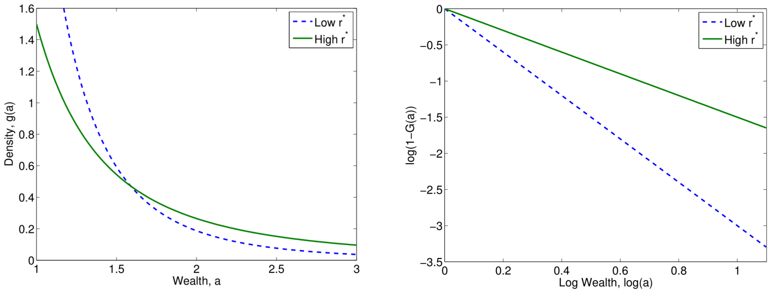
- Probability of holding wealth of at least a is then given by:

$$\Pr[\text{wealth} > a] = \Pr[\text{age} > x(a)] = e^{-bx(a)}$$

$$= \exp\left[-\frac{b}{r-g-\tau-\alpha} \cdot \ln\left(\frac{a_t}{(d/b)\,k_t}\right)\right] = \left[\frac{a_t}{(d/b)\,k_t}\right]^{-\frac{b}{r-g-\tau-\alpha}}$$

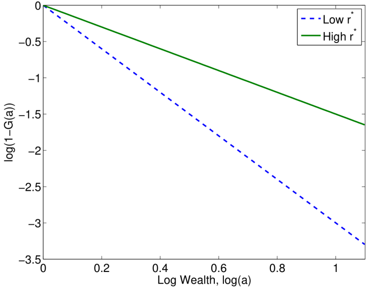
- Wealth is Pareto distributed with $\eta = \dfrac{r-g-\tau-\alpha}{n+d}$

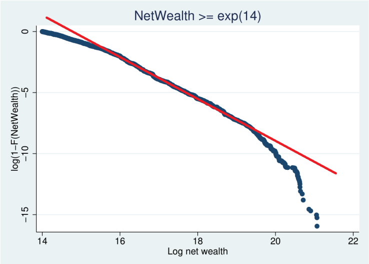
Wealth distribution (Partial Equilibrium)



Ben Moll's lecture

Wealth distribution (Partial Equilibrium)

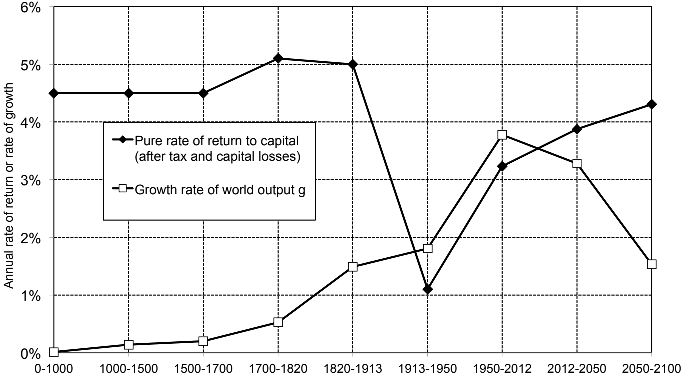




Ben Moll's lecture

Piketty (2014): importance of $r\,(-\tau) - g\,(-n)$

Figure 10.10. After tax rate of return vs. growth rate at the world level, from Antiquity until 2100



The rate of return to capital (after tax and capital losses) fell below the growth rate during the 20th century, and may again surpass it in the 21st century. Sources and series : see piketty.pse.ens.fr/capital21c

Piketty (2014) Capital in the Twenty-First Century

$r > g$ t-shirts

r > g t-shirts

Wealth inequality (Partial Equilibrium)

- Wealth is Pareto distributed with $\eta = \dfrac{r - g - \tau - \alpha}{n + d}$
- Those lucky to live a long life (members of long-lived dynasties) will accumulate greater stocks of wealth
- Piketty (2014): increase in $r - g$ $(-n)$ increases wealth inequality
- 19th century: low g and low $n \rightarrow$ high inequality
- Middle 20th century: high g and $n \rightarrow$ low inequality
- 21st century: declining g and $n \rightarrow$ back to 19th century (?)
- Piketty's prescription: increase τ to counteract g and n

Wealth inequality (General Equilibrium)

- Relationship between aggregate capital and individual wealth:

$$K_t = \sum_{x=0}^{\infty} sh(x)\, N_t \cdot a_t(x) = \sum_{x=0}^{\infty} b(1-b)^x N_t \cdot \frac{dk_t}{b}(1+r-g-\tau-\alpha)^x$$

$$\approx dK_t \sum_{x=0}^{\infty} (1+r-g-\tau-\alpha-b)^x = \frac{dK_t}{1-(1+r-g-\tau-\alpha-b)}$$

- Real interest rate under General Equilibrium is given by:

$$d = -(r-g-\tau-\alpha-d-n) \quad \rightarrow \quad r = n+g+\tau+\alpha$$

- Wealth inequality coefficient under General Equilibrium:

$$\eta = \frac{r-g-\tau-\alpha}{n+d} = \frac{n+g+\tau+\alpha-g-\tau-\alpha}{n+d} = \frac{n}{n+d}$$

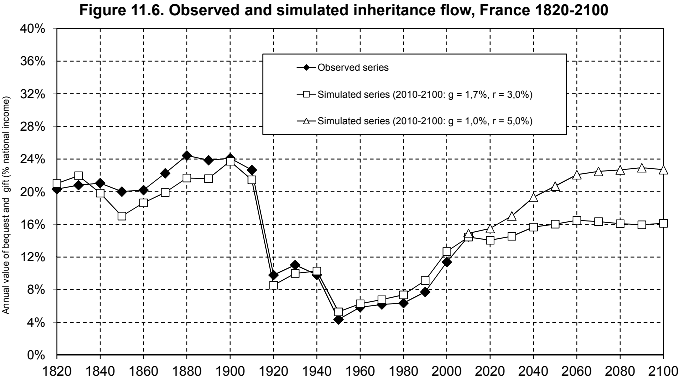
- Wealth inequality is determined purely by demography!

Takeaway

$$\eta^{PE} = \frac{r - g - \tau - \alpha}{n + d} \quad \text{vs} \quad \eta^{GE} = \frac{n}{n + d}$$

- Both results hinge on assumptions regarding inheritance
- If $n > 0$, more people are born than die,
  and newborns inherit less than the average wealth per capita
- If $n = 0$, there is no inequality under General Equilibrium!
- Need for richer framework, including bequests, social mobility, progressive taxation, micro- and macroeconomic shocks
- Piketty is right to highlight the link between $r - g$,
  population growth, taxes and top wealth inequality (under PE)
- But these results are fragile and can disappear under GE
  $\hookrightarrow$ more research needed (empirics & theory)

Either way, inheritance is key



Simulations based upon the theoretical model indicate that the level of the inheritance flow in the 21st century will depend upon the growth rate and the net rate of return to capital. Sources and series: see piketty.pse.ens.fr/capital21c.

Piketty (2014) Capital in the Twenty-First Century

Precautionary savings and borrowing constraints

Household's problem under uncertainty I

- Consider a two-period expected utility maximization problem:

$$\begin{aligned} \max_{c_1,\, c_2,\, a} \quad & U = \ln\left(c_1\right) + \beta E_1\left[\ln\left(c_2\right)\right] \\ \text{subject to} \quad & c_1 + a = y_1 \\ & c_2 = y_2 + \left(1+r\right) a \end{aligned}$$

- First period income is certain and equals y
- Second period income will be equal to either $y + e$ or $y - e$, with 50%-50% probability:

$$y_2 = \begin{cases} y + e & \text{w. prob. } 1/2 \\ y - e & \text{w. prob. } 1/2 \end{cases}$$

Household's problem under uncertainty II

- Assume $\beta = 1$ and $r = 0$ for simplicity
- Use budget constraints to express consumption levels:

$$c_1 = y - a$$

$$c_2 = \begin{cases} y + e + a & \text{w. prob. } 1/2 \\ y - e + a & \text{w. prob. } 1/2 \end{cases}$$

- Rewrite the problem as choosing the optimal a alone:

$$\max_a \quad U = \ln(y - a) + \frac{1}{2}\ln(y + e + a) + \frac{1}{2}\ln(y - e + a)$$

- First order condition:

$$-\frac{1}{y - a} + \frac{1}{2}\frac{1}{y + e + a} + \frac{1}{2}\frac{1}{y - e + a} = 0$$

- Solution: ▸ full solution

$$a = \frac{1}{2}\left(\sqrt{y^2 + 2e^2} - y\right)$$

Precautionary savings

$$a = \frac{1}{2}\left(\sqrt{y^2 + 2e^2} - y\right)$$

When second period income is certain ($e = 0$) then
(given $\beta = 1$ and $r = 0$) the household holds no assets in optimum.
It allows them to smooth consumption over time, since $c_1 = c_2 = y$.

When there is uncertainty about second period income ($e > 0$),
the household accumulates **precautionary savings** to self-insure
against the scenario of low income in the second period.

We can easily demonstrate that the more variable second period
income is, the higher is the stock of accumulated assets:

$$\frac{\partial a}{\partial e} = \frac{1}{2} \cdot \frac{1}{2\sqrt{y^2 + 2e^2}} \cdot 2 \cdot 2e = \frac{e}{\sqrt{y^2 + 2e^2}} > 0$$

Incomplete markets and borrowing constraints

- But what if they could purchase insurance against shocks?
  - insurance pays $+e$ under negative shock and $-e$ under positive
  - consumption always equals y, no matter the state of the world
  - expected insurance payout = 0, should be available at low cost
  - its absence evidence for market incompleteness
  - real-world example: no full unemployment insurance
- HH could not borrow in period 2 to smooth out the shocks
- Can be shown that (assuming $\beta(1+r)=1$) infinitely lived household does not change consumption under temporary income shocks when borrowing constraints are absent
  - asset holdings follow random walk
  - wealth distribution indeterminate
- Market incompleteness and borrowing constraints generate stationary wealth distribution under idiosyncratic shocks
  - typically under GE $\beta(1+r)<1 \rightarrow$ "excess" savings
  - capital taxation may increase welfare if finances insurance

Solution to precautionary savings problem I

Rewrite the FOC:

$$
\begin{aligned}
\frac{1}{y-a} &= \frac{1}{2}\frac{1}{y+e+a} + \frac{1}{2}\frac{1}{y-e+a} \quad \Big| \quad \cdot 2 \\
\frac{2}{y-a} &= \frac{1}{y+e+a} + \frac{1}{y-e+a} \\
\frac{2}{y-a} &= \frac{y-e+a+y+e+a}{(y+e+a)(y-e+a)} \\
\frac{2}{y-a} &= \frac{2y+2a}{y^2-ye+ya+ey-e^2+ea+ay-ae+a^2} \\
\frac{2}{y-a} &= \frac{2(y+a)}{y^2+2ay-e^2+a^2} \\
\frac{1}{y-a} &= \frac{y+a}{y^2+2ay-e^2+a^2}
\end{aligned}
$$

Solution to precautionary savings problem II

Cross-multiply the above equation to get:

$$\begin{aligned} y^2 + 2ay - e^2 + a^2 &= (y+a)(y-a) \\ y^2 + 2ay - e^2 + a^2 &= y^2 - a^2 \\ 2ay + 2a^2 - e^2 &= 0 \end{aligned}$$

The result is the following quadratic equation for a:

$$a^2 + ay - \frac{e^2}{2} = 0$$

The above quadratic equation has two roots: back

$$a = \frac{-y + \sqrt{y^2 + 2e^2}}{2} \quad \text{or} \quad a = \frac{-y - \sqrt{y^2 + 2e^2}}{2}$$

Discard the second root – in this case for $e = 0$ we get $a = -y$ and $c_2 = 0$, which is clearly not the solution of the consumer's problem

Income and wealth inequality – De Nardi (2015)

Basic infinitely-lived Bewley model

- Framework proposed by Bewley (1977)
- Labor market status z_t (e.g. $z_t = \{0, 1\}$) evolves according to the transition matrix P (with stationary distribution $\bar{P}$)
- Households want to maximize lifetime expected utility:

$$
\begin{aligned}
\max_{\{c_t\}_{t=0}^{\infty}} \quad & U = E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \right] \\
\text{subject to} \quad & c_t + a_{t+1} = z_t w + (1 + r)\, a_t \\
& a_{t+1} \geq \underline{a} \\
& z_{t+1} \sim P\left(z_t\right)
\end{aligned}
$$

- Solution: infinite sequence of consumption plans $\{c_t\}_{t=0}^{\infty}$
- Can rewrite this problem as: choosing today's consumption and tomorrow's assets only, conditional on today's assets and labor market status

Recursive formulation of household's problem

- We can rewrite the utility function into the **value function**:

$$V\left(a_t, z_t\right) = \max_{c_t,\, a_{t+1}} \left\{u\left(c_t\right) + \beta E_t\left[V\left(a_{t+1}, z_{t+1}\right)|z_t\right]\right\}$$

$$\text{subject to} \quad c_t + a_{t+1} = z_t w + \left(1+r\right) a_t$$

$$a_{t+1} \geq \underline{a}$$

- Even more compactly:

$$V\left(a_t, z_t\right) = \max_{a_{t+1} \geq \underline{a}} \left\{u\left(z_t w + \left(1+r\right) a_t - a_{t+1}\right) + \beta E_t\left[V\left(a_{t+1}, z_{t+1}\right)|z_t\right]\right\}$$

- Solution is the **policy function** A which maps from $\left(a_t, z_t\right)$ to a_{t+1}:

$$a_{t+1} = A\left(a_t, z_t\right)$$

- We can also use the budget constraint to obtain the policy function C which maps from $\left(a_t, z_t\right)$ to c_t:

$$c_t = C\left(a_t, z_t\right) = z_t w + \left(1+r\right) a_t - A\left(a_t, z_t\right)$$

Simplified analytical example: setup

- Based on section "No-trade equilibria" in Ragot (2018)
- Agents either employed ($z_t = 1$) or unemployed ($z_t = 0$)
- Probabilities of flows:
  - employed to unemployed: s
  - unemployed to employed: p
- Employed receive wage w
- Unemployed generate "home production" b
- Capital-less economy: only assets are borrowing contracts
- Since $\underline{a} = 0$, unemployed can't borrow and employed can't save
- Unemployed are borrowing constrained, but employed are not
  $\hookrightarrow$ Euler eq. of employed will determine the real interest rate

Simplified analytical example: problem of employed

$$V^E(a_t) = \max_{c_t,\, a_{t+1}} \left\{ u(c_t) + \beta \left[(1-s) V^E(a_{t+1}) + s V^U(a_{t+1}) \right] \right\}$$

$$\text{subject to} \quad c_t + a_{t+1} = w + (1+r) a_t$$

Incorporate constraint directly into the objective:

$$V^E(a_t) = \max_{a_{t+1}} \left\{ \begin{array}{c} u\left(w + (1+r) a_t - a_{t+1}\right) \\ +\beta \left[(1-s) V^E(a_{t+1}) + s V^U(a_{t+1}) \right] \end{array} \right\}$$

First order condition (with respect to a_{t+1}):

$$0 = -u'\left(c_t^E\right) + \beta \left[(1-s) \frac{\partial V^E(a_{t+1})}{\partial a_{t+1}} + s \frac{\partial V^U(a_{t+1})}{\partial a_{t+1}} \right]$$

Envelope condition (with respect to a_t):

$$\frac{\partial V^E(a_t)}{\partial a_t} = (1+r) u'\left(c_t^E\right)$$

Simplified analytical example: problem of unemployed

$$V^U(a_t) = \max_{c_t,\, a_{t+1}} \left\{ u(c_t) + \beta \left[p V^E(a_{t+1}) + (1-p) V^U(a_{t+1}) \right] \right\}$$

$$\text{subject to} \quad c_t + a_{t+1} = b + (1+r) a_t$$

$$a_{t+1} \geq 0$$

Incorporate constraints directly into the objective:

$$V^U(a_t) = \max_{a_{t+1}} \left\{ \begin{array}{c} u(b + (1+r) a_t - a_{t+1}) \\ +\beta \left[p V^E(a_{t+1}) + (1-p) V^U(a_{t+1}) \right] \end{array} \right\} + \mu a_{t+1}$$

First order condition (with respect to a_{t+1}):

$$0 = -u'\left(c_t^U\right) + \beta \left[p \frac{\partial V^E(a_{t+1})}{\partial a_{t+1}} + (1-p) \frac{\partial V^U(a_{t+1})}{\partial a_{t+1}} \right] + \mu$$

Envelope condition (with respect to a_t):

$$\frac{\partial V^U(a_t)}{\partial a_t} = (1+r) u'\left(c_t^U\right)$$

Simplified analytical example: joint problem

Optimality conditions:

$$
\begin{aligned}
u'\left(c_t^E\right) &= \beta\left[(1-s)\frac{\partial V^E\left(a_{t+1}\right)}{\partial a_{t+1}} + s\frac{\partial V^U\left(a_{t+1}\right)}{\partial a_{t+1}}\right] \\
u'\left(c_t^U\right) &= \beta\left[p\frac{\partial V^E\left(a_{t+1}\right)}{\partial a_{t+1}} + (1-p)\frac{\partial V^U\left(a_{t+1}\right)}{\partial a_{t+1}}\right] + \mu \\
\frac{\partial V^E\left(a_{t+1}\right)}{\partial a_{t+1}} &= (1+r)\, u'\left(c_{t+1}^E\right) \\
\frac{\partial V^U\left(a_{t+1}\right)}{\partial a_{t+1}} &= (1+r)\, u'\left(c_{t+1}^U\right)
\end{aligned}
$$

Resulting in (recall that $\mu > 0$):

$$
\begin{aligned}
u'\left(c_t^E\right) &= \beta(1+r)\left[(1-s)\, u'\left(c_{t+1}^E\right) + s u'\left(c_{t+1}^U\right)\right] \\
u'\left(c_t^U\right) &> \beta(1+r)\left[p u'\left(c_{t+1}^E\right) + (1-p)\, u'\left(c_{t+1}^U\right)\right]
\end{aligned}
$$

Simplified analytical example: real interest rate

Real interest rate is pinned down by the Euler eq. of employed:

$$u'\left(c_t^E\right) = \beta\left(1+r\right)\left[\left(1-s\right)u'\left(c_{t+1}^E\right) + su'\left(c_{t+1}^U\right)\right]$$

There is no borrowing or saving, so that:

$$c^E = w \quad \text{and} \quad c^U = b$$

Assume CRRA utility function:

$$u\left(c\right) = \frac{c^{1-\sigma}}{1-\sigma} \quad \rightarrow \quad u'\left(c\right) = c^{-\sigma}$$

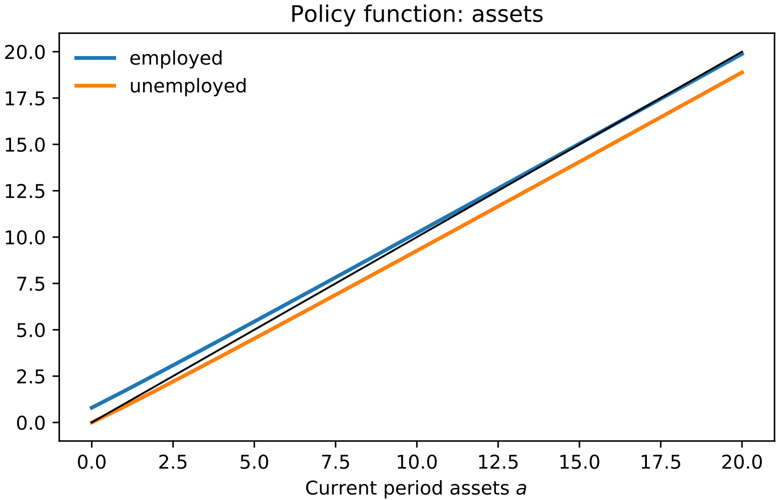
After some algebra:

$$1+r = \frac{1}{\beta}\left[1-s+s\left(\frac{w}{b}\right)^{\sigma}\right]^{-1} < \frac{1}{\beta}$$

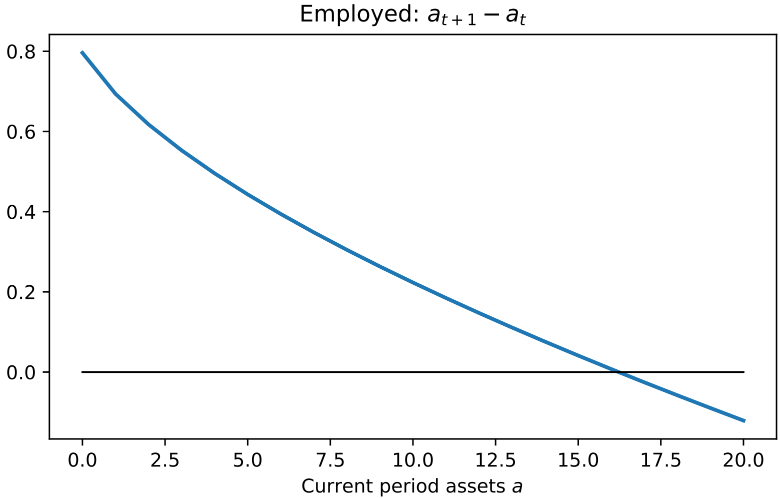
Numerical example

- Solutions are obtained numerically
  using a variety of computational methods
- Example model:
  - households have low ("unemployed")
    or high ("employed") labor productivity
  - low productivity is 10% of high productivity
  - $z_t = [0.1, 1]$, $P = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$, $\bar{P} = [0.5,\ 0.5]$
  - borrowing constraint $\underline{a} = 0$
  - $u(c) = \ln c$, $\beta = 0.96$
  - $r = 2\%$

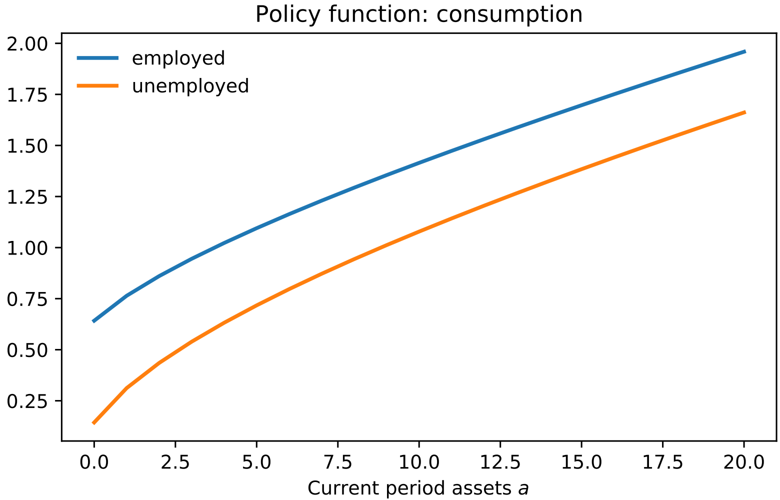
Policy functions (Partial Equilibrium)



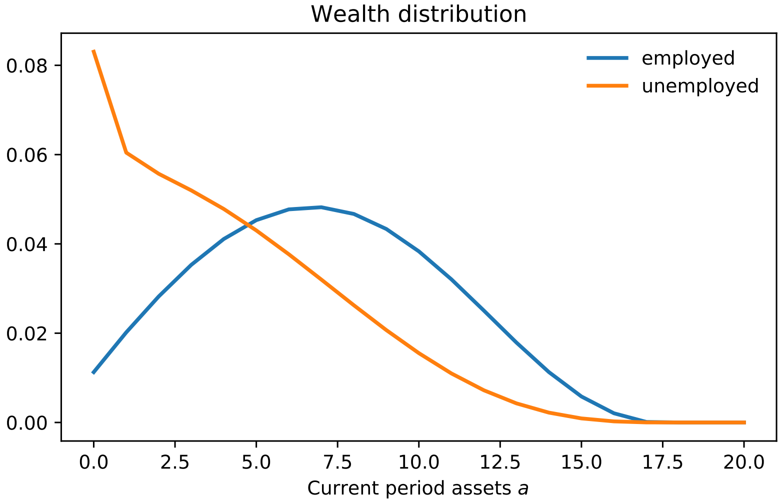
Policy functions (Partial Equilibrium)



Policy functions (Partial Equilibrium)



Wealth distribution (Partial Equilibrium)



General Equilibrium

- Households (and firms) take prices w and r as given
- Assume standard production function:

$$Y = K^{\alpha} L^{1-\alpha}$$

- Prices depend on the supply of factors of production:

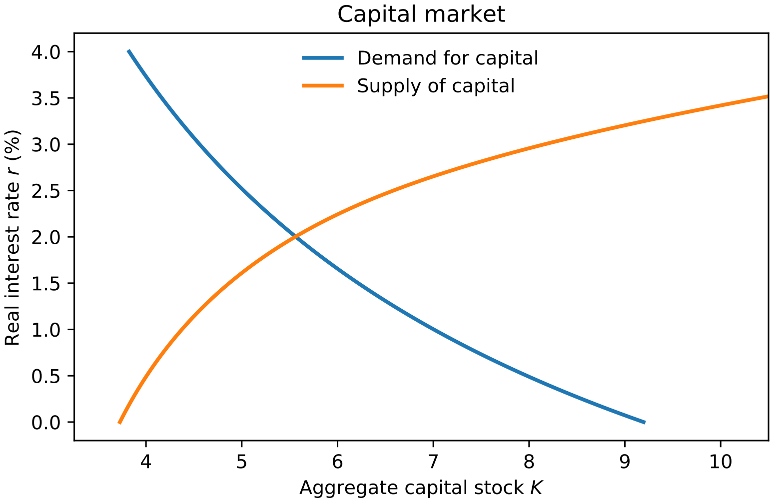
$$L = N \cdot \bar{P} \times z^{T}$$
$$K = \int_{\underline{a}}^{\infty} a \, dg(a)$$

- Expressions for prices:

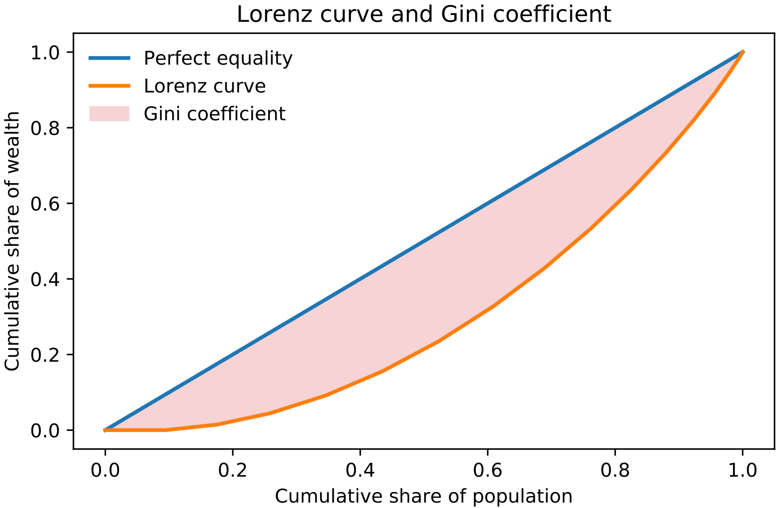
$$w = \alpha K^{\alpha} L^{-\alpha}$$
$$r = (1-\alpha) K^{\alpha-1} L^{1-\alpha} - \delta$$

- Market clearing: equalize capital supply (households) with capital demand (firms) $\rightarrow$ equilibrium prices

Capital market equilibrium



Lorenz curve and Gini coefficient for wealth



Some issues

- Since under General Equilibrium $\beta(1+r) < 1$, households do not want to save without bound (good for computational reasons)
- Households are willing to hold positive assets because:
  - there is a borrowing constraint
  - they don't want to have low assets when "unemployed" $\rightarrow$ very low consumption
  - no reason to increase assets if the above possibility is small and in distant future
- Hard to generate households with very high wealth
- Inequality does not matter much for aggregate outcomes
  - Policy functions close to linear
  - Households with low assets have low consumption $\rightarrow$ impact on aggregate consumption small
  - Not that many of borrowing-constrained households

Aiyagari (1994)

- Aiyagari (1994) approximates the earnings process of US workers by an AR(1) process in logs:

$$\ln z_t = \rho \ln z_{t-1} + \varepsilon_t$$

- Autocorrelation $\rho = 0.6$ and standard deviation $\sigma_\varepsilon = 0.2$
- Labor productivity can take 7 values

	% wealth in top		
Gini	1%	5%	20%
U.S. data, 1989 SCF			
.78	29	53	80
Aiyagari Baseline			
.38	3.2	12.2	41.0
Aiyagari higher variability			
.41	4.0	15.6	44.6

Huggett (1996)

- Huggett (1996): overlapping generations variant of the Bewley model
- Households can live for up to T periods and face age-dependent survival probability s_t
- Value function is age-dependent:

$$V_t\left(a_t, z_t\right) = \max_{c_t,\, a_{t+1}} \left\{u\left(c_t\right) + \beta s_{t+1} E_t\left[V_{t+1}\left(a_{t+1}, z_{t+1}\right)|z_t\right]\right\}$$

$$\text{subject to} \quad c_t + a_{t+1} = e_t\left(z_t\right) w + (1+r)\, a_t + b_t$$

$$a_{t+1} \geq \underline{a}$$

where b_t are bequest from the deceased (redistributed equally) plus Social Security payments to retirees

- Partial Equilibrium very easy to solve since V_T is known

De Nardi (2004)

- De Nardi (2004): Huggett model with intergenerational links
  - voluntary bequests from parents to children (utility from giving)
  - transmission of labor productivity from parents to children

Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	40%	60%	Percentage with negative or zero wealth
				U.S. data, 1989 SCF			
.60	.78	29	53	80	93	98	5.8–15.0
				Equal bequests to all (Huggett)			
.67	.67	7	27	69	90	98	17
				Unequal bequests to children (unintentional)			
.38	.68	7	27	69	91	99	17
				Parent's bequest motive			
.55	.74	14	37	76	95	100	19
				Parent's bequest motive and productivity inheritance			
.60	.76	18	42	79	95	100	19

Lifetime wealth profiles

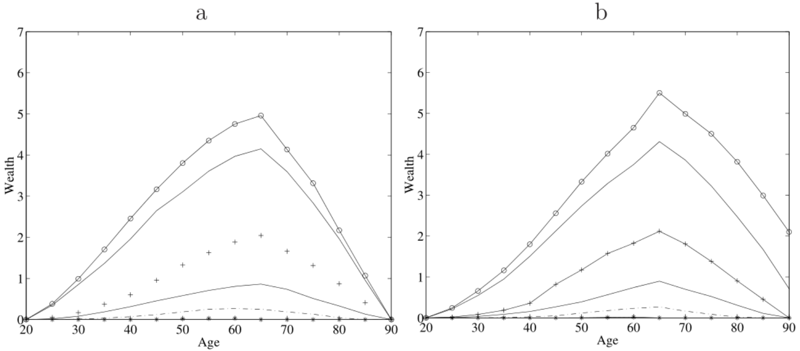


Figure 2: Wealth .1, .3, .5, .7, .9, .95 quantiles. No links, equal bequests to all, panel a, and Bequest motive, panel b .

De Nardi (2015)

Cagetti and De Nardi (2006)

- Entrepreneurs: households who declare being self-employed, own a privately held business (or a share of one), and have an active management role in it
- Small fraction of the population, but hold a large share of total net worth

Top %	1	5	10	20
Whole population				
percentage of total net worth held	30	54	67	81
Entrepreneurs				
percentage of households in a given percentile	63	49	39	28
percentage of net worth held in a given percentile	68	58	53	47

Cagetti and De Nardi (2006)

- Cagetti and De Nardi (2006): Altruistic agents care about their children
- Every period, agents decide whether to run a business or work for a wage
- Entrepreneurial production function depends on entrepreneurial ability and working capital
- Borrowing for working capital is constrained by agents' assets
- Rationale for holding high levels of wealth

Wealth Gini	Fraction of entrepreneurs	Percentage wealth in the top 1%	5%	20%	40%
		U.S. data			
0.8	7.55%	30	54	81	94
		Baseline model with entrepreneurs			
0.8	7.50%	31	60	83	94

Takeaway

- Intergenerational linkages and entrepreneurship can account for the observed wealth inequality
- Caution: changes in these assumptions can yield vastly different welfare effects of policies!
- To get macroeconomic effects of inequality usually at least two assets are needed: Ahn et al. (2017)
  - think liquid assets and housing
  - “wealthy hand-to-mouth” agents: low liquid asset holdings and mortgaged house
  - consumption choices of these agents matter for aggregate consumption $\rightarrow$ they consume a lot and are quite numerous (up to 50%)