UNIVERSITY OF WARSAW
Faculty of Economic Sciences

Introduction to modern macroeconomics

Advanced Macroeconomics IE: Lecture 1

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Course organization

Website & contact information

- Website: http://coin.wne.uw.edu.pl/mbielecki/
  $\hookrightarrow$ Advanced Macroeconomics IE
- Lecture slides and/or notes available prior to the lecture
- E-mail: mbielecki@wne.uw.edu.pl
- Office hours: Wednesday after lecture, by appointment

Assessment

You will be graded on the basis of two exams

- Midterm exam (50) in April
- Final exam (50) in June
- Additional points (15) can be earned
  by completing homework assignments

Points from the exams and homeworks add up

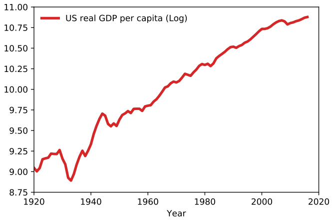
You need at least 50 to pass the course

Score	[0, 50)	[50, 60)	[60, 70)	[70, 80)	[80, 90)	[90, 100]
Grade	2	3	3.5	4	4.5	5

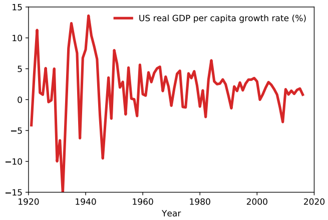
Topics of interest

We want to understand the mechanisms behind

Long-run growth



Business cycles



using the tools of modern macroeconomics

Course structure I

- Microeconomic Foundations
  - Consumption
  - Government sector
  - Neoclassical labor markets
  - Investment
- Economic Growth
- Business Cycles and Labor Markets

Course structure II

- Microeconomic Foundations
- Economic Growth
  - Neoclassical Growth Theory
    - Solow-Swan model
    - Growth empirics
    - Overlapping generations model
    - Ramsey-Cass-Koopmans model
  - New Growth Theory
- Business Cycles and Labor Markets

Course structure III

- Microeconomic Foundations
- Economic Growth
  - Neoclassical Growth Theory
  - New Growth Theory
    - AK models and externalities
    - Expanding product variety models
    - Improving product quality models
    - Diffusion of technology
- Business Cycles and Labor Markets

Course structure IV

- Microeconomic Foundations
- Economic Growth
- Business Cycles and Labor Markets
  - Real Business Cycles model
  - Models of unemployment
  - Models of income and wealth inequality
  - New Keynesian model and monetary policy
  - Coordination failures and financial frictions

Questions?

Modern macroeconomics

Modern macroeconomics[1]

- Why is it so different from your previous macro courses?
- Cornerstone of “modern” (post-1970s) macroeconomics

Macroeconomics is microeconomics

(at a high level of aggregation)

[1]This section heavily borrows from Lutz Hendricks.

An "old" macroeconomic model

- Consumption function: $C = C_0 + cY$
- Investment function: $I = I_0 - bi$
- Identity: $Y = C + I + G$
- IS curve:

$$(1 - c)\,Y = C_0 + I_0 + G - bi$$

- Real money demand: $L = L_0 + kY - hi$
- Real money supply: M/P
- LM curve:

$$M/P = L_0 + kY - hi$$

Shortcomings of the IS-LM model

- Government spending always raises output
  - Supply side constraints are missing
- Consuming more / saving less always raises output
  - The model lacks capital
- Behavior depends on parameters: c, b, k, h and C_0, I_0, L_0
  - Which parameters are stable?
  - Can policy affect these parameters?
- Expectations are not modeled

Modern macroeconomic models

- Behavior of agents is **derived** from the solutions to their optimization problems
  - Often involving time and uncertainty
- Agents have model-consistent **endogenous expectations**
- **Aggregate** outcomes result from **individual** decisions
- The economy is in **general equilibrium**
  - Which does not mean it is “at rest”
  - Nor that the outcomes are desirable

What do we gain from this approach?

- Consistency
  - Aggregate relationships satisfy all individual constraints
- Transparency
  - Assumptions about the fundamentals are clearly stated
- Non-arbitrary behavior
  - In old macro, results depend on the assumed behavior
  - In modern macro, behavior is derived
- Testability
  - Models can be tested against both macro and micro data
- Welfare analysis
  - It is possible to evaluate how a policy change affects the welfare (utility) of each agent

How to build a model in 4 simple steps

Step 1: Describe the economy

- List the agents (e.g.: households, firms, governments)
- For each agent define (examples after colon mark)
  - Demographics: population grows at rate n
  - Preferences: households maximize utility $u(c)$
  - Endowments: each household has one unit of time
  - Technologies: output is produced using $f(k)$
- Define the markets in which agents interact (examples)
  - Households work for firms (labor market)
  - Households purchase goods from firms (goods market)

How to build a model in 4 simple steps

Step 2: Solve each agent's problem

- Write down the maximization problem each agent solves
  - Households maximize utility, subject to budget constraints
  - Firms maximize profits, subject to production functions
- Derive a set of equations determining the agent's choice
  - Households' consumption and saving functions
  - Firms' demands for factors of production

How to build a model in 4 simple steps

Step 3: Market clearing – for each market

- Calculate supply and demand by each agent
- Aggregate supply = $\sum$ individual supplies
- Aggregate demand = $\sum$ individual demands
- Aggregate supply = Aggregate demand

Step 4: Define the equilibrium

- Collect all endogenous objects
    - Consumption, output, wage rate, . . .
- Collect all equations
    - First order conditions, market clearing conditions
- You should have N equations to solve for N variables
    - Quantities and prices

Before we run, we walk

- We will not consider these general equilibrium models in the first couple of lectures
- First, let us familiarize with Steps 1 and 2
- Very similar to the partial equilibrium approach in microeconomics

Example 1: One period, two goods

Step 1: Describe the economy

- Agents and their demographics
  - A household who lives for one period
- Preferences
  - The household values consumption of two goods with preferences described by the utility function $U(c_1, c_2)$
- Endowments
  - The household receives endowments of two goods (y_1, y_2)
- Technology
  - There is no production, but goods can be traded
- Markets
  - There are competitive markets for the two goods
  - The prices of the two goods are p_1 and p_2

Step 2: Solve the agent's problem

- The household maximizes $U(c_1, c_2)$ subject to the budget constraint
- Household takes as given the following state variables
  - Market prices for the two goods, p_1 and p_2
  - Endowments y_1 and y_2
- The choice variables are c_1 and c_2
- We can normalize the price of good 1 to unity: $p_1 = 1$ and call the relative price $p \equiv p_2/p_1$

Households' utility maximization problem (UMP)

- Budget constraint:
  Value of consumption $\leq$ Value of endowments

- The household solves the problem

$$\begin{aligned} \max \quad & U(c_1, c_2) \\ \text{subject to} \quad & c_1 + pc_2 \leq y_1 + py_2 \end{aligned}$$

- A solution to the problem is the pair (c_1, c_2), conditional on the relative price p and endowments (y_1, y_2)
  - Can we replace the symbol "$\leq$" with "$=$" ?
  - Ideas on how to solve this problem?

Method of Lagrange multipliers

- Many good approaches to solving such simple problems
- Fewer good approaches as problems get more complex
- One tool to rule them all – Lagrange function (Lagrangian)

Solving the household's problem

- Set up the **Lagrangian**

$$\mathcal{L} = U\left(c_1, c_2\right) + \lambda\left[y_1 + py_2 - c_1 + pc_2\right]$$

- Derive the **first order conditions** (FOCs)

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{\partial U\left(c_1, c_2\right)}{\partial c_1} - \lambda = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = \frac{\partial U\left(c_1, c_2\right)}{\partial c_2} - \lambda p = 0 \quad (2)$$

- The **Lagrange multiplier** λ has a useful interpretation
    - It is the marginal utility of relaxing the constraint a bit
    - In this example λ is the marginal utility of wealth
- The solution is then a vector (c_1, c_2, λ) that satisfies
    - FOCs (1), (2) and the budget constraint $c_1 + pc_2 = y_1 + py_2$

Simplify the conditions

- Convenient to substitute out the Lagrange multiplier λ
- The ratio of the FOCs implies

$$\frac{\partial U\left(c_1, c_2\right)/\partial c_2}{\partial U\left(c_1, c_2\right)/\partial c_1} \equiv \frac{U_2}{U_1} = p = \frac{p_2}{p_1} \tag{3}$$

- This is the familiar tangency condition
  - marginal rate of substitution equals relative price
- Now the solution is a pair (c_1, c_2) that satisfies (3) and the budget constraint $c_1 + pc_2 = y_1 + py_2$

Let's make our example more concrete

- Assume logarithmic utility

$$U\left(c_1, c_2\right) = \ln c_1 + \beta \ln c_2$$

- The FOCs are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_1} &= \frac{1}{c_1} - \lambda = 0 \quad &\rightarrow \quad \lambda &= \frac{1}{c_1} \\ \frac{\partial \mathcal{L}}{\partial c_2} &= \beta \frac{1}{c_2} - \lambda p = 0 \quad &\rightarrow \quad \lambda p &= \beta \frac{1}{c_2} \end{aligned}$$

- Ratio of FOCs

$$p = \beta \frac{c_1}{c_2} \quad \rightarrow \quad c_2 = \frac{\beta}{p} c_1$$

Let’s make our example more concrete

- Ratio of FOCs

$$c_2 = \frac{\beta}{p} c_1$$

- Plug into the budget constraint

$$\begin{aligned} c_1 + pc_2 &= y_1 + py_2 \\ c_1 + \beta c_1 &= y_1 + py_2 \end{aligned}$$

- Optimal consumption levels

$$c_1 = \frac{1}{1+\beta}\left[y_1 + py_2\right] \quad \text{and} \quad c_2 = \frac{\beta}{1+\beta}\left[\frac{y_1}{p} + y_2\right]$$

Example 2: Two periods, one good

Step 1: Describe the economy

- Agents and their demographics
  - A household who lives for two periods
- Preferences
  - The household values consumption in two **time periods** with preferences described by the utility function $U(c_1, c_2)$
- Endowments
  - The household receives **income** in two time periods (y_1, y_2)
- Technology
  - There is no production, but the agent can **save or borrow**
- Markets
  - Competitive **financial market**: one unit of first period good saved delivers $(1 + r)$ units of second period good
  - First period good costs 1, second period good costs $\frac{1}{1+r}$

Step 2: Solve the agent's problem

- The household maximizes

$$U(c_1, c_2) = \ln c_1 + \beta \ln c_2$$

subject to the budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

- Household takes the following state variables as given
  - Interest rate r
  - Incomes y_1 and y_2
- The choice variables are c_1 and c_2

Solving the household's problem

- Set up the Lagrangian

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left[y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right]$$

- Derive the first order conditions (FOCs)

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial c_2} = \beta \frac{1}{c_2} - \lambda \frac{1}{1+r} = 0$$

- Simplify

$$\lambda = \frac{1}{c_1} \quad \text{and} \quad \lambda = \beta \left(1+r\right) \frac{1}{c_2}$$

- Obtain the **intertemporal** condition / **Euler equation**

$$\frac{1}{c_1} = \beta \left(1+r\right) \frac{1}{c_2} \quad \rightarrow \quad c_2 = \beta \left(1+r\right) c_1$$

Solving the household's problem

- Obtain the **intertemporal** condition / **Euler equation**

$$c_2 = \beta (1 + r) c_1$$

- Combine it with the budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$c_1 + \frac{\beta (1+r) c_1}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$c_1 + \beta c_1 = y_1 + \frac{y_2}{1+r}$$

- Optimal consumption levels

$$c_1 = \frac{1}{1+\beta} \left[y_1 + \frac{y_2}{1+r} \right]$$

$$c_2 = \frac{\beta}{1+\beta} \left[(1+r) y_1 + y_2 \right]$$