Marcin Bielecki, Advanced Macroeconomics IE, Spring 2019
Homework 7 - Business Cycle Models

Problem 1

Consider the following model with no labor disutility ($h_t = 1$). Households maximize expected utility subject to the standard budget constraint:

$$\begin{aligned} \max \quad & U_t = E_t \left[\sum_{i=0}^{\infty} \beta^i \ln c_{t+i} \right] \\ \text{subject to} \quad & c_t + k_{t+1} = w_t h_t + (1 + r_t) k_t + div_t \end{aligned}$$

Firms maximize profits subject to the Cobb-Douglas production function:

$$\begin{aligned} \max \quad & div_t = y_t - w_t h_t - (r_t + \delta) k_t \\ \text{subject to} \quad & y_t = z_t k_t^{\alpha} h_t^{1-\alpha} \end{aligned}$$

where $\delta = 1$ (capital deprecates fully). The technology constant z evolves according to the process:

$$z_t = (1 - \rho_z) + \rho_z z_{t-1} + \varepsilon_{z,t}$$

(a) Derive the first order conditions of the households.

(b) Derive the first order conditions of the firm.

(c) Find the steady state of the system.

(d) Assuming that household behavior can be expressed as $c_t = (1 - s) y_t$ where s is a constant, find the value of s as a function of model parameters.

(e) Find the expression for k_{t+1} as a function of variables at time t.

Problem 2

Suppose that you have the following simplified log-linearized (around 0 inflation steady state) New Keynesian model. The two main non-policy equations of the model can be written:

$$\begin{aligned} x_t &= E_t x_{t+1} - (i_t - E_t \pi_{t+1}) \\ \pi_t &= \kappa x_t + \beta E_t \pi_{t+1} \end{aligned}$$

where x is output gap, i is the nominal interest rate, π is inflation rate, β is the households' discount rate and $\kappa > 0$ is a constant that depends on model parameters. The central bank obeys a strict inflation targeting rule. In particular, let π_t^* be an exogenous inflation target. The central bank will adjust i_t so that $\pi_t = \pi_t^*$ is consistent with these equations holding. Assume that the inflation target follows an exogenous AR(1) process:

$$\pi_t^* = \rho_\pi \pi_{t-1}^* + \varepsilon_{\pi,t}, \quad \rho_\pi \in (0, 1)$$

(a) Derive an analytic expression for i_t as a function of π_t^*.

(b) Suppose that ρ is 0. In which direction must the central bank adjust i_t in order to achieve an increase in π_t^*?

(c) If ρ is sufficiently close to 1, is it possible that i_t must increase in order to implement an increase in π_t^*? How does this answer depend on the value of κ?

(d) Provide intuition behind the difference in results in (b) and (c).

Problem 3

In Lecture 17 Slides we considered a model where permanent changes to marginal productivity of labor reduced the unemployment rate. This would imply that with trend productivity growth unemployment would disappear over time. Let's modify our model so that it is consistent with stationary unemployment in face of trend productivity growth. Suppose that the flow cost of a vacancy κ and the imputed value of free time b are functions of the wage rate w (instead being exogenous). In particular, assume that $\kappa_t = \kappa_0 w_t$ and $b_t = b_0 w_t$.

(a) Determine the formula for job creation and wage setting along the balanced growth path (steady state).

(b) How do θ and wages along the balanced growth path react to productivity changes?

(c) Does a continuous growth of productivity lead to a decrease in the long run unemployment rate?