Marcin Bielecki, Advanced Macroeconomics IE, Spring 2019
Homework 3 – Investment and Introduction to Neoclassical Growth

Problem 1

Consider a problem of choosing the level of capital stock by two firms: A and B. For simplicity we will assume that production requires capital only. We will follow the convention established at the lecture that in time period t the level of capital K_t is predetermined, but the firm can choose its future level of capital, K_{t+1}.

Firm A does not own its capital stock, but instead rents it at price r^K, the rental cost of capital. The problem of maximizing the value of firm A is given by:

$$\max_{\{K_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[F(K_t) - r^K K_t\right]$$

Firm B owns its capital stock, and can adjust its level via investment. The problem of maximizing the value of firm B is given by:

$$\begin{aligned} \max_{\{K_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[F(K_t) - I_t\right] \\ \text{subject to} \quad & K_{t+1} = (1-\delta) K_t + I_t \end{aligned}$$

(a) Derive the first order condition of firm A.

(b) Derive the first order condition of firm B.

(c) What condition has to be satisfied for both firms to choose the same level of K_{t+1}?

(d) Imagine you are the owner of firm C, which rents capital goods to firm A. What would be the maximal level of r^K that you could charge this firm?

(e) What would happen if you demanded higher rental rate than one found in (d)?

Problem 2

Consider the following problem of a manager maximizing the value of the firm:

$$\begin{aligned} \max_{\left\{L_t, I_t^n, K_{t+1}\right\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[(1-\tau)\left(A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - \delta K_t - I_t^n\right) - \frac{\chi}{2}\frac{(I_t^n)^2}{K_t}\right] \\ \text{subject to} \quad & K_{t+1} = I_t^n + K_t \qquad \forall t = 0, 1, \ldots, \infty \end{aligned}$$

where τ is a tax levied on firm's profits, A is the level of technology, K is firm's capital stock, L are firm's employees, $\alpha \in (0,1)$ is output elasticity w.r.t. capital, I^n is net investment and $\delta \in (0,1)$ stands for capital depreciation. Parameter χ describes the magnitude of capital installation costs. Note that the tax code does not treat installation costs as tax deductible.

(a) Write down the problem in the Lagrangian form and derive the first order conditions.

(b) Find the steady state level of q (the Lagrange multiplier). Is it equal to 1? *Hint: by definition in the steady state firm's capital stock is constant and net investment is 0.*

(c) Find the desired level of firm's capital stock per employee, K/L, treating interest rate r as given.

(d) Suppose the tax on firm's profits is reduced. What happens with the firm's investment if its level of capital stock per employee was at the level from (c) prior to the tax change?

(e) What happens with the firm's investment if its level of capital stock per employee was lower than the level from (c) prior to the tax change?

Problem 3

The social planner solves the following discrete time utility maximization problem:

$$\begin{aligned}
\max_{\{c_t,\, k_{t+1}\}_{t=0}^{\infty}} \quad & U = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \\
\text{subject to} \quad & c_t + k_{t+1} = A_t k_t^{\alpha} + (1-\delta)\, k_t \\
& k_0 > 0
\end{aligned}$$

where ρ is the household's discount rate.

(a) Find the first order conditions for the choice of c_t and k_{t+1}. Obtain the Euler equation.

(b) Compute the steady state values of k and c.

(c) How do they change in response to changes in A, δ and ρ?

(d) Use the following assumptions $\sigma = 1$ and $\delta = 1$. Assuming that household behavior can be expressed as $c_t = (1-s)\, y_t$, where s is a constant, find the value of s. *Hint: use the Euler equation first.*

(e) Find the expression for k_{t+1} as a function of k_t and model parameters.

Problem 4

The social planner solves the following continuous time utility maximization problem:

$$\begin{aligned}
\max_{\{c_t,\, k_t\}_{t=0}^{\infty}} \quad & U = \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma}\, \mathrm{d}t \\
\text{subject to} \quad & \dot{k}_t = A_t k_t^{\alpha} - \delta k_t - c_t \\
& k_0 > 0
\end{aligned}$$

where ρ is the household's discount rate.

(a) Set up the Hamiltonian function.

(b) Find the first order conditions characterizing the optimal choice.

(c) Obtain the Euler equation.

(d) Compute the steady state values of k and c.

(e) How do they change in response to changes in A, δ and ρ?