Marcin Bielecki, Advanced Macroeconomics IE, Spring 2019
Homework 2 – Ricardian Equivalence and Labor Markets

Problem 1

Suppose an economy with households who live for two time periods. At the same time, there are people who are young and old (this is the overlapping generations setup that we'll encounter later in the course). There is no production; each agent receives an endowment in each period. However, agents are able to buy government bonds when they are young and redeem them when they are old. Let y denote the value of a variable when an agent in young and o denote the value of a variable when an agent is old. There are equally many young and old agents. The interest rate is constant over time. Young agents born at time t maximize:

$$U_t = \ln c_t^y + \beta \ln c_{t+1}^o$$

subject to the following constraints:

$$c_t^y + b_{t+1} = y^y - \tau_t^y$$
$$c_{t+1}^o = y^o - \tau_{t+1}^o + (1+r)\, b_{t+1}$$

where $\beta = 1/(1+r)$ and $y^y - \tau_t^y > y^o - \tau_{t+1}^o$. Old agents in period t (who were young in period $t-1$) consume all resources they have at hand:

$$c_t^o = y^o - \tau_t^o + (1+r)\, b_t$$

Suppose the government reduces the taxes of period t old by issuing bonds to the period t young and retires the bonds in $t+1$ by raising the taxes on the period $t+1$ old. In short,

$$\Delta\tau_t^o = -\Delta b_{t+1} = -\Delta T$$
$$\Delta\tau_{t+1}^o = -(1+r)\,\Delta\tau_t^o = (1+r)\,\Delta T$$

where ΔT is the amount by which the taxes on period t old are reduced.

(a) Find the optimal levels of consumption of agents who are young in period t.

(b) What would be the value of aggregate period t consumption $C_t = c_t^y + c_t^o$, if the tax change was not implemented?

(c) What happens to aggregate period t consumption if taxes are changed at the start of period t, i.e., what is $\Delta C_t/\Delta T$?

(d) Given (b) and (c), does the Ricardian equivalence hold? Why or why not?

Problem 2

Consider the following single-period model with labor supply choice and consumption (τ_c) and wage income taxation (τ_w):

$$\max_{c,\,h} \quad U = \ln c + \phi \ln(1-h)$$
$$\text{subject to} \quad (1+\tau_c)\, c = (1-\tau_w)\, wh + R$$

where h is hours worked, w is wage, $\phi \geq 0$ is a parameter describing the preference for leisure and total available hours are normalized to one. R denotes non-labor income.

(a) Find optimal c and h as functions of preference parameter ϕ, wage w and tax rates τ_c and τ_w.

(b) In the $(h,\, w)$ space draw the labor supply curve. Show how it reacts to changes in taxes.

(c) Find the reservation wage $\bar{w}$, that is such wage level for which $h = 0$.

(d) What is the effect of an increase in consumption tax τ_c on optimal c, h and reservation wage $\bar{w}$? Provide intuition for this result.

(e) What is the effect of an increase in wage income tax τ_w on optimal c, h and reservation wage $\bar{w}$? Provide intuition for this result.

Problem 3

Consider a simple general equilibrium model. We assume that the household cannot change its capital stock k and that the depreciation rate $\delta = 0$, so that the capital rental cost is equal to the interest rate. The household solves the following utility maximization problem:

$$\begin{aligned} \max_{c,\,h} \quad & U = \ln c + \phi \ln (1 - h) \\ \text{subject to} \quad & c = wh + rk + d \end{aligned}$$

while the firm solves the following profit maximization problem (in per worker terms):

$$\begin{aligned} \max_{y,\,h,\,k} \quad & d = y - wh - (r + \delta)\, k \\ \text{subject to} \quad & y = Ak^{\alpha} h^{1-\alpha} \end{aligned}$$

where real wage w and real interest rate r are taken as given for both the firm and the household and will adjust to balance supply and demand in the labor and capital market, respectively.

(a) Using the Lagrangian method derive the first order conditions of the household.

(b) Using the Lagrangian method derive the first order conditions of the firm.

(c) In the $(h,\, w)$ space draw the labor supply curve derived from household's problem and the labor demand curve derived from firm's problem.

(d) Assume that goods market clears and that the entire output of the firm is consumed by the household, $y = c$. Find the level of hours worked by the household. Find the equilibrium level of wage w and output y.

(e) How do the hours worked, wage and output level change if A improves? How do they change if ϕ goes up (households value leisure more)?