Marcin Bielecki, Applied Macroeconomics: RCK model and taxes

1 Ramsey-Cass-Koopmans model

The "workhorse model" of modern macroeconomics.

Authors: **Frank Ramsey (1928)**, **David Cass (1965)**, **Tjalling Koopmans (1965)**

Assumptions similar to the Solow-Swan model:

- Closed economy
- No government (for now)
- Single, homogenous final good with its price normalized to 1 in each period (all variables are expressed in real terms)
- All households supply a unit of labor, number of people equal to number of workers
- Output is produced according to a neoclassical production function
- Two types of representative, optimizing agents:
  - Firms
  - Households
- Households are solving a utility maximizing problem – we have a well defined welfare measure

1.1 Firms

Representative firms produce according to the Cobb-Douglas production function:

$$Y_t = K_t^\alpha \left(A_t L_t\right)^{1-\alpha}$$

where Y is real GDP, K is capital stock, A denotes the technology/productivity level and L is employment. For now Firms want to maximize their profits:

$$\max \quad \Pi_t = Y_t - w_t L_t - r_t^k K_t$$

where r^k denotes the rental cost of capital. The rental cost of capital is related to the real interest rate in the financial market in the following way:

$$r_t = r_t^k - \delta$$

The households are indifferent between allocating assets in the financial market, yielding return r, and owning physical capital, yielding net return of $r^k - \delta$.

Let us restate the problem of the firm and solve it:

$$\max \quad \Pi_t = K_t^\alpha \left(A_t L_t\right)^{1-\alpha} - w_t L_t - r_t^k K_t$$

First order conditions:

$$\frac{\partial \Pi_t}{\partial L_t} = \left(1-\alpha\right) K_t^\alpha A_t^{1-\alpha} \left(1-\alpha\right) L_t^{-\alpha} - w_t = 0$$
$$\frac{\partial \Pi_t}{\partial K_t} = \alpha K_t^{\alpha-1} \left(A_t L_t\right)^{1-\alpha} - r_t^k = 0$$

Simplify and rewrite:

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{A_t L_t}\right)^{\alpha}$$
$$r_t^k = \alpha \left(\frac{K_t}{A_t L_t}\right)^{\alpha-1}$$

The real interest rate is given by:

$$r_t = r_t^k - \delta = \alpha \left(\frac{K_t}{A_t L_t}\right)^{\alpha-1} - \delta$$

When we abstract from technological progress, we assume $A = 1$ and then the prices are equal to:

$$w_t = (1-\alpha) k_t^{\alpha}$$
$$r_t = \alpha k_t^{\alpha-1} - \delta$$

where $k_t \equiv \frac{K_t}{L_t}$ denotes capital per worker.

1.2 Households

Representative, infinitely lived households (dynasties) solve the following utility maximization problem:

$$\max \quad U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$
$$\text{subject to} \quad Assets_{t+1} = w_t L_t + (1 + r_t) Assets_t - C_t, \quad t = 0, 1, \ldots, \infty$$

where β is the discount (impatience) factor, c stands for consumption per capita, $\sigma \geq 0$ is a parameter which we will discuss in a moment, $Assets$ denote total assets of the household sector, r is the real interest rate, w denotes the wage earned by workers and $C = c \cdot N$ is the total consumption of households. We also assume that all agents are working, so that $L_t = N_t$.

The instantaneous utility function, $\left(c^{1-\sigma} - 1\right) / (1-\sigma)$ is a Constant Relative Risk Aversion (CRRA) function and can be thought of as a generalization of the familiar logarithmic function. One can show using L'Hôpital's rule (H) that as $\sigma \to 1$, the CRRA function becomes the logarithmic function:

$$\lim_{\sigma \to 1} \frac{c^{1-\sigma} - 1}{1-\sigma} = \lim_{\sigma \to 1} \frac{\exp\left[(1-\sigma)\ln c\right] - 1}{1-\sigma} = (H) = \lim_{\sigma \to 1} \frac{\exp\left[(1-\sigma)\ln c\right] \cdot (-\ln c)}{-1} = \ln c$$

The parameter σ measures risk aversion of the agent. In our context this will mean that an agent with high σ will prefer smooth and stable paths of consumption relative to ones that imply large changes in consumption over time.

We will assume that the population grows at a constant rate n, that is:

$$N_{t+1} = (1+n) N_t \quad \text{and} \quad N_t = (1+n)^t N_0$$

Now we will rewrite the original problem in per capita terms. The budget constraint is easy to reformulate, just be careful about the distinction between periods t and $t+1$:

$$Assets_{t+1} = w_t L_t + (1+r_t) Assets_t - C_t \quad | \quad : N_t$$
$$\frac{Assets_{t+1}}{N_t} = \frac{w_t L_t}{N_t} + \frac{(1+r_t) Assets_t}{N_t} - \frac{C_t}{N_t}$$
$$\frac{Assets_{t+1}}{N_{t+1}} \frac{N_{t+1}}{N_t} = w_t + (1+r_t) a_t - c_t$$
$$(1+n) a_{t+1} = w_t + (1+r_t) a_t - c_t$$

where we define per capita assets $a_t \equiv Assets_t/N_t$. Notice how $Assets_{t+1}/N_t \neq a_{t+1}$ and we have to take into account population growth. We are now ready to solve the problem using the Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \sum_{t=0}^{\infty} \lambda_t \left[w_t + (1+r_t)\, a_t - c_t - (1+n)\, a_{t+1} \right]$$

In each time period t the choice variables for this problem are consumption per capita in the current period c_t and assets per capita in the next period a_{t+1}. It may be easier to derive the first order conditions if we expand the Lagrangian first:

$$\begin{aligned} \mathcal{L} = \ldots + \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \ldots + \lambda_t \left[w_t + (1+r_t)\, a_t - c_t - (1+n)\, a_{t+1} \right] \\ + \lambda_{t+1} \left[w_{t+1} + (1+r_{t+1})\, a_{t+1} - c_{t+1} - (1+n)\, a_{t+2} \right] + \ldots \end{aligned}$$

First order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} &= \beta^t c_t^{-\sigma} + \lambda_t \left[-1\right] = 0 \\ \frac{\partial \mathcal{L}}{\partial a_{t+1}} &= \lambda_t \left[-\left(1+n\right) \right] + \lambda_{t+1} \left[\left(1+r_{t+1}\right) \right] = 0 \end{aligned}$$

Simplify and rewrite:

$$\begin{aligned} \lambda_t &= \beta^t c_t^{-\sigma} \\ (1+n)\, \lambda_t &= \lambda_{t+1} \left(1+r_{t+1}\right) \end{aligned}$$

Resulting Euler equation:

$$\begin{aligned} (1+n)\, \beta^t c_t^{-\sigma} &= \beta^{t+1} c_{t+1}^{-\sigma} \left(1+r_{t+1}\right) \\ \left(\frac{c_{t+1}}{c_t} \right)^{\sigma} &= \frac{\beta \left(1+r_{t+1}\right)}{1+n} \end{aligned}$$

Notice that if $\sigma = 1$ and $n = 0$, the equation can be written in the form we have previously seen for the logarithmic utility function:

$$c_{t+1} = \beta \left(1+r_{t+1}\right) c_t$$

To gain more intuition, we introduce the discount rate ρ which is related to the discount factor β in the following way:

$$\beta = \frac{1}{1+\rho}$$

The discount rate can be interpreted as a "psychological" interest rate of an agent. The Euler equation now can be written as:

$$\frac{c_{t+1}}{c_t} = \left[\frac{1+r_{t+1}}{(1+\rho)(1+n)} \right]^{1/\sigma}$$

and it implies that consumption increases over time if the real interest rate exceeds the "psychological" interest rate (multiplied by the rate of population growth). In the situation where $1+r_{t+1} > (1+\rho)(1+n)$, the dynasty is willing to save and it means that it consumes less in the present to consume more in the future. The parameter σ will affect how strongly the difference between real and "psychological" interest rates influences changes in consumption. The higher the σ, the smaller are the changes of consumption in response to the difference between the interest rates.

1.3 General Equilibrium

In a closed economy, the only asset in positive net supply is capital, because all the borrowing and lending must cancel out within the economy. Hence, equilibrium in the asset market requires $a = k$. We will modify the households' budget constraint accordingly and in the next step plug in the expressions for prices:

$$\begin{aligned}
(1+n)\,a_{t+1} &= w_t + (1+r_t)\,a_t - c_t \\
(1+n)\,k_{t+1} &= w_t + (1+r_t)\,k_t - c_t \\
(1+n)\,k_{t+1} &= (1-\alpha)\,k_t^{\alpha} + \left(1+\alpha k_t^{\alpha-1} - \delta\right) k_t - c_t \\
(1+n)\,k_{t+1} &= k_t^{\alpha} + (1-\delta)\,k_t - c_t
\end{aligned}$$

If we "reverse-engineer" this equation into form with aggregate terms by multiplying both sides by L_t, we get:

$$\begin{aligned}
L_t \frac{L_{t+1}}{L_t}\frac{K_{t+1}}{L_{t+1}} &= L_t \left(\frac{K_t}{L_t}\right)^{\alpha} + (1-\delta)\,L_t \frac{K_t}{L_t} - L_t \frac{C_t}{N_t} \\
K_{t+1} &= K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)\,K_t - C_t \\
Y_t &= C_t + I_t
\end{aligned}$$

where we used the accounting definition of gross investment, $I_t = K_{t+1} - (1-\delta)\,K_t$. Unsurprisingly, the above equation is a variant of the national accounting identity, $Y = C + I + G + NX$, for a closed economy ($NX = 0$) with no government sector ($G = 0$).

If we go back to the Euler equation, we can replace the real interest rate with the marginal product of capital net of depreciation:

$$\begin{aligned}
\frac{c_{t+1}}{c_t} &= \left[\frac{1+r_{t+1}}{(1+\rho)\,(1+n)}\right]^{1/\sigma} \\
\frac{c_{t+1}}{c_t} &= \left[\frac{1+\alpha k_{t+1}^{\alpha-1} - \delta}{(1+\rho)\,(1+n)}\right]^{1/\sigma}
\end{aligned}$$

The dynamics of the entire economy are summed up by the following two equations:

$$\begin{aligned}
\text{Euler equation} \quad & \frac{c_{t+1}}{c_t} = \left[\frac{1+\alpha k_{t+1}^{\alpha-1} - \delta}{(1+\rho)\,(1+n)}\right]^{1/\sigma} \\
\text{Resource constraint} \quad & (1+n)\,k_{t+1} = k_t^{\alpha} + (1-\delta)\,k_t - c_t
\end{aligned}$$

1.4 Steady state and transition dynamics

Without technology improvements, variables per person stabilize over time. We can easily find values of c and k that put the system at rest. Start with the Euler equation:

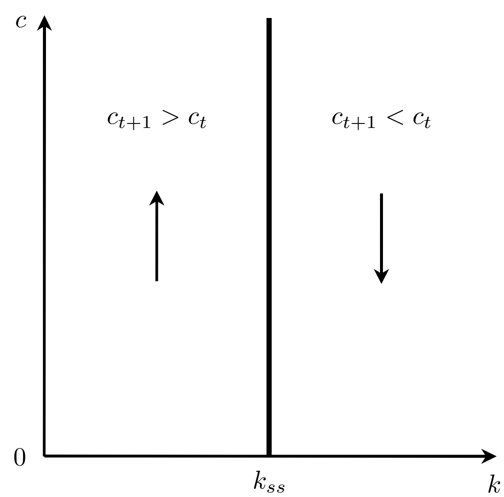
$$
\begin{aligned}
\frac{c}{c} &= \left[\frac{1+\alpha k^{\alpha-1}-\delta}{(1+\rho)(1+n)}\right]^{1/\sigma} \\
(1+\rho)(1+n) &= 1-\delta+\alpha k^{\alpha-1} \\
(1+\rho)(1+n)-(1-\delta) &= \alpha k^{\alpha-1} \\
k^{1-\alpha} &= \frac{\alpha}{(1+\rho)(1+n)-(1-\delta)}
\end{aligned}
$$

$$
\begin{aligned}
k^* &= \left[\frac{\alpha}{(1+\rho)(1+n)-(1-\delta)}\right]^{\frac{1}{1-\alpha}} \\
k^* &\approx \left[\frac{\alpha}{\rho+n+\delta}\right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

If we go back to the Euler equation in the form relating real interest rate and discount rate of households:

$$
\frac{c_{t+1}}{c_t} = \left[\frac{1+r_{t+1}}{(1+\rho)(1+n)}\right]^{1/\sigma}
$$

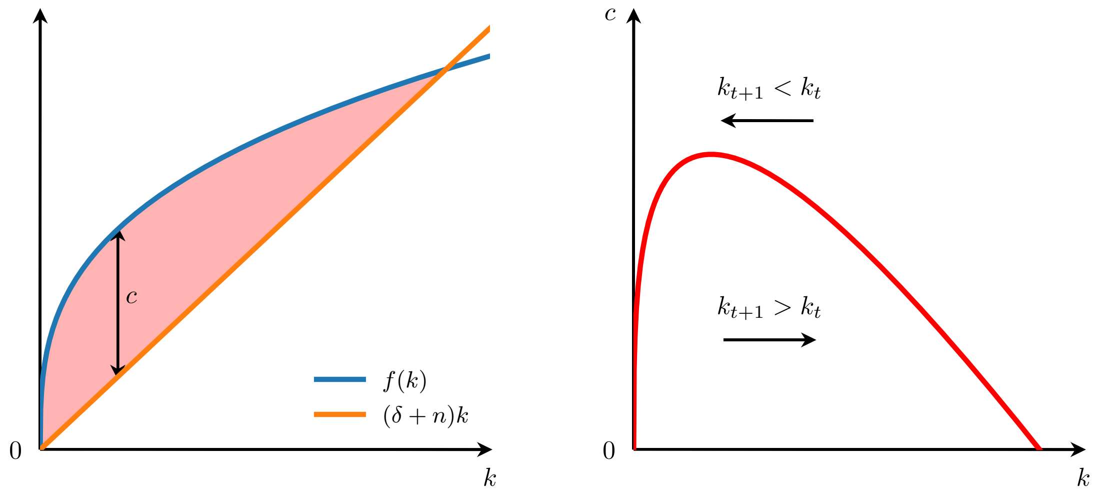
we can see that consumption will increase over time whenever $1+r_{t+1} > (1+\rho)(1+n)$. If the marginal product of capital exceeds its steady state value, consumption increases over time. A glance at the figure below reveals that consumption will increase over time for $k < k^*$ and by analogy will decrease over time for $k > k^*$:



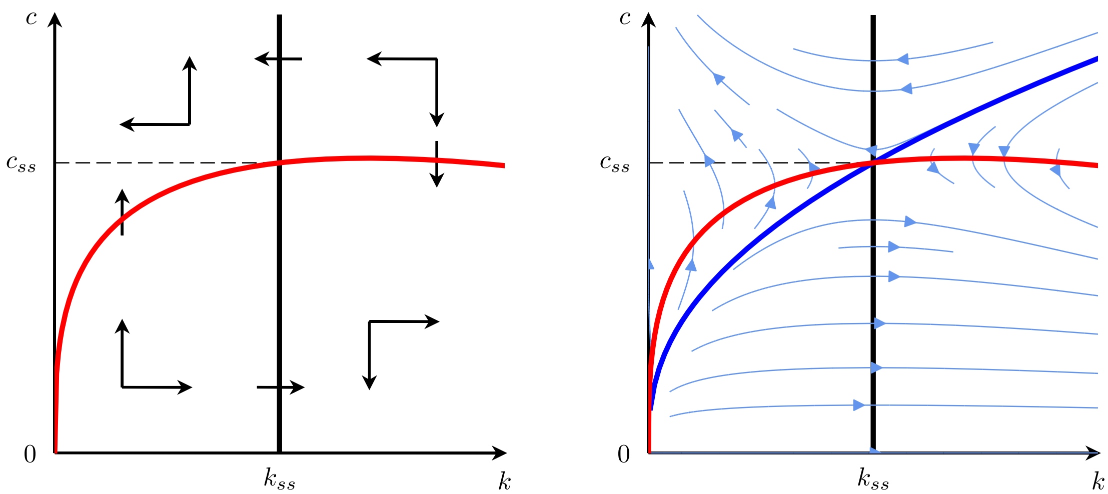
If we rearrange the resource constraint, we will find the expression for c given that the capital stock does not change over time:

$$
\begin{aligned}
(1+n)k &= k^\alpha + (1-\delta)k - c \\
c &= k^\alpha - (\delta+n)k
\end{aligned}
$$

If consumption will be chosen above the quantity that implies constant capital, capital will decrease over time as investment will be lower than depreciation. If consumption will be chosen below the constant capital schedule, capital will increase over time. The relationship is displayed in the graphs below:



We can now join the two schedules and produce the full phase diagram:



2 Optimal taxation in the long run

For simplicity, we will assume $n = g = 0$ and $N = A = 1$. We will consider two uses for the taxes – lump-sum transfers to households v and per capita government expenditures g (note the notation change!). We assume that in each period the government's budget is balanced.

2.1 Households

Utility maximization problem:

$$\begin{aligned} \max \quad & U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \\ \text{subject to} \quad & a_{t+1} = \left(1 + \left(1 - \tau_t^a\right) r_t\right) a_t + \left(1 - \tau_t^w\right) w_t - \left(1 + \tau_t^c\right) c_t - \tau_t + v_t \qquad \forall t = 0, 1, \ldots, \infty \end{aligned}$$

where τ^a is capital gains tax, τ^w is labor income tax, τ^c is consumption tax and τ is lump-sum tax.

Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \sum_{t=0}^{\infty} \lambda_t \left[\left(1 + \left(1 - \tau_t^a\right) r_t\right) a_t + \left(1 - \tau_t^l\right) w_t - \left(1 + \tau_t^c\right) c_t - \tau_t + v_t - a_{t+1} \right]$$

FOCs:

$$\begin{aligned} c_t \quad &: \quad \beta^t c_t^{-\sigma} - \lambda_t \left(1 + \tau_t^c\right) = 0 \longrightarrow \lambda_t = \frac{c_t^{-\sigma}}{1 + \tau_t^c} \\ a_{t+1} \quad &: \quad -\lambda_t + \lambda_{t+1} \left(1 + \left(1 - \tau_{t+1}^a\right) r_{t+1}\right) = 0 \end{aligned}$$

Euler equation:

$$\begin{aligned} \frac{c_t^{-\sigma}}{1 + \tau_t^c} &= \beta \frac{c_{t+1}^{-\sigma}}{1 + \tau_{t+1}^c} \left(1 + \left(1 - \tau_{t+1}^a\right) r_{t+1}\right) \\ \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} &= \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \beta \left(1 + \left(1 - \tau_{t+1}^a\right) r_{t+1}\right) \end{aligned}$$

2.2 Firms

Profit maximization problem:

$$\max \quad \Pi_t = \left(1 - \tau_t^f\right) \left[K_t^{\alpha} L_t^{1-\alpha} - \delta K_t - w_t L_t\right] - r_t K_t \qquad \forall t = 0, 1, \ldots, \infty$$

where τ^f is tax on firms' accounting profits.

FOCs:

$$\begin{aligned} \frac{\partial \Pi_t}{\partial K_t} &= \left(1 - \tau_t^f\right) \left[\alpha K_t^{\alpha-1} L_t^{1-\alpha} - \delta\right] - r_t = 0 \quad \longrightarrow \quad r_t = \left(1 - \tau_t^f\right) \left[\alpha k_t^{\alpha-1} - \delta\right] \\ \frac{\partial \Pi_t}{\partial L_t} &= \left(1 - \tau_t^f\right) \left[\left(1 - \alpha\right) K_t^{\alpha} L_t^{-\alpha} - w_t\right] = 0 \quad \longrightarrow \quad w_t = \left(1 - \alpha\right) k_t^{\alpha} \end{aligned}$$

The tax on firm's earnings lowers the return on capital and incentivises firms to hold less capital.

2.3 Government sector

The government maintains balanced budget. In per capita terms:

$$g_t + v_t = \tau_t^f \left[k_t^\alpha - \delta k_t - w_t\right] + \tau_t^a r_t a_t + \tau_t^c c_t + \tau_t^w w_t + \tau_t$$
$$v_t = \tau_t^f \left[k_t^\alpha - \delta k_t - w_t\right] + \tau_t^a r_t a_t + \tau_t^c c_t + \tau_t^w w_t + \tau_t - g_t$$

2.4 General equilibrium

Market clearing for capital:

$$k_t = a_t$$

Rewrite households' budget constraint to get capital accumulation equation:

$$k_{t+1} = \left(1 + \left(1 - \tau_t^a\right) r_t\right) k_t + \left(1 - \tau_t^w\right) w_t - \left(1 + \tau_t^c\right) c_t - \tau_t + v_t$$
$$k_{t+1} = \left(1 + r_t\right) k_t + w_t - c_t - \left(\tau_t^a r_t k_t + \tau_t^w w_t + \tau_t^c c_t + \tau_t - v_t\right)$$
$$k_{t+1} = \left(1 + r_t\right) k_t + w_t - c_t - \left(\tau_t^a r_t k_t + \tau_t^w w_t + \tau_t^c c_t + \tau_t - \left(\tau_t^f \left[k_t^\alpha - \delta k_t - w_t\right] + \tau_t^a r_t a_t + \tau_t^c c_t + \tau_t^w w_t + \tau_t - g_t\right)\right)$$
$$k_{t+1} = \left(1 + \left(1 - \tau_t^f\right)\left[\alpha k_t^{\alpha-1} - \delta\right]\right) k_t + \left(1 - \alpha\right) k_t^\alpha - c_t - \left(g_t - \tau_t^f \left[k_t^\alpha - \delta k_t - \left(1 - \alpha\right) k_t^\alpha\right]\right)$$
$$k_{t+1} = \left(1 - \delta\right) k_t + \alpha k_t^\alpha - \tau_t^f \left[\alpha k_t^\alpha - \delta k_t\right] + \left(1 - \alpha\right) k_t^\alpha - c_t - \left(g_t - \tau_t^f \left[\alpha k_t^\alpha - \delta k_t\right]\right)$$
$$k_{t+1} = \left(1 - \delta\right) k_t + k_t^\alpha - c_t - g_t$$

Rewrite the Euler equation:

$$\left(\frac{c_{t+1}}{c_t}\right)^\sigma = \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \beta \left(1 + \left(1 - \tau_{t+1}^a\right)\left(1 - \tau_t^f\right)\left[\alpha k_{t+1}^{\alpha-1} - \delta\right]\right)$$

2.5 Steady state

Assume constant tax rates:

$$1 = \beta \left(1 + \left(1 - \tau^a\right)\left(1 - \tau^f\right)\left[\alpha k^{\alpha-1} - \delta\right]\right)$$
$$\frac{1}{\beta} - 1 = \left(1 - \tau^a\right)\left(1 - \tau^f\right)\left[\alpha k^{\alpha-1} - \delta\right]$$
$$\frac{\rho}{\left(1 - \tau^a\right)\left(1 - \tau^f\right)} = \alpha k^{\alpha-1} - \delta$$
$$\alpha k^{\alpha-1} = \frac{\rho}{\left(1 - \tau^a\right)\left(1 - \tau^f\right)} + \delta$$
$$k^* = \left[\frac{\alpha}{\frac{\rho}{\left(1-\tau^a\right)\left(1-\tau^f\right)} + \delta}\right]^{\frac{1}{1-\alpha}}$$

$$c^* = \left(k^*\right)^\alpha - \delta k^* - g$$

Government expenditures lower private consumption but do not affect steady state capital per worker.

Capital gains and firm earnings taxes lower steady state capital per worker which then translates to lower steady state private consumption.

2.6 Chamley-Judd result – redistribution impossibility theorem

Again assume $n = g = 0$ and $N = A = 1$ for simplicity. Divide population into two groups: workers and capitalists. Workers do not save and consume their wages and any transfers they receive. Capitalists both save and consume. The government wants to redistribute between capitalists and workers. It levies tax on capital gains and distributes the proceeds to workers. Since the proof of the theorem is easier when using a generic neoclassical production function, I do not impose any specific functional form.

Worker households

$$\begin{aligned} \max \quad & U^w = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^w)^{1-\sigma} - 1}{1-\sigma} \\ \text{subject to} \quad & c_t^w = w_t + v_t \qquad \forall t = 0, 1, \ldots, \infty \end{aligned}$$

Solution:

$$c_t^w = w_t + v_t$$

Capitalist households

$$\begin{aligned} \max \qquad & U^c = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^c)^{1-\sigma} - 1}{1-\sigma} \\ \text{subject to} \qquad & a_{t+1} = (1 + (1-\tau^a) r_t) a_t - c_t^c \qquad \forall t = 0, 1, \ldots, \infty \end{aligned}$$

Solution:

$$\left(\frac{c_{t+1}^c}{c_t^c}\right)^{\sigma} = \beta (1 + (1-\tau^a) r_{t+1})$$

Firms

$$\begin{aligned} \max \quad & \Pi_t = F(K_t, L_t) - w_t L_t - r_t^k K_t \\ \max \quad & \Pi_t = L_t \left[f(k_t) - w_t - r_t^k k_t \right] \end{aligned}$$

Solution:

$$\begin{aligned} r_t^k &= f'(k_t) \longrightarrow r_t = f'(k_t) - \delta \\ w_t &= f(k_t) - f'(k_t) k_t \end{aligned}$$

Government sector

$$v_t = \frac{N^c}{N^w} \tau^a r_{t+1} a_{t+1}$$

General equilibrium

Capital market equilibrium:

$$k_t = \frac{N^c}{N^w} a_t \longrightarrow v_t = \tau^a r_{t+1} k_{t+1}$$

Steady state capital per worker:

$$\begin{aligned} 1 &= \beta (1 + (1-\tau^a) [f'(k) - \delta]) \\ f'(k) &= \frac{\rho}{1-\tau^a} + \delta \end{aligned}$$

Steady state capitalists' consumption:

$$\begin{aligned} a &= (1 + (1-\tau^a) r) a - c^c \\ c^c &= (1-\tau^a) ra = (1-\tau^a) r \frac{N^w}{N^c} k \\ c^c &= (1-\tau^a) \frac{N^w}{N^c} [f'(k) - \delta] k \end{aligned}$$

Steady state workers' consumption:

$$c^w = f(k) - f'(k) k + \tau^a [f'(k) - \delta] k$$

To demonstrate the result, it suffices to show that workers' consumption depends positively on the steady state stock of capital per worker:

$$\begin{aligned} \frac{\partial c^w}{\partial k} &= f'(k) - [f''(k) k + f'(k)] + \tau^a [f''(k) k + f'(k) - \delta] \\ &= -f''(k) k + \tau^a [f''(k) k + f'(k) - \delta] \\ &= \underbrace{(\tau^a - 1)}_{<0} \underbrace{f''(k)}_{<0} k + \tau^a \frac{\rho}{1-\tau^a} > 0 \end{aligned}$$

It turns out that it is impossible to increase steady state consumption of workers by taxing capitalists. Taxing capitalists reduces steady state capital stock and lowers wages. Even if all of the revenue from taxation is given to workers in transfer, the loss in wages is greater than the gain from the transfer, due to the dynamic deadweight loss from taxation.

Note that the above result holds only in the long run. In the short run, when the capital stock is (almost) fixed, from the worker's perspective it is welfare improving to tax capital, as the wages remain the same and they receive additional transfers. However, the situation where we would like to impose positive taxes now but zero taxes in the future is an example of a **time-inconsistent policy**, and the policymakers have no access to a credible commitment device (like in the case of Odysseus tying himself to the mast to be able to hear the Sirens) that would allow them to enact such policy.

Take also a look **here** for conditions under which the Chamley-Judd result might not hold. For example, **Aiyagari (1995)** shows that with incomplete insurance markets and borrowing constraints, the optimal capital gains tax rate is positive, even in the long run.