Monopolistic competition
Price stickiness
New Keynesian model

Applied Macroeconomics

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Spring 2018

Monopolistic competition: introduction

- Most (if not all) sectors of the economy are not perfectly competitive
- There is a significant markup on prices, averaging around 33%
- Monopolistic competition allows us to introduce a new shock
- Is a stepping stone for nominal frictions models

Empirical evidence on markups

Christopoulou and Vermeulen (2008) Markups in the Euro area and the US over the period 1981–2004: a comparison of 50 sectors

Table 1. Weighted average markup, 1981-2004

Country	Manufacturing & Construction		Market Services		All (Manufacturing, Construction & Market Services)	
Germany	1.16	(0.01)*	1.54	(0.03)*	1.33	(0.01)*
France	1.15	(0.01)*	1.26	(0.02)*	1.21	(0.01)*
Italy	1.23	(0.01)*	1.87	(0.02)*	1.61	(0.01)*
Spain	1.18	(0.00)*	1.37	(0.01)*	1.26	(0.01)*
Netherlands	1.13	(0.01)*	1.31	(0.02)*	1.22	(0.01)*
Belgium	1.14	(0.00)*	1.29	(0.01)*	1.22	(0.01)*
Austria	1.20	(0.02)*	1.45	(0.03)*	1.31	(0.02)*
Finland	1.22	(0.01)*	1.39	(0.02)*	1.28	(0.01)*
Euro Area	1.18	(0.01)*	1.56	(0.01)*	1.37	(0.01)*
USA	1.28	(0.02)*	1.36	(0.03)*	1.32	(0.02)*

Monopolistic competition: setup

- Two sectors of producers – final and intermediate goods
- Final goods sector is perfectly competitive
- Intermediate goods sector is monopolistically competitive and produces differentiated goods
- There is a degree of market power captured by $\mu \geq 1$
- If $\mu = 1$ then we are in perfect competition
- Higher μ indicates higher monopoly power
- Final goods production function expressed as Dixit-Stiglitz aggregator

$$
\begin{aligned}
y_t &= \left(\sum_i y_t(i)^{\frac{1}{\mu}} \right)^{\mu} \\
y_t &= \left(\int_0^1 y_t(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right)^{\mu}
\end{aligned}
$$

- For small μ goods are close substitutes
- For large μ goods are complementary

Final goods producing firm I

Profit maximization problem

$$\begin{aligned} \text{max} \quad & P_t y_t - \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i \\ \text{subject to} \quad & y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu} \end{aligned}$$

Lagrangian

$$\mathcal{L} = P_t y_t - \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i + \lambda_t \left[\left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu} - y_t \right]$$

FOCs

$$\begin{aligned} y_t \quad &: \quad P_t - \lambda_t = 0 \\ y_t(i) \quad &: \quad -P_t(i) + \lambda_t \left[\mu \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu - 1} \cdot \frac{1}{\mu} y_t(i)^{\frac{1}{\mu} - 1} \right] = 0 \end{aligned}$$

Final goods producing firm II

Result

$$P_t(i) = P_t \left(\int_0^1 y_t(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right)^{\mu-1} y_t(i)^{\frac{1-\mu}{\mu}}$$

$$P_t(i)^{\frac{\mu}{1-\mu}} = P_t^{\frac{\mu}{1-\mu}} \left(\int_0^1 y_t(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right)^{-\mu} y_t(i)$$

$$P_t(i)^{\frac{\mu}{1-\mu}} = P_t^{\frac{\mu}{1-\mu}} y_t^{-1} y_t(i)$$

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} y_t$$

Aggregate price index derivation

$$P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}} \, \mathrm{d}i \right)^{1-\mu}$$

Intermediate goods producing firm (simplified) I

For now let's consider production function linear in hours

$$y_t(i) = z_t h_t(i)$$

Cost minimization problem

$$\begin{aligned} \min \quad & tc_t(i) = w_t h_t(i) \\ \text{subject to} \quad & y_t(i) = z_t h_t(i) \end{aligned}$$

Lagrangian

$$\mathcal{L} = -w_t h_t(i) + mc_t(i)\left(z_t h_t(i) - y_t(i)\right)$$

FOC

$$h_t(i) \quad : \quad -w_t + mc_t(i)\, z_t = 0$$

Marginal cost is identical across firms

$$mc_t(i) = mc_t = \frac{w_t}{z_t}$$

Intermediate goods producing firm (simplified) II

Profit maximization problem

$$\max \quad P_t(i)\, y_t(i) - mc_t y_t(i)$$

$$\text{subject to} \quad y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t$$

Rewrite

$$\max \quad P_t(i)^{1+\frac{\mu}{1-\mu}}\, P_t^{\frac{\mu}{\mu-1}} y_t - mc_t P_t(i)^{\frac{\mu}{1-\mu}}\, P_t^{\frac{\mu}{\mu-1}} y_t$$

FOC

$$\left(\frac{1}{1-\mu}\right) P_t(i)^{\frac{\mu}{1-\mu}}\, P_t^{\frac{\mu}{\mu-1}} y_t - mc_t \left(\frac{\mu}{1-\mu}\right) P_t(i)^{\frac{\mu}{1-\mu}-1}\, P_t^{\frac{\mu}{\mu-1}} y_t = 0$$

$$P_t(i) = \mu \cdot mc_t$$

Identical marginal costs $\rightarrow$ identical prices (and we normalize $P = 1$)

$$mc_t = 1/\mu \quad \text{and} \quad w_t = z_t/\mu$$

Intermediate goods producing firm

In the case of production function with capital

$$y_t(i) = z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha}$$

we get

$$mc_t = \frac{1}{\mu}$$

and

$$w_t = \frac{(1-\alpha)}{\mu} z_t k_t(i)^{\alpha} h_t(i)^{-\alpha}$$
$$r_t = \frac{\alpha}{\mu} z_t k_t(i)^{\alpha-1} h_t(i)^{1-\alpha} - \delta$$

- The rest of the model is unchanged relative to the basic RBC model
- We will introduce shocks to vary market power parameter over time

Full set of equilibrium conditions

System of 9 equations and 9 unknowns: $\{c, h, y, r, w, k, i, z, \mu\}$

$$
\begin{aligned}
\text{Euler equation} &: \quad 1 = \beta E_t \left[\frac{c_t}{c_{t+1}} (1 + r_{t+1}) \right] && (1) \\
\text{Consumption-hours choice} &: \quad h_t = 1 - \phi \frac{c_t}{w_t} && (2) \\
\text{Production function} &: \quad y_t = z_t k_t^{\alpha} h_t^{1-\alpha} && (3) \\
\text{Real interest rate} &: \quad r_t = \frac{\alpha}{\mu_t} z_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta && (4) \\
\text{Real hourly wage} &: \quad w_t = \frac{(1-\alpha)}{\mu_t} z_t k_t^{\alpha} h_t^{-\alpha} && (5) \\
\text{Investment} &: \quad i_t = k_{t+1} - (1-\delta) k_t && (6) \\
\text{Output accounting} &: \quad y_t = c_t + i_t && (7) \\
\text{TFP AR(1) process} &: \quad \ln z_t = \rho \ln z_{t-1} + \varepsilon_t && (8) \\
\text{Markup AR(1) process} &: \quad \ln \mu_t = (1-\rho_\mu) \ln \mu + \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu,t} && (9)
\end{aligned}
$$

Steady state

- The only thing to be careful about is the monopoly wedge
- Other than that steady state is identical to the basic RBC case

$$(8) \qquad z = 1$$

$$(9) \qquad \mu = \mu$$

$$(1) \qquad r = 1/\beta - 1$$

$$(4) \qquad \frac{k}{h} = \left(\frac{\alpha/\mu}{r + \delta} \right)^{\frac{1}{1-\alpha}}$$

$$(3) \qquad \frac{y}{h} = \left(\frac{k}{h} \right)^{\alpha}$$

$$(5) \qquad w = \frac{(1-\alpha)}{\mu} \frac{y}{h}$$

$$(6) \qquad \frac{i}{h} = \delta \frac{k}{h}$$

$$(7) \qquad \frac{c}{h} = \frac{y}{h} - \frac{i}{h}$$

$$(2) \qquad h = \left(1 + \frac{\phi}{w} \frac{c}{h} \right)^{-1}$$

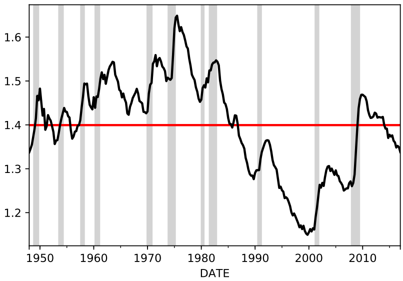
Parameters

From the consumption-labor choice and wage equations we get

$$\log(1-h_t) = \log\phi + \log c_t - (-\log\mu_t + \log(1-\alpha) + \log y_t - \log h_t)$$

$$\log\mu_t = -\log\phi + \log(1-\alpha) + \log(1-h_t) - \log h_t + \log y_t - \log c_t$$

All variables on the RHS are observable. The result is plotted below



Regression on the above markup implies $\rho_\mu = 0.99$ and $\varepsilon_\mu = 0.011$

Model comparison

	Rel. S. D.			Corr. w. y			Autocorr.		
	Data	RBC	MC	Data	RBC	MC	Data	RBC	MC
y	1.00	1.00	1.00	1.00	1.00	1.00	0.85	0.72	0.73
c	0.53	0.38	0.38	0.78	0.94	0.63	0.82	0.78	0.82
i	2.75	3.11	4.99	0.76	0.99	0.96	0.87	0.71	0.71
h	1.17	0.44	1.09	0.80	0.98	0.85	0.91	0.71	0.72
w	0.55	0.58	0.71	0.08	0.99	0.99	0.68	0.74	0.76
$\frac{y}{h}$	0.60	0.58	0.59	0.44	0.99	0.13	0.71	0.74	0.74
μ	1.08	–	0.88	-0.48	–	-0.72	0.83	–	0.72

Monopolistic competition – summary

- Introduction of second shock reduces model's reliance on TFP
- It improves hours volatility by a lot
- Markup shock does not have a good economic interpretation
- May be a result of many factors unrelated to monopoly power – increases sharply in recessions
- Chari, Kehoe, and McGrattan (2007) perform "business cycle accounting", where they identify "wedges" (residuals) from the first order conditions of a very basic RBC model
- We just have obtained the measure for the labor wedge

Aggregate price index derivation

Perfect competition in the final goods sector implies

$$
\begin{aligned}
P_t y_t &= \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i \\
P_t y_t &= \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t \mathrm{d}i \\
P_t y_t &= P_t^{-\frac{\mu}{1-\mu}} y_t \cdot \int_0^1 P_t(i)^{1+\frac{\mu}{1-\mu}}\, \mathrm{d}i \\
P_t^{1+\frac{\mu}{1-\mu}} &= \int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i \\
P_t &= \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i\right)^{1-\mu}
\end{aligned}
$$

back

Marginal cost for production function with capital I

Cost minimization problem

$$\begin{aligned} \min \quad & tc_t(i) = w_t h_t(i) + r_t^k k_t(i) \\ \text{subject to} \quad & y_t(i) = z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} \end{aligned}$$

Lagrangian

$$\mathcal{L} = -\left(w_t h_t(i) + r_t^k k_t(i)\right) + mc_t(i)\left(z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} - y_t(i)\right)$$

FOCs

$$\begin{aligned} h_t(i) \quad &: \quad w_t = mc_t(i)(1-\alpha) z_t k_t(i)^{\alpha} h_t(i)^{-\alpha} \\ k_t(i) \quad &: \quad r_t^k = mc_t(i)\, \alpha z_t k_t(i)^{\alpha-1} h_t(i)^{1-\alpha} \end{aligned}$$

Divide

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{k_t(i)}{h_t(i)} \longrightarrow \frac{k_t(i)}{h_t(i)} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \longrightarrow h_t(i) = \frac{1-\alpha}{\alpha}\frac{r_t^k}{w_t} k_t(i)$$

All firms have identical k/h ratio

Marginal cost for production function with capital II

Production function

$$\begin{aligned} y_t(i) &= z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} = z_t k_t(i)^{\alpha} \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} k_t(i) \right)^{1-\alpha} \\ &= z_t k_t(i) \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} \right)^{1-\alpha} \longrightarrow k_t(i) = \frac{y_t(i)}{z_t} \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} \right)^{\alpha-1} \end{aligned}$$

Total cost

$$\begin{aligned} tc_t(i) &= w_t h_t(i) + r_t^k k_t(i) = \frac{\alpha}{1-\alpha} r_t^k k_t(i) + r_t^k k_t(i) \\ &= \left(\frac{1-\alpha}{\alpha} + 1 \right) r_t^k k_t(i) = \frac{1}{\alpha} r_t^k k_t(i) \\ &= \frac{1}{\alpha} r_t^k \frac{y_t(i)}{z_t} \left(\frac{r_t^k}{w_t} \frac{1-\alpha}{\alpha} \right)^{\alpha-1} = \frac{y_t(i)}{z_t} \frac{\left(r_t^k\right)^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} \end{aligned}$$

Marginal cost is identical across firms back

$$mc_t(i) = \frac{\partial tc_t(i)}{\partial y_t(i)} = \frac{1}{z_t} \frac{\left(r_t^k\right)^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} = mc_t$$

Flexible vs sticky prices

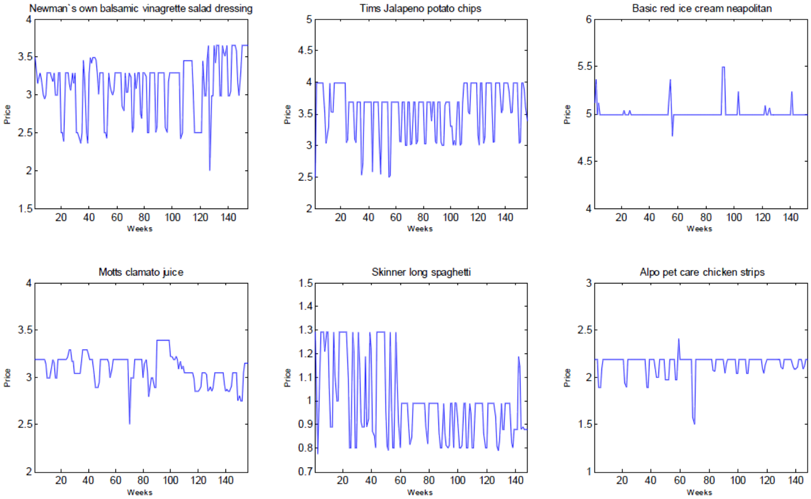
- Central assumption of the (new) classical economics
- Prices (of goods and factor services) are fully flexible
  - An increase in money supply increases prices 1:1 immediately
  - Money is (super)neutral, monetary policy has no power – classical dichotomy
  - In previous models we abstracted from money and nominal variables
- (New) Keynesian economics
  - Prices are sticky (inertial), do not adjust instantly
  - Classical dichotomy no longer holds - nominal variables affect real
  - Scope for monetary policy
  - Additional propagation channels for other shocks

Sticky prices: empirical evidence

- Price duration
  - US: average time between price changes is 2-4 quarters
    Blinder et al. (1998), Bils and Klenow (2004),
    Klenow and Kryvstov (2008), Nakamura and Steinsson (2008)
  - Euro area: average time between price changes is 4-5 quarters
    Rumler and Vilmunen (2005), Altissimo et al. (2006)
  - Poland: average time between price changes is 4 quarters
    Macias and Makarski (2013)
- The higher inflation, the more frequently price changes occur
- Cross-industry heterogeneity
  - Prices of tradables less sticky than those of nontradables
  - Retail prices usually more sticky than producer prices

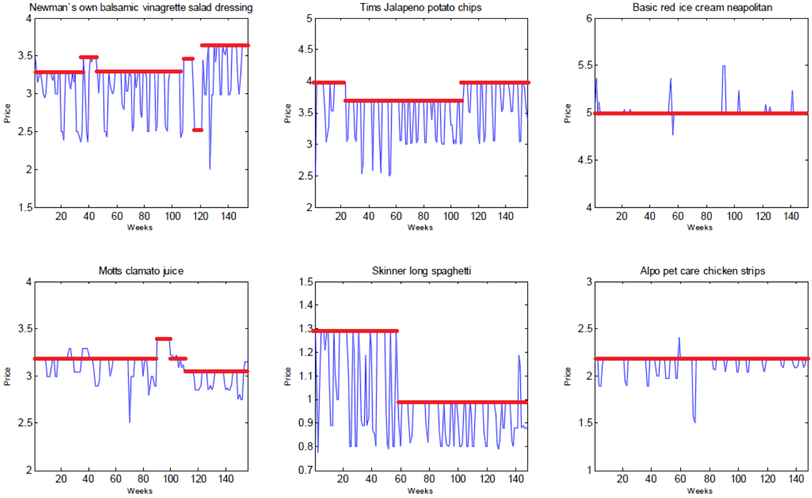
Example retail prices behavior

Raw retail scanner data



Example retail prices behavior

After "controlling" for short-lived sales prices – reference prices



Theories on price stickiness

- Lucas (1972) – imperfect information
  - When faced with a higher nominal demand for product a firm does not know whether real demand or price level went up
  - If it's real demand firm should increase output
  - If it's inflation firm should increase prices
  - If inflation expectations are low – rational to leave prices unchanged
- Extensions
  - Sticky information – Mankiw and Reis (2007)
  - Rational inattention – Sims (2003), Maćkowiak and Wiederholt (2009)
- Behavioral reasons – psychological pricing, judging quality by price
- Costs of changing prices (explicit or implicit)
  - Menu costs – Sheshinski and Weiss (1977), Akerlof and Yellen (1985), Mankiw (1985)
  - Explicit contracts which are costly to renegotiate
  - Long-term relationships with customers – price changes less frequent in sectors with more monopoly power
- "Good" causes of price stickiness – in a stable economic environment agents trust in price stability

Price stickiness depends on sector

Altissimo, Ehrmann and Smets (2006)
Inflation persistence and price-setting behaviour in the euro area

Table 4.1 Frequency of consumer price changes by product type, in %

Country	Unprocessed food	Processed food	Energy (oil products)	Non-energy industrial goods	Services	Total, country weights	Total, Euro area weights
Belgium	31.5	19.1	81.6	5.9	3.0	17.6	15.6
Germany	25.2	8.9	91.4	5.4	4.3	13.5	15.0
Spain	50.9	17.7	n.a.	6.1	4.6	13.3	11.5
France	24.7	20.3	76.9	18.0	7.4	20.9	20.4
Italy	19.3	9.4	61.6	5.8	4.6	10.0	12.0
Luxembourg	54.6	10.5	73.9	14.5	4.8	23.0	19.2
The Netherlands	30.8	17.3	72.6	14.2	7.9	16.2	19.0
Austria	37.5	15.5	72.3	8.4	7.1	15.4	17.1
Portugal	55.3	24.5	15.9	14.3	13.6	21.1	18.7
Finland	52.7	12.8	89.3	18.1	11.6	20.3	-
Euro Area	28.3	13.7	78.0	9.2	5.6	15.1	15.8

Source: Dhyne et al. (2005). Figures presented in this table are computed on the basis of the 50 product sample, with the only exception of Finland for which figures based on the entire CPI are presented. The total with country weights is calculated using country-specific weights for each item, the total with euro area weights using common euro area weights for each sub-index. No figures are provided for Finland because of a lack of comparability of the sample of products used in this country.

Survey assessment of price stickiness theories

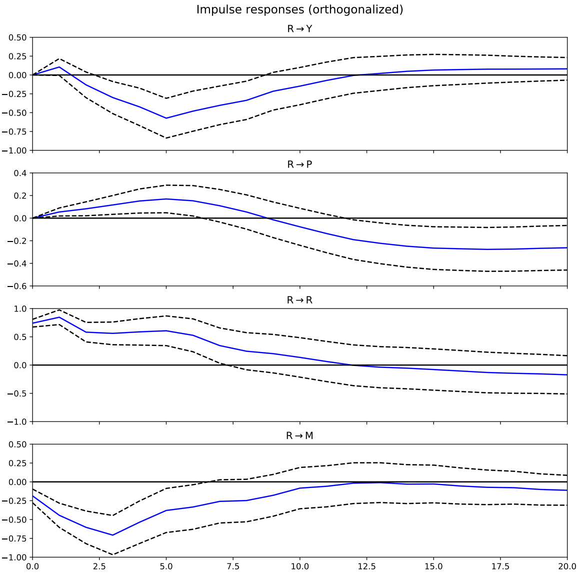
Altissimo, Ehrmann and Smets (2006)
Inflation persistence and price-setting behaviour in the euro area

Table 4.6 Ranking of theories explaining price stickiness

	Belgium	Germany	Spain	France	Italy	Luxem-bourg	Nether-lands	Austria	Portugal	Euro Area
Implicit contracts	2.5		2.6	2.2		2.7	2.7	3.0	3.1	2.7
Explicit contracts	2.4	2.4	2.3	2.7	2.6	2.8	2.5	3.0	2.6	2.6
Cost-based pricing	2.4			2.5		2.7		2.6	2.7	2.6
Co-ordination failure	2.2	2.2	2.4	3.0	2.6	2.1	2.2	2.3	2.8	2.4
Judging quality by price	1.9		1.8			2.2	2.4	1.9	2.3	2.1
Temporary shocks	1.8	1.9	1.8	2.1	2.0	1.7	2.4	1.5	2.5	2.0
Change non-price factors	1.7		1.3			1.9	1.9	1.7		1.7
Menu costs	1.5	1.4	1.4	1.4	1.6	1.8	1.7	1.5	1.9	1.6
Costly information	1.6		1.3			1.8		1.6	1.7	1.6
Pricing thresholds	1.7		1.5	1.6	1.4	1.8	1.8	1.3	1.8	1.6

Source: Fabiani et al. (2005). Euro area figures are unweighted averages of country scores.

Effects of price stickiness – influence of nominal variables



Results from a 4-variable 6-lag vector autoregression

New Keynesian model – introduction

- NK model is an RBC model with
  - Monopolistic competition
  - Sticky prices
  - Monetary policy authority
- Model price stickiness via Calvo (1983) assumption
  - A firm can change its price only if it receives a signal
  - Firm does not receive the signal with probability θ
  - Expected (average) price duration is $\frac{1}{1-\theta}$

Households – problem

For simplicity consider a model without physical capital

$$\max \qquad E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{c_t^{1-\sigma}}{1-\sigma}-\phi\frac{h_t^{1+\eta}}{1+\eta}\right)\right]$$

$$\text{subject to} \qquad P_t c_t + B_t = W_t h_t + R_{t-1}B_{t-1} + P_t div_t$$

where nominal bonds B yield the gross nominal interest rate R

Rewrite budget constraint in real terms

$$\begin{aligned} c_t + \frac{B_t}{P_t} &= \frac{W_t}{P_t}h_t + R_{t-1}\frac{P_{t-1}}{P_t}\frac{B_{t-1}}{P_{t-1}} + div_t \\ c_t + b_t &= w_t h_t + \frac{R_{t-1}}{\Pi_t}b_{t-1} + div_t \end{aligned}$$

where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate

Households – solution

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_0 \left[\begin{array}{c} \frac{c_t^{1-\sigma}}{1-\sigma} - \phi \frac{h_t^{1+\eta}}{1+\eta} \\ +\lambda_t \left(w_t h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + div_t - c_t - b_t \right) \end{array} \right]$$

FOCs

$$\begin{aligned} c_t &: \quad c_t^{-\sigma} - \lambda_t = 0 \\ h_t &: \quad -\phi h_t^{\eta} + \lambda_t w_t = 0 \\ b_t &: \quad -\lambda_t + \beta E_t \left[\lambda_{t+1} \left(R_t / \Pi_{t+1} \right) \right] = 0 \end{aligned}$$

Resulting

$$\begin{aligned} \text{Intratemporal choice } (c+h) &: \quad c_t^{-\sigma} w_t = \phi h_t^{\eta} \\ \text{Intertemporal choice } (c+b) &: \quad c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} \left(R_t / \Pi_{t+1} \right) \right] \end{aligned}$$

Final goods producing firm I

Profit maximization problem

$$\max \qquad P_t y_t - \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i$$

$$\text{subject to} \qquad y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu}$$

Lagrangian

$$\mathcal{L} = P_t y_t - \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i + \lambda_t \left[\left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu} - y_t \right]$$

FOCs

$$\begin{aligned} y_t \quad &: \quad P_t - \lambda_t = 0 \\ y_t(i) \quad &: \quad -P_t(i) + \lambda_t \left[\mu \left(\int_0^1 y_t(i)^{\frac{1}{\mu}}\, \mathrm{d}i \right)^{\mu-1} \cdot \frac{1}{\mu} y_t(i)^{\frac{1}{\mu}-1} \right] = 0 \end{aligned}$$

Final goods producing firm II

Result

$$
\begin{aligned}
P_t(i) &= P_t \left(\int_0^1 y_t(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right)^{\mu-1} y_t(i)^{\frac{1-\mu}{\mu}} \\
P_t(i)^{\frac{\mu}{1-\mu}} &= P_t^{\frac{\mu}{1-\mu}} \left(\int_0^1 y_t(i)^{\frac{1}{\mu}} \, \mathrm{d}i \right)^{-\mu} y_t(i) \\
P_t(i)^{\frac{\mu}{1-\mu}} &= P_t^{\frac{\mu}{1-\mu}} y_t^{-1} y_t(i) \\
y_t(i) &= \left(\frac{P_t(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} y_t
\end{aligned}
$$

Aggregate price index derivation

$$
P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}} \, \mathrm{d}i \right)^{1-\mu}
$$

Intermediate goods producing firm I

Production function is linear in hours

$$y_t(i) = z_t h_t(i)$$

Cost minimization problem

$$\begin{aligned} \min \quad & tc_t(i) = w_t h_t(i) \\ \text{subject to} \quad & y_t(i) = z_t h_t(i) \end{aligned}$$

Lagrangian

$$\mathcal{L} = -w_t h_t(i) + mc_t(i)\left(z_t h_t(i) - y_t(i)\right)$$

FOC

$$w_t = mc_t(i)\, z_t$$

Marginal cost is identical across firms

$$mc_t(i) = mc_t = \frac{w_t}{z_t}$$

Intermediate goods producing firm II

Profit maximization problem (where $\Lambda_{0,t} = \lambda_t / \lambda_0$)

$$\max \quad E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_{0,t} \left(\frac{\tilde{P}_0(i)}{P_t} y_t(i) - mc_t y_t(i) \right) \right]$$

$$\text{subject to} \quad y_t(i) = \left(\frac{\tilde{P}_0(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} y_t$$

Define $\tilde{p}_0(i) = \tilde{P}_0(i) / P_0$ and $\Pi_{0,t} = P_t / P_0 = \Pi_1 \cdot \ldots \cdot \Pi_t$. Then

$$\frac{\tilde{P}_0(i)}{P_t} = \frac{\tilde{P}_0(i)}{P_0} \frac{P_0}{P_t} = \tilde{p}_0(i) \frac{1}{\Pi_{0,t}} = \frac{\tilde{p}_0(i)}{\Pi_{0,t}}$$

Rewrite

$$\max \qquad E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_{0,t} \left(\left(\frac{\tilde{p}_0(i)}{\Pi_{0,t}} \right)^{1+\frac{\mu}{1-\mu}} y_t - mc_t \left(\frac{\tilde{p}_0(i)}{\Pi_{0,t}} \right)^{\frac{\mu}{1-\mu}} y_t \right) \right]$$

Intermediate goods producing firm III

$$\max \quad E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{\lambda_t}{\lambda_0} \left(\tilde{p}_0(i)^{1+\frac{\mu}{1-\mu}} \Pi_{0,t}^{\frac{\mu}{\mu-1}-1} y_t - mc_t \tilde{p}_0(i)^{\frac{\mu}{1-\mu}} \Pi_{0,t}^{\frac{\mu}{\mu-1}} y_t \right) \right]$$

FOC

$$E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{\lambda_t}{\lambda_0} \left(\frac{1}{1-\mu} \right) \tilde{p}_0(i)^{\frac{\mu}{1-\mu}} \Pi_{0,t}^{\frac{\mu}{\mu-1}-1} y_t \right] =$$

$$= E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \frac{\lambda_t}{\lambda_0} mc_t \left(\frac{\mu}{1-\mu} \right) \tilde{p}_0(i)^{\frac{\mu}{1-\mu}-1} \Pi_{0,t}^{\frac{\mu}{\mu-1}} y_t \right]$$

Optimal relative price

$$\tilde{p}_0(i) = \mu \cdot \frac{E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \lambda_t mc_t \Pi_{0,t}^{\frac{\mu}{\mu-1}} y_t \right]}{E_0 \left[\sum_{t=0}^{\infty} (\beta\theta)^t \lambda_t \Pi_{0,t}^{\frac{\mu}{\mu-1}-1} y_t \right]}$$

Intermediate goods producing firm IV

Optimal relative price is the same across all firms resetting prices

$$\tilde{p}_t = \mu \cdot \frac{E_t \left[\sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+j} mc_{t+j} \Pi_{t,t+j}^{\frac{\mu}{\mu-1}} y_{t+j} \right]}{E_t \left[\sum_{j=0}^{\infty} (\beta\theta)^j \lambda_{t+j} \Pi_{t,t+j}^{\frac{1}{\mu-1}} y_{t+j} \right]}$$

This expression has a convenient recursive representation derivation

$$\tilde{p}_t = \mu \frac{Num_t}{Den_t}$$

$$Num_t = \lambda_t mc_t y_t + \beta\theta E_t \left[\Pi_{t+1}^{\frac{\mu}{\mu-1}} Num_{t+1} \right]$$

$$Den_t = \lambda_t y_t + \beta\theta E_t \left[\Pi_{t+1}^{\frac{1}{\mu-1}} Den_{t+1} \right]$$

If prices are not sticky $(\theta = 0)$ then

$$\tilde{p}_t = \mu \cdot mc_t$$

NK collapses to RBC with monopolistic competition if prices are flexible

Inflation dynamics

Recall the formula for aggregate price index

$$
\begin{aligned}
P_t &= \left(\int_0^1 P_t\left(i\right)^{\frac{1}{1-\mu}}\,\mathrm{d}i\right)^{1-\mu} \\
P_t^{\frac{1}{1-\mu}} &= \int_0^\theta P_{t-1}\left(i\right)^{\frac{1}{1-\mu}}\,\mathrm{d}i + \int_\theta^1 \tilde{P}_t^{\frac{1}{1-\mu}}\mathrm{d}i \\
P_t^{\frac{1}{1-\mu}} &= \theta P_{t-1}^{\frac{1}{1-\mu}} + \left(1-\theta\right)\tilde{P}_t^{\frac{1}{1-\mu}} \quad \Big| \quad : P_{t-1}^{\frac{1}{1-\mu}} \\
\left(\frac{P_t}{P_{t-1}}\right)^{\frac{1}{1-\mu}} &= \theta\left(\frac{P_{t-1}}{P_{t-1}}\right)^{\frac{1}{1-\mu}} + \left(1-\theta\right)\left(\frac{\tilde{P}_t}{P_t}\frac{P_t}{P_{t-1}}\right)^{\frac{1}{1-\mu}} \\
\Pi_t^{\frac{1}{1-\mu}} &= \theta + \left(1-\theta\right)\left(\tilde{p}_t\Pi_t\right)^{\frac{1}{1-\mu}} \\
\Pi_t &= \left[\theta / \left(1-\left(1-\theta\right)\tilde{p}_t^{\frac{1}{1-\mu}}\right)\right]^{1-\mu}
\end{aligned}
$$

Market clearing

Factor markets clear

$$h_t = \int_0^1 h_t(i)\,\mathrm{d}i$$

Intermediate goods markets are in equilibrium

$$z_t h_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t$$

$$\int_0^1 z_t h_t(i)\,\mathrm{d}i = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} y_t \mathrm{d}i$$

$$z_t h_t = y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$$

$$z_t h_t = y_t \Delta_t$$

where price dispersion Δ creates inefficiency

$$y_t = \frac{z_t h_t}{\Delta_t}$$

Output accounting

Dividends

$$div_t = y_t - w_t h_t$$

Budget constraint

$$c_t + b_t = w_t h_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} + div_t$$

In equilibrium representative agent holds 0 bonds ($b_t = b_{t-1} = 0$)

$$\begin{aligned} c_t &= w_t h_t + y_t - w_t h_t \\ c_t &= y_t \end{aligned}$$

Price dispersion: source of inefficiency

Define

$$\Delta_t = \int_0^1 \left(\frac{P_t\,(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$$

Dynamics

$$\Delta_t = \int_0^\theta \left(\frac{P_{t-1}\,(i)}{P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{\frac{\mu}{1-\mu}} \mathrm{d}i + \int_\theta^1 \left(\frac{\tilde{P}_t}{P_t} \right)^{\frac{\mu}{1-\mu}} \mathrm{d}i$$

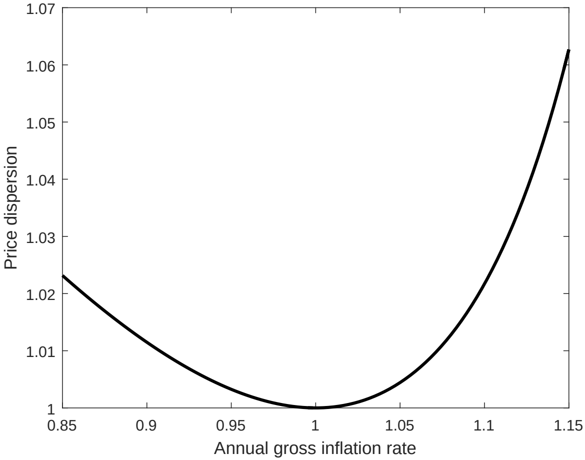
$$\Delta_t = \theta \Delta_{t-1} \Pi_t^{\frac{\mu}{\mu-1}} + (1-\theta)\, \tilde{p}_t^{\frac{\mu}{1-\mu}}$$

One can show that $\Delta_t \geq 1$ and in consequence

$$y_t \leq z_t h_t$$

Costs of non-zero inflation

Price dispersion as a function of steady state gross annual inflation
Parameters used: $\mu = 1.33$, $\theta = 0.75$



Costs of non-zero inflation

- Inflation is more harmful than deflation
- Costs of inflation are convex
  - An annual inflation of 2% causes about 0.05% loss in GDP
  - An annual inflation of 5% causes about 0.4% loss in GDP
  - An annual inflation of 10% causes about 2% loss in GDP
  - An annual inflation of 15% causes about 6% loss in GDP
  - An annual inflation of 20% causes about 15% loss in GDP
- For high levels of inflation the model breaks down
  – not suitable for analysing hyperinflations
- Even before that firms would probably change prices more often
  – Calvo pricing is a modeling shortcut, not microfounded
- Despite efficiency losses from price dispersion,
  higher inflation target lowers probability of hitting ZLB
- Before the crisis the consensus for inflation target was 2%
- After the crisis: Blanchard, Ball and others propose 4%

Equilibrium conditions

$$
\begin{aligned}
\text{Euler equation} &: \quad 1 = \beta E_t \left(c_t / c_{t+1}\right)^{\sigma} \left(R_t / \Pi_{t+1}\right) && (10) \\
\text{Consumption-hours} &: \quad w_t = \phi h_t^{\eta} c_t^{\sigma} && (11) \\
\text{Real wages} &: \quad w_t = mc_t z_t h_t && (12) \\
\text{Production function} &: \quad y_t = z_t h_t / \Delta_t && (13) \\
\text{Price dispersion} &: \quad \Delta_t = \theta \Delta_{t-1} \Pi_t^{\frac{\mu}{\mu-1}} + (1-\theta)\, \tilde{p}_t^{\frac{\mu}{1-\mu}} && (14) \\
\text{Inflation dynamics} &: \quad \Pi_t = \left[1/\theta - (1/\theta - 1)\, \tilde{p}_t^{\frac{1}{1-\mu}}\right]^{\mu-1} && (15) \\
\text{Optimal reset price} &: \quad \tilde{p}_t = \mu \cdot \left(Num_t / Den_t\right) && (16) \\
\text{Numerator} &: \quad Num_t = c_t^{-\sigma} mc_t y_t + \beta\theta E_t \Pi_{t+1}^{\frac{\mu}{\mu-1}} Num_{t+1} && (17) \\
\text{Denominator} &: \quad Den_t = c_t^{-\sigma} y_t + \beta\theta E_t \Pi_{t+1}^{\frac{1}{\mu-1}} Den_{t+1} && (18) \\
\text{Output accounting} &: \quad y_t = c_t && (19) \\
\text{TFP AR(1) process} &: \quad \ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t} && (20)
\end{aligned}
$$

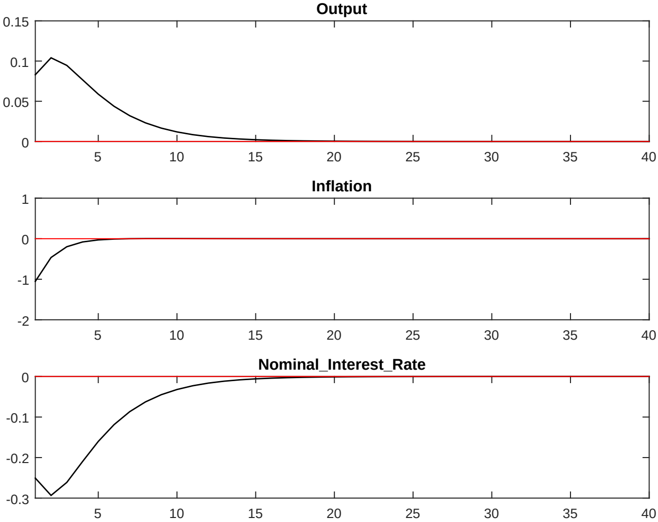
Monetary policy

- There are 11 equations but 12 variables!
  $\{y, c, h, w, z, mc, R, \Pi, \Delta, \tilde{p}, Num, Den\}$
- Need another equation to close the model
- Model is closed by adding a monetary policy rule
- Often the Taylor rule is used

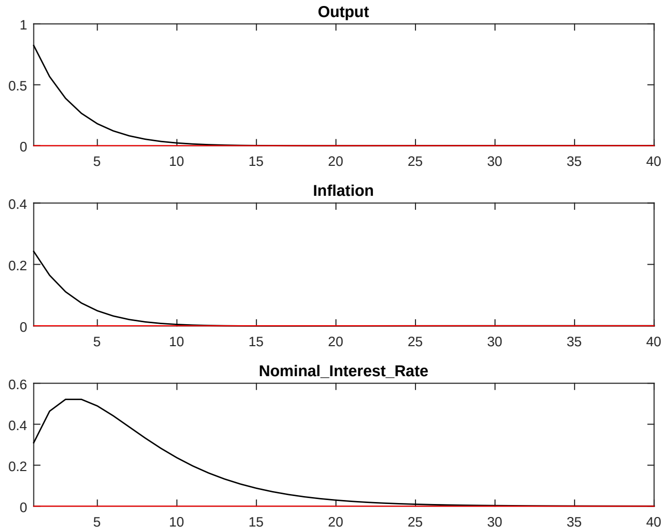
$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{y_t}{y}\right)^{\gamma_y}\right)^{1-\gamma_R} \varepsilon_{R,t} \tag{21}$$

- Taylor principle – $\gamma_\Pi > 1$
- The model can then be reduced to just three equations:
  for output, inflation and nominal interest rate
  (3-equation NK model)

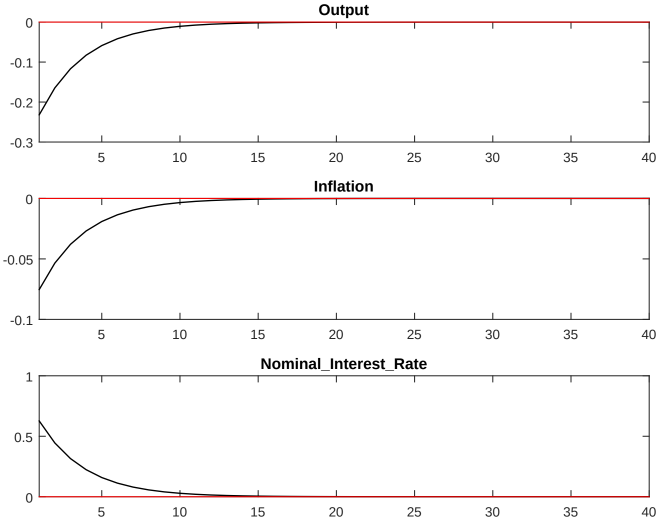
Positive supply shock



Positive demand shock

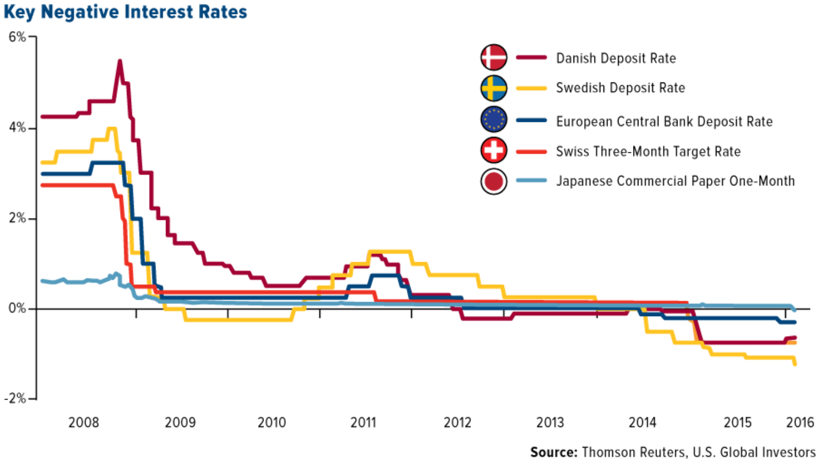


Monetary shock

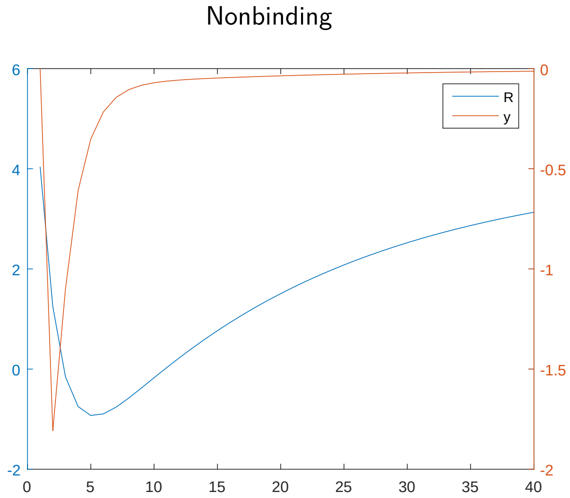


Zero Lower Bound (liquidity trap)

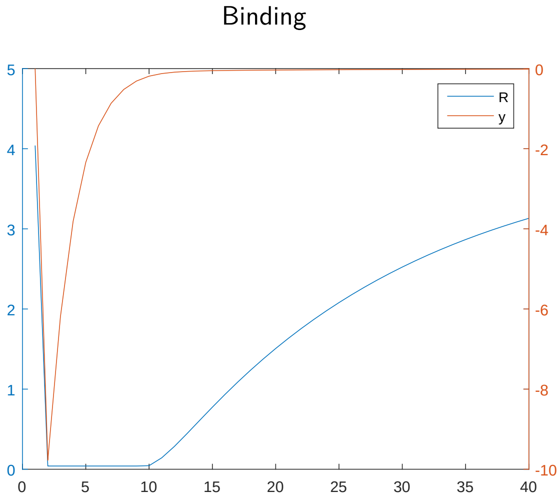
Literally zero?



Zero Lower Bound (liquidity trap)



Zero Lower Bound (liquidity trap)



Aggregate price index derivation

Perfect competition in the final goods sector implies

$$
\begin{aligned}
P_t y_t &= \int_0^1 P_t(i)\, y_t(i)\, \mathrm{d}i \\
P_t y_t &= \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t} \right)^{\frac{\mu}{1-\mu}} y_t \mathrm{d}i \\
P_t y_t &= P_t^{-\frac{\mu}{1-\mu}} y_t \cdot \int_0^1 P_t(i)^{1+\frac{\mu}{1-\mu}}\, \mathrm{d}i \\
P_t^{1+\frac{\mu}{1-\mu}} &= \int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i \\
P_t &= \left(\int_0^1 P_t(i)^{\frac{1}{1-\mu}}\, \mathrm{d}i \right)^{1-\mu}
\end{aligned}
$$

back

Optimal reset price – numerator

$$
\begin{aligned}
&Num_t = \\
&= E_t\left[\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+j}mc_{t+j}\Pi_{t,t+j}^{\frac{\mu}{\mu-1}}y_{t+j}\right] \\
&= \lambda_t mc_t y_t + E_t\left[\sum_{j=1}^{\infty}(\beta\theta)^j\,\lambda_{t+j}mc_{t+j}\Pi_{t,t+j}^{\frac{\mu}{\mu-1}}y_{t+j}\right] \\
&= \lambda_t mc_t y_t + E_t\left[\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+1+j}mc_{t+1+j}\Pi_{t,t+1+j}^{\frac{\mu}{\mu-1}}y_{t+1+j}\right] \\
&= \lambda_t mc_t y_t + E_t\left[\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+1+j}mc_{t+1+j}\left(\Pi_{t,t+1}\cdot\Pi_{t+1,t+1+j}\right)^{\frac{\mu}{\mu-1}}y_{t+1+j}\right] \\
&= \lambda_t mc_t y_t + E_t\left[\Pi_{t,t+1}^{\frac{\mu}{\mu-1}}\cdot\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+1+j}mc_{t+1+j}\Pi_{t+1,t+1+j}^{\frac{\mu}{\mu-1}}y_{t+1+j}\right] \\
&= \lambda_t mc_t y_t + E_t\left[\Pi_{t,t+1}^{\frac{\mu}{\mu-1}}\cdot Num_{t+1}\right]
\end{aligned}
$$

Optimal reset price – denominator

$$
\begin{aligned}
Den_t &= E_t\left[\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+j}\Pi_{t,t+j}^{\frac{1}{\mu-1}}y_{t+j}\right] \\
&= \lambda_t y_t + E_t\left[\sum_{j=1}^{\infty}(\beta\theta)^j\,\lambda_{t+j}\Pi_{t,t+j}^{\frac{1}{\mu-1}}y_{t+j}\right] \\
&= \lambda_t y_t + E_t\left[\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+1+j}\Pi_{t,t+1+j}^{\frac{1}{\mu-1}}y_{t+1+j}\right] \\
&= \lambda_t y_t + E_t\left[\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+1+j}\left(\Pi_{t,t+1}\cdot\Pi_{t+1,t+1+j}\right)^{\frac{1}{\mu-1}}y_{t+1+j}\right] \\
&= \lambda_t y_t + E_t\left[\Pi_{t,t+1}^{\frac{1}{\mu-1}}\cdot\sum_{j=0}^{\infty}(\beta\theta)^j\,\lambda_{t+1+j}\Pi_{t+1,t+1+j}^{\frac{1}{\mu-1}}y_{t+1+j}\right] \\
&= \lambda_t y_t + E_t\left[\Pi_{t,t+1}^{\frac{1}{\mu-1}}\cdot Den_{t+1}\right]
\end{aligned}
$$

back