Marcin Bielecki, Applied Macroeconomics, Spring 2018
Models of financial frictions: main mechanisms

1 Maturity transformation

Diamond & Dybvig (1983): there are three periods: 0, 1 and 2. In period 0 all consumers (of measure 1) are endowed with a unit of consumption good. With probability π each consumer will be "impatient" and want to consume in period 1, and with probability $1-\pi$ each consumer will be "patient" and want to consume in period 2. The expected utility of a consumer is given by:

$$U = \pi \cdot u(c_1) + (1-\pi) \cdot u(c_2)$$

The good can be invested and then the investment delivers $R > 1$ goods in period 2. The investment can be "interrupted" in period 1 and then yields $d < 1$ goods in period 1. The amount invested in period 0 is denoted with $i \leq 1$.

When agents do not trade with each other (autarky), then each of them invests i units of consumption good in period 0. Those who become impatient interrupt the investment and consume in period 1 $c_1^A = (1-i) + di = 1 - (1-d)\,i \leq 1$. Those who become patient consume in period 2 $c_2^A = (1-i) + Ri = 1 + (R-1)\,i \leq R$. Note that it is not possible to choose at the same time $c_1 = 1$ and $c_2 = R$.

Consider now what would constitute an optimal allocation. We would like to allocate the amount $(1-i)$ to the agents who are impatient and amount Ri to the agents who are patient:

$$\pi c_1 = 1 - i$$
$$(1-\pi)\,c_2 = Ri$$

$$\pi c_1 + \frac{(1-\pi)\,c_2}{R} = 1$$

Note that the feasibility constraint allows for $c_1 = 1$ and $c_2 = R$, and obviously $U(1,R) > U\left(c_1^A, c_2^A\right)$. This illustrates the gains from "insurance" across agents.

Imagine now that consumers have access to a perfectly competitive bank. The deposit contract is as follows: if an agent withdraws deposit in period 1, she receives c_1^B and if she withdraws in period 2, she receives c_2^B. A bank then stores the amount $x = \pi c_1^B$ and invests $1-x$, with $(1-\pi)\,c_2^B = R(1-x)$. We will assume that the consumers' utility function is logarithmic. An optimal contract is constructed as follows:

$$\max \quad U = \pi \log c_1^B + (1-\pi)\log c_2^B$$
$$\text{subject to} \quad \pi c_1^B + \frac{(1-\pi)\,c_2^B}{R} = 1$$

Lagrangian:

$$\mathcal{L} = \pi \log c_1^B + (1-\pi)\log c_2^B + \lambda\left[1 - \pi c_1^B - \frac{(1-\pi)\,c_2^B}{R}\right]$$

FOCs:

$$c_1^B \quad : \quad \frac{\pi}{c_1^B} - \lambda\pi = 0$$
$$c_2^B \quad : \quad \frac{1-\pi}{c_2^B} - \lambda\frac{1-\pi}{R} = 0$$

Simplify and rewrite:

$$\lambda = \frac{1}{c_1^B}$$

$$\lambda = \frac{R}{c_2^B}$$

It is now easy to verify that $c_1^B = 1$ and $c_2^B = R$ satisfies both the feasibility constraint and the optimality condition:

$$\frac{1}{c_1^B} = \frac{R}{c_2^B}$$

If we generalize our result to the CRRA utility function, with $\sigma \geq 1$, we will get that:

$$1 \leq c_1^B < c_2^B \leq R$$

This implies the following:

- Patient agents will not in general want to withdraw in period 1 as $c_2^B > c_1^B$.
- Banks are able to create liquidity and insure the consumers against risk: the utility increases.
- But banks are subject to bank runs. If for any reason patient consumers believe that other patient consumers will want to withdraw at period 1, they will also want to withdraw. But the bank does not have enough resources available at period 1 to service everyone in period 1.
- Banks are then inherently fragile! This creates a strong rationale for the lender of last resort institution (central bank). But the presence of the lender of last resort may incentivize banks to take excessive investment risk.

2 Action verification

Holmstrom & Tirole (1997): there are three agents: entrepreneurs, investors and bankers. All are risk neutral. Investors have resources available at period 0 to invest. Entrepreneurs differ with respect to their net worth k, but all have opportunities to invest i in investment projects. A successful project yields return R, while a failed one yields 0. The project is successful with probability p_H. However, the entrepreneurs can also use another investment technology, which has lower success probability $p_L < p_H$ but then they get private benefit B. The optimal lending contract between the investors and entrepreneurs incentivizes the entrepreneurs to not be "lazy" and has the following components:

- The total return R is split between the investors' return R^f and the entrepreneurs' return R^e
- Participation constraint (PC): the investors' return is at least equal to the amount lent: $p_H R^f \geq i - k$
- Incentive compatibility constraint (IC): the entrepreneurs' return is big enough for them to not be "lazy": $p_H R^e \geq p_L R^e + B$

The conditions jointly imply that:

$$(p_H - p_L) R^e \geq B \quad \longrightarrow \quad (p_H - p_L)\left(R - R^f\right) = (p_H - p_L)\left(R - \frac{i+k}{p_H}\right) \geq B$$

$$k \geq p_H \left(R + \frac{B}{p_H - p_L}\right) - i$$

The above condition states that only entrepreneurs with net worth above the threshold determined on the RHS will get financing.

Imagine now that a bank can act as an intermediary between the investors and entrepreneurs. Banks are able to monitor the investment projects at cost m. When they do so, private benefit of "lazy" entrepreneurs is lower than without monitoring: $b < B$. The optimal contract now involves:

- The total return R is split between the investors' return R^f, the entrepreneurs' return R^e and monitoring costs m
- Participation constraint (PC): the investors' return is at least equal to the amount lent: $p_H R^f \geq i - k$
- Incentive compatibility constraint (IC): the entrepreneurs' return is big enough for them to not be "lazy": $p_H R^e \geq p_L R^e + b$

The conditions jointly imply that:

$$(p_H - p_L) R^e \geq b \quad \longrightarrow \quad (p_H - p_L)\left(R - m - R^f\right) = (p_H - p_L)\left(R - m - \frac{i+k}{p_H}\right) \geq b$$

$$k \geq p_H \left(R - m + \frac{b}{p_H - p_L}\right) - i$$

The above condition implies that some entrepreneurs, who previously were not able to get financing, can obtain it now. In both cases, when entrepreneurs' net worth decreases, some of them do not get financing anymore and aggregate investment falls.

3 State verification

Townsend (1979): there are two agents: investors and entrepreneurs. Entrepreneurs have no net worth but have access to the investment technology, which requires investment i in period 0 and pays sR in period 1, with s being a random variable drawn from distribution S. In period 1 entrepreneurs know s but investors do not. An entrepreneur can declare bankruptcy and not return the loan, but then their investment is seized by investors. The investors have to pay a proportional audit cost m to learn the value of s. The optimal contract now results in the following separating equilibrium: projects with $s \leq \bar{s}$ are declared bankrupt and are seized by the investor. Here I will specify only the participation constraint and its consequences:

$$i = \beta R \left[(1-m) \int_0^{\bar{s}} s \mathrm{d}S + \bar{s}\left(1 - S(\bar{s})\right)\right]$$

Assume that S is a uniform distribution on $(0,1)$. Then:

$$i = \beta R \left[(1-m)\left(\frac{s^2}{2}\right)_0^{\bar{s}} + \bar{s}(1-\bar{s})\right] = \beta R \left[(1-m)\frac{\bar{s}^2}{2} - \bar{s}^2 + \bar{s}\right] = \beta R \left[\bar{s} - \frac{1+m}{2}\bar{s}^2\right]$$

The maximal value of RHS is when:

$$1 - (1+m)\bar{s} = 0 \quad \longrightarrow \quad \bar{s} = \frac{1}{1+m}$$

and then its value is equal to:

$$\beta R \left[\frac{1}{1+m} - \frac{1+m}{2}\left(\frac{1}{1+m}\right)^2\right] = \frac{\beta R}{1+m}$$

Investment will be made only if:

$$i \leq \frac{\beta R}{1+m}$$

Auditing costsexclude some investment opportunities and drive a wedge between lending and borrowing interest rates. It is also possible to show that if we allow for entrepreneurs to have net worth, negative shocks to it will lower aggregate investment.

4 Collateral constraints

Kiyotaki & Moore (1997): borrowers cannot commit to repay the loan. To incentivize them to do so, they need to hold some tradeable collateral which can be seized and sold by the lender. Focus on the example of housing. The maximal amount of the loan L can be given by:

$$RL \leq E\left[p\right]$$

where R is the interest rate on mortgage and $E\left[p\right]$ denotes expected house price in the future. If the house prices are expected to increase, banks easier extend mortgage loans. This in turn increases demand for housing, driving their prices up in a self-fulfilling loop.

References

Diamond, D. W. & Dybvig, P. H. (1983). Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy*, 91(3), 401–419.

Holmstrom, B. & Tirole, J. (1997). Financial Intermediation, Loanable Funds, and The Real Sector. *The Quarterly Journal of Economics*, 112(3), 663–691.

Kiyotaki, N. & Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105(2), 211–248.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2), 265–293.