Blind Single Channel Deconvolution using Nonstationary Signal Processing

Reverberation Cancellation in Acoustic Environments

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Introduction to Blind Deconvolution

- Blind deconvolution fundamental in signal processing
- Observation, \( x = \{x(t), t \in T\} \), modelled as the convolution of unknown source, \( \{s(t), t \in T\} \), with unknown distortion operator, \( A \); i.e. \( x(t) \triangleq s(t) \ast A \)

![Image of room transfer function diagram]

- Estimate \( A \), or \( \hat{s}(t) = a s(t - \tau) \), a scaled shifted version of \( s(t) \), where \( a, \tau \in \mathbb{R} \), given only the observations, \( x \)
Acoustic Reverberation Cancellation

- Normal hearing: can concentrate on original sound despite:
  - reverberation
Normal hearing: can concentrate on original sound despite:

- reverberation
- environmental noise
Acoustic Reverberation Cancellation

- Normal hearing: can concentrate on original sound despite:
  - reverberation
  - environmental noise
- Hearing aid users unable to distinguish one voice from another
- Sensori-neuro loss cannot be compensated for by simple amplifying hearing aids or surgery
Bayesian Blind Deconvolution
Bayes’s Theorem

The posterior probability, \( p(\theta \mid x, I) \), of the system parameters, \( \theta \), given the state of the system, \( x \), and an underlying model, \( I \), is given by Bayes’ theorem:

\[
p(\theta \mid x, I) = \frac{p(x \mid \theta, I)p(\theta \mid I)}{p(x \mid I)}
\]
Bayes’s Theorem

The posterior probability, \( p(\theta | x, \mathcal{I}) \), of the system parameters, \( \theta \), given the state of the system, \( x \), and an underlying model, \( \mathcal{I} \), is given by Bayes’ theorem:

\[
p(\theta | x, \mathcal{I}) = \frac{p(x | \theta, \mathcal{I}) p(\theta | \mathcal{I})}{p(x | \mathcal{I})}
\]

\( p(x | \theta, \mathcal{I}) \) is the likelihood.
Bayes’s Theorem

The posterior probability, $p(\theta \mid x, I)$, of the system parameters, $\theta$, given the state of the system, $x$, and an underlying model, $I$, is given by Bayes’ theorem:

$$p(\theta \mid x, I) = \frac{p(x \mid \theta, I) p(\theta \mid I)}{p(x \mid I)}$$

- $p(x \mid \theta, I)$ is the likelihood
- $p(\theta \mid I)$ represents prior knowledge
Bayes’s Theorem

- The posterior probability, \( p(\theta \mid x, \mathcal{I}) \), of the system parameters, \( \theta \), given the state of the system, \( x \), and an underlying model, \( \mathcal{I} \), is given by Bayes’ theorem:

\[
p(\theta \mid x, \mathcal{I}) = \frac{p(x \mid \theta, \mathcal{I})p(\theta \mid \mathcal{I})}{p(x \mid \mathcal{I})}
\]

- \( p(x \mid \theta, \mathcal{I}) \) is the likelihood

- \( p(\theta \mid \mathcal{I}) \) represents prior knowledge

- \( p(x \mid \mathcal{I}) \) is the evidence and, although usually regarded as a normalising constant, is of interest for model selection
Likelihood Function for System

The likelihood function for the observed signal, \( x \), is:

\[
p(x \mid \theta, \phi, I) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi}\sigma_i)^T_i} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}
\]

Where \( s(t) \equiv s(t, a, x) \) given by:

\[
s(t) = x(t) + \sum_{p \in \mathcal{P}} a(p) x(t - p)
\]
Likelihood Function for System

The likelihood function for the observed signal, $x$, is:

$$ p(x \mid \theta, \phi, T) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi}\sigma_i)^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\} $$

- Data vector, $s_i$: $[s_i]_{t-t_i+1} = s(t), \ t \in T_i, \ i \in M$
The likelihood function for the observed signal, $x$, is:

$$p(x \mid \theta, \phi, \mathcal{I}) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi}\sigma_i)^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}$$

- Data vector, $s_i$: $[s_i]_{t-t_i+1} = s(t), t \in \mathcal{T}_i, i \in \mathcal{M}$
- Data matrix, $S_i$: $[S_i]_{t-t_i+1,q} = s(t-q), t \in \mathcal{T}_i, q \in \mathcal{Q}_i, i \in \mathcal{M}$
Likelihood Function for System

The likelihood function for the observed signal, \( x \), is:

\[
\begin{align*}
    p(x | \theta, \phi, \mathcal{T}) &= \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi\sigma_i})^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\} \\

    \text{Data vector, } s_i : [s_i]_{t-t_i+1} &= s(t), \ t \in \mathcal{T}_i, \ i \in \mathcal{M} \\

    \text{Data matrix, } S_i : [S_i]_{t-t_i+1,q} &= s(t-q), \ t \in \mathcal{T}_i, q \in \mathcal{Q}_i, \ i \in \mathcal{M} \\

    \text{Source parameters, } b = \{b_i, i \in \mathcal{M}\} : [b_i]_q = b_i(q), \ q \in \mathcal{Q}_i
\end{align*}
\]
Likelihood Function for System

The likelihood function for the observed signal, $x$, is:

$$
p(x \mid \theta, \phi, \mathcal{T}) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi}\sigma_i)^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}
$$

- Data vector, $s_i: [s_i]_{t-t_i+1} = s(t), t \in \mathcal{T}_i, i \in \mathcal{M}$
- Data matrix, $S_i: [S_i]_{t-t_i+1,q} = s(t-q), t \in \mathcal{T}_i, q \in \mathcal{Q}_i, i \in \mathcal{M}$
- Source parameters, $b = \{b_i, i \in \mathcal{M}\}: [b_i]_q = b_i(q), q \in \mathcal{Q}_i$
- Excitation Variances, $\sigma = \{\sigma_i^2, i \in \mathcal{M}\}$
The likelihood function for the observed signal, $x$, is:

$$p(x | \theta, \phi, T) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi} \sigma_i)^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}$$

- Data vector, $s_i$: $[s_i]_{t-t_i+1} = s(t)$, $t \in T_i$, $i \in \mathcal{M}$
- Data matrix, $S_i$: $[S_i]_{t-t_i+1,q} = s(t-q)$, $t \in T_i$, $q \in Q_i$, $i \in \mathcal{M}$
- Source parameters, $b = \{b_i, i \in \mathcal{M}\}$, $[b_i]_q = b_i(q)$, $q \in Q_i$
- Excitation Variances, $\sigma = \{\sigma_i^2, i \in \mathcal{M}\}$
- All source and channel parameters, $\theta = \{a, \sigma, b\}$
Likelihood Function

The likelihood function for the observed signal, $x$, is:

$$p(x | \theta, \phi, \mathcal{I}) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi}\sigma_i)^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}$$

Other parameters: $\phi = \{\tau, \Xi, \delta, \nu, \gamma\}$
Likelihood Function

The likelihood function for the observed signal, $x$, is:

$$p(x | \theta, \phi, \mathcal{I}) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi}\sigma_i)^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}$$

- Other parameters: $\phi = \{\tau, \Xi, \delta, \nu, \gamma\}$
- Vector of changepoints: $\tau = \{t_i, i \in \mathcal{M}\}$
Likelihood Function

The likelihood function for the observed signal, \( x \), is:

\[
p(x \mid \theta, \phi, \mathcal{I}) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi\sigma_i})^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}
\]

- Other parameters: \( \phi = \{\tau, \Xi, \delta, \nu, \gamma\} \)
- Vector of changepoints: \( \tau = \{t_i, i \in \mathcal{M}\} \)
- Vector of model orders: \( \Xi = \{Q_i, i \in \mathcal{M}\} \)
Likelihood Function

The likelihood function for the observed signal, $x$, is:

$$p(x | \theta, \phi, \mathcal{I}) = \prod_{i=1}^{M} \frac{1}{(2\pi \sigma_i T_i)^{\frac{3}{2}}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}$$

Other parameters: $\phi = \{\tau, \Xi, \delta, \nu, \gamma\}$

- vector of changepoints: $\tau = \{t_i, i \in \mathcal{M}\}$
- vector of model orders: $\Xi = \{Q_i, i \in \mathcal{M}\}$
- vectors of hyperparameters: $\delta = \{\delta_i, i \in \mathcal{M}\}$
Likelihood Function

The likelihood function for the observed signal, $x$, is:

$$p(x \mid \theta, \phi, \mathcal{I}) = \prod_{i=1}^{M} \frac{1}{(\sqrt{2\pi \sigma_i})^{T_i}} \exp \left\{ -\frac{(s_i + S_i b_i)^T (s_i + S_i b_i)}{2\sigma_i^2} \right\}$$

- Other parameters: $\phi = \{\tau, \Xi, \delta, \nu, \gamma\}$
  - vector of changepoints: $\tau = \{t_i, i \in \mathcal{M}\}$
  - vector of model orders: $\Xi = \{Q_i, i \in \mathcal{M}\}$
  - vectors of hyperparameters: $\delta = \{\delta_i, i \in \mathcal{M}\}$
  - vectors of hyper-hyperparameters: $\nu = \{\nu_i, i \in \mathcal{M}\}$, and $\gamma = \{\gamma_i, i \in \mathcal{M}\}$
Posterior Distribution for System

Apply Bayes’s rule to obtain the posterior pdf for the unknown parameters $\theta$ (assuming $\phi$ is known):

$$p(\theta \mid x, \phi, I) \propto p(x \mid \theta, \phi, I) \ p(\theta \mid \phi, I)$$
Posterior Distribution for System

- Apply Bayes’s rule to obtain the posterior pdf for the unknown parameters $\theta$ (assuming $\phi$ is known):

$$p(\theta \mid x, \phi, I) \propto p(x \mid \theta, \phi, I) p(\theta \mid \phi, I)$$

- Assuming $\{b_j, \sigma_j\}$ are independent between blocks, the assigned priors are:

$$b_j \mid \sigma_j^2 \sim \mathcal{N}(0, \sigma_j^2 \delta_j^2 I_{Q_j}), \delta_j \in \mathbb{R}^+ \text{ and } \sigma_j^2 \sim \mathcal{IG}\left(\frac{\nu_j}{2}, \frac{\gamma_j}{2}\right)$$

- Hence:

$$p(\theta \mid \phi, I) = p(a \mid \phi, I) p(b \mid \sigma, \phi, I) p(\sigma \mid \phi, I)$$
Only interested in estimating the channel, \( a \), so marginalise the *nuisance* parameters \( b \) and \( \sigma \) by integrating over \( \theta_{-a} \):

\[
p(\theta_{-b} | x) = \int_{\mathbb{R}^{Q_1}} \cdots \int_{\mathbb{R}^{Q_M}} p(\theta_{-b}, b | x) \, db_M \cdots db_1
\]

\[
p(a | x) = \int_{0}^{\infty} \cdots \int_{0}^{\infty} p(a, \sigma | x) \, d\sigma^2_M \cdots d\sigma^2_1
\]
Posterior Distribution for Channel

Yields the posterior density for the channel parameters $a$:

$$p(a | x, \phi, I) \propto p(a | \phi, I)$$

$$\times \prod_{i=1}^{M} \left\{ \gamma_i + s_i^T s_i - s_i^T S_i (S_i^T S_i + \delta_i^{-2} I_{Q_i})^{-1} S_i^T s_i \right\}^{-R_i}$$

where $R_i = \frac{T_i + \nu_i + 1}{2}$, $i \in M$

Written in terms of $s(t)$ to emphasise that it can be efficiently calculated by ‘inverse filtering’ the data, $x(t)$

MMAP estimate used i.e. $\arg_{a} \max p(a | x, \phi, P, I)$
Principle Revisited

\[ P = 2, \quad Q_i = 2, \quad N = 10 \quad \text{and} \quad T_i = 1000, \quad \forall i \in \mathcal{M} \]

- Phase and magnitude of the pole locations for this BSAR(2) process change linearly with block number
Principle Revisited

\[ P = 2, \quad Q_i = 2, \quad N = 10 \quad \text{and} \quad T_i = 1000, \quad \forall i \in M \]

- Phase and magnitude of the pole locations for this BSAR(2) process change linearly with block number

\[ \ln \hat{\rho}_1 (r_a | x_1, x_0, \phi, \mathcal{I}) \]
Principle Revisited

\[ P = 2, \quad Q_i = 2, \quad N = 10 \quad \text{and} \quad T_i = 1000, \ \forall i \in \mathcal{M} \]

- Phase and magnitude of the pole locations for this BSAR(2) process change linearly with block number

\[
\ln \hat{p}_7(\mathbf{r}_a | \mathbf{x}_7, \mathbf{x}_6, \phi, \mathcal{I})
\]
Principle Revisited

\[ P = 2, \quad Q_i = 2, \quad N = 10 \quad \text{and} \quad \quad T_i = 1000, \quad \forall i \in \mathcal{M} \]

- Phase and magnitude of the pole locations for this BSAR(2) process change linearly with block number

\[ \ln \hat{p}(r_a | \mathbf{x}, \phi, \mathcal{T}) \]
A simple acoustic environment

\[ \nu_i = \gamma_i = 0, \delta_i \sim 10^6, Q_i = 100, T_i = 1000 \text{ and } M = 100 \]