Lecture 2
The NAIRU model

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Lecture outline:

**The inflation-unemployment trade-off**
- A simple model
- The Phillips curve

**The NAIRU**
- Consequences of macroeconomic policy

**Nominal rigidities: the critique by Friedman and Lucas**

**Unemployment persistence and hysteresis**

**Next time**
A simple model

The economy comprises three ’goods’: labour – offered by households and utilized by firm in production; a good, representing all goods and services produced by firms and consumed by households; and money – the numeraire, which is storable and created by the state to serve as a medium of exchange.

Let $y_t$ be the logarithm of the aggregate output, $m_t$ – log of money supply and $p_t$ the log of price level. Equality of supply and demand in the goods and services market yields:

$$y_t = m_t - p_t.$$  (1)
A simple model

Firms produce with a CRS technology, represented by the following production function:

\[ y_t = a_t + l_t, \]  

where \( a_t > 0 \) denotes productivity parameter and \( l_t \) stands for the log of employment.

The prices are set by firms, which are assumed to have some market power (prices is marginal costs multiplied by a mark-up). Denoting the log of nominal wages by \( w_t \) we get:

\[ p_t = w_t - a_t + \mu, \]  

where \( \mu \) increases with the market power of firms and with costs of capital and payroll taxes.
A simple model

The log of labour supply of households, denoted by $l^s_t$ is an increasing function of real wages:

$$l^s_t = \bar{l} + \eta(w_t - p_t).$$  \hspace{1cm} (4)

$\bar{l}$ and $\eta$ are assumed constant. These four equations have five unknowns: $l_t, l^s_t, y_t, p_t$ and $w_t$. Therefore one equation needed to determine the equilibrium values of unknowns is missing.
Equilibrium: the classical approach

- In the classical view: real wages maintain the labour market equilibrium at full employment. The equation to close the model is then $l_t = l_t^s$ for all $t$

- The price rule (3) and labour supply (4) make it possible to find the equilibrium employment $l_t^*$ as:

$$l_t^* = \bar{l} + \eta(a_t - \mu).$$

(5)

All other unknowns are then determined.
Features of the classical equilibrium

- No voluntary unemployment (contradicts with empirical observations)
- Flexible prices (short run vs. the long run)
- Flexible real wages (classical model predicts too much volatility)
- Neutrality of money (money expansion has no real effect on real wages, employment and production)
Keynesian approach: inflation-unemployment trade-off

- Nominal wage is rigid in the short run
- Therefore there is not necessarily equilibrium between labour supply and demand at every instant
- The process of wage adjustment is represented by the Phillips relation
- To describe functioning of the labour market, earliest Keynesian works adopted a process of wage formation that depicted a negative relation between the rate of growth of nominal wage and the unemployment rate (Phillips curve)
- The simplest interpretation of this curve is to consider that unemployment exerts downward pressure on nominal wages
- When there are few unemployed, workers can obtain higher wages, because of higher competition between employers
The Phillips curve

- For the sake of simplicity, we assume that the labour supply is inelastic. If we set $\eta = 0$ in equation (4), labour supply is equal to constant $\bar{l}$
- We also assume that a part of this supply is not satisfied at current wage, while all firms are on their labour demand lines. Therefore $l_t < \bar{l}$
- Let $u_t$ be unemployment rate. If it is close to zero, we have:
  
  $u_t = (\bar{L} - L_t)/\bar{L} \approx \log(\bar{L}/L_t)$, so that we can write:

  $$u_t = \bar{l} - l_t.$$  (6)
The Phillips curve

- In the Keynesian model based on Phillips curve, unemployment comes from the fact that nominal wages do not react immediately in such a way as to close the gap between supply and demand in the labour market.
- Yet, nominal wages are not totally rigid, because the Phillips curve postulates a negative relation between the growth rate of wages and unemployment.
- In this setting, the unemployment rate is not the only variable capable of guiding movements in nominal wages. In many countries they are indexed to inflation.
- Adding inflation rate to Phillips curve yields so called – 'augmented' Phillips curve.
The Phillips curve

- Basing on many empirical works we can write the Phillips curve in the following form:

\[ \Delta w_t = \lambda_0 + (1 - \lambda_1) \Delta p_t + \lambda_1 \Delta p_{t-1} - \lambda_2 u_t + \lambda_3 \Delta a_t \]  

- \( \Delta w_t \) – growth rate of nominal wages
- \( \Delta p_t \) – growth rate of prices (inflation)
- \( \Delta a_t \) – growth rate of productivity
- This specification allows for study of nominal and real rigidities
Nominal rigidities

\[ \Delta w_t = \lambda_0 + (1 - \lambda_1) \Delta p_t + \lambda_1 \Delta p_{t-1} - \lambda_2 u_t + \lambda_3 \Delta a_t \]

- The notion of nominal rigidity refers to the degree to which nominal wages are sensitive to movements in the price. What may be reasons for nominal rigidity?
- money illusion
- costs of negotiating wage contracts, which prevent wages from being perfectly indexed to prices
- In equation (7), parameter \( \lambda_1 \), representing the average length of time that wage adjustment takes, supplies the measure of the degree of nominal rigidity
- If it is close to unity, the degree of nominal rigidity is high in the sense that an increase in current inflation entails only a small adjustment of wages in the period
Real rigidities

In order to grasp real rigidity it is more convenient to rewrite equation (7) in the form:

\[ \Delta(w_t - p_t) = \lambda_0 - \lambda_1(\Delta p_t - \Delta p_{t-1}) - \lambda_2 u_t + \lambda_3 \Delta a_t \quad (8) \]

- Real rigidity portrays the reaction of the real wages growth to the level of unemployment.
- Looking at our equation, we can observe that the influence of the unemployment rate on real wage growth increases with \( \lambda_2 \).
- That’s why we can consider \( 1/\lambda_2 \) as a measure of the degree of real rigidity.
- Finally, parameter \( \lambda_3 \), generally lying between 0 and 1, represents the degree to which real wages are indexed to productivity gains.
The NAIRU

The Keynesian model comprises five unknowns: $l_t, u_t, y_t, w_t$ and $p_t$. The equilibrium values of them are obtained by solving five equations: (1), (2), (3), (6) and (7). The price setting rule (3) and the Phillips curve (7) allow for defining a relationship between unemployment rate and inflation. This in turn allows for writing a relation between variation in the inflation rate and the unemployment level in the following form:

$$\lambda_1(\Delta p_t - \Delta p_{t-1}) = \lambda_0 - \lambda_2 u_t - (1 - \lambda_3) \Delta a_t. \quad (9)$$

This equation allows us to define the unemployment rate $\overline{u}_t$ compatible with constant inflation rate ($\Delta p_t - \Delta p_{t-1} = 0$).
The NAIRU

This unemployment rate $\bar{u}_t$, compatible with constant inflation rate ($\Delta p_t - \Delta p_{t-1} = 0$) is commonly called the NAIRU, which is *Non-Accelerating Inflation Rate of Unemployment*.

It is sometimes called the natural rate of unemployment or equilibrium unemployment, since it also represents the long-run equilibrium value of unemployment rate. Setting $\Delta p_t - \Delta p_{t-1} = 0$ in equation (9) gives:

$$\bar{u}_t = \frac{\lambda_0 - (1 - \lambda_3)\Delta a_t}{\lambda_2}.$$  (10)
The NAIRU

\[ \bar{u}_t = \frac{\lambda_0 - (1 - \lambda_3) \Delta a_t}{\lambda_2}. \] (10)

It appears that NAIRU increases with the degree of real rigidity \((1/\lambda_2)\) and that it depends on the rate of growth – not the level – of productivity. If nominal wages are not perfectly indexed to productivity gains (so that \(0 \leq \lambda_3 < 1\)), a slowing of productivity growth (negative \(\Delta a_t\)) will result in an increase in NAIRU.

Bringing the value of \(\bar{u}_t\) into the Phillips curve (9), we can write a new form of Phillips curve, linking current unemployment, the NAIRU and the acceleration of inflation.
The NAIRU

\[ u_t = \bar{u}_t - \frac{\lambda_1}{\lambda_2} (\Delta p_t - \Delta p_{t-1}) \] \hspace{1cm} (11)

In the absence of nominal rigidity \((\lambda_1 = 0)\), the current unemployment rate would always equal NAIRU. Conversely, when \(\lambda_1 > 0\), the current unemployment rate is lower than the NAIRU only if inflation rate increases \((\Delta p_t > \Delta p_{t-1})\).
The NAIRU

\[ u_t = \bar{u}_t - \frac{\lambda_1}{\lambda_2} (\Delta p_t - \Delta p_{t-1}) \]  

Equation (11) shows that the unemployment rate can only be lowered by an increase in the inflation rate. Conversely, it is evident that a reduction in the inflation rate must necessarily lead to a transitory increase in unemployment. From this perspective, the ratio \( \lambda_1/\lambda_2 \), called the sacrifice ratio, measures the increase in the unemployment that is needed to reduce inflation by one percentage point. The stronger the nominal and real rigidities are, the greater this ratio is.
Consequences of macroeconomic policy

Macroeconomic policies that act on aggregate demand, should have in principle little effect on the long-run employment, but they might have a positive impact in the short run.

Conversely, policies that act on supply side have structural effects that alter long-run equilibrium of the labour market.
Demand side policies

We shall compare the properties of the long-run and the short-run equilibria. For simplicity we assume that productivity and money supply grow at a constant rate ($\Delta a_t = \Delta a$ and $\Delta m_t = \Delta m$ for all $t$). This assumption means that NAIRU remains constant at the level $\overline{u}$, given by equation (10).

The new form of Phillips curve (11) yields first interesting relationship between the inflation rate and unemployment rate. Using (1) and (2), we find equation defining employment as a function of aggregate demand:

$$l_t = m_t - a_t - p_t.$$
Demand side policies

Applying the difference operator and defining $\pi = \Delta m - \Delta a$ as the stationary inflation rate, we get $\Delta p_t = \pi - \Delta l_t$. Assuming that the labour force is constant, relation (6) between unemployment rate and employment becomes: $\Delta u_t = -\Delta l_t$. In sum, we get a new version of aggregate demand function that directly ties the inflation rate to the unemployment rate:

$$\Delta p_t = \pi + u_t - u_{t-1}. \quad (12)$$

At date $t$, the variables inherited from the past: $u_{t-1}$ and $\Delta p_{t-1}$ are known and the short-run equilibrium values of the unemployment rate and inflation rate correspond to the intersection of the two curves defined by relations (11) and (12) for given $u_{t-1}$ and $\Delta p_{t-1}$.
Demand side policies

The Phillips curve described by (11), reflects the mode of the wage setting ($WS$ line). Relation (12) portrays the mechanism of price setting ($PS$ line). For given values of $u_{t-1}$ and $\Delta p_{t-1}$ we obtain short-run lines $WS_t$ and $PS_t$. Therefore the short-run equilibrium lies on the intersection of these two lines.

Long-run equilibrium is obtained by looking for stationary values of unemployment rate and inflation rate. Setting $u_t = u_{t-1} = u$ and $\Delta p_t = \Delta p_{t-1} = \Delta p$ in (11) and (12), we find that $u = \bar{u}$ and $\Delta p = \pi = \Delta m - \Delta a$. Therefore long-run $WS$ line is vertical and $PS$ line is horizontal.
Figure 1. Short-run vs long-run equilibrium
Short-run vs long-run equilibrium

In the short run, the economy may operate with unemployment rate that is lower than the NAIRU, but only in case of increasing inflation. We see it in our graph by having $\Delta p_t > \Delta p_{t-1}$.

However in the long run the economy stabilizes at the long-run stationary value of inflation, which is $\pi = \Delta m - \Delta a$, and with unemployment rate equal $\bar{u}$. The vertical line $WS$ indicates that there is no more trade-off between inflation and unemployment rate in the long run, and this represents the long-run Phillips curve.
Short-run vs long-run equilibrium

Increasing aggregate demand in the long run is ineffective, since the long-run equilibrium value of unemployment (NAIRU) depends exclusively on the structural components of the economy. But what about the short run?

Suppose the economy initially \((t = 0)\) is in the steady-state equilibrium with inflation \(\pi\) corresponding to the growth of money supply \(\Delta m\). Assume that in some point in time \((t=1)\) government decides to permanently increase the rate of growth of money supply to \(\Delta m' > \Delta m\). Using equations (11) and (12) we can calculate new values of inflation \(\Delta p_1\) and unemployment rate \(u_1\) for that \(t = 1\) period.
Short-run vs long-run equilibrium

Setting $\pi' = \Delta m' - \Delta a$ we get:

$$\Delta p_1 = \frac{\pi' + \frac{\lambda_1}{\lambda_2} \pi}{1 + \frac{\lambda_1}{\lambda_2}} > \pi$$

and

$$u_1 = \bar{u} + \frac{\Delta m - \Delta m'}{1 + \frac{\lambda_2}{\lambda_1}} < \bar{u}. \quad (13)$$

This has very interesting implications!
Short-run vs long-run equilibrium

We can see that the impact of permanent increase of the money supply growth rate on inflation is greater if the nominal rigidity is low. At the limit, with no nominal rigidity, i.e. if $\lambda_1 \to 0$, the inflation rate jumps immediately to its new stationary value $\pi'$. But the unemployment rate would be $u_1 = \overline{u}$!

If we observe that there is nominal rigidity, i.e. $\lambda_1 > 0$, monetary expansion has significant impact on unemployment (it lowers it below the natural rate) in the short run.
Transitory dynamics

Transitory dynamics are governed by the following two equations:

\[ u_t = \frac{\overline{u_t} - \frac{\lambda_1}{\lambda_2} (\Delta m_t - \Delta a_t - u_{t-1} - \Delta p_{t-1})}{1 + \frac{\lambda_1}{\lambda_2}} \]

\[ \Delta p_t = \Delta m - \Delta a + u_t - u_{t-1} \]
Transitory dynamics

Suppose we have regressed the Phillips curve obtaining results for US economy: $\lambda_0 = 0.03$, $\lambda_1 = 0.46$, $\lambda_2 = 0.34$, $\lambda_3 = 0.38$.

Suppose that money supply ($\Delta m$) grows at a constant rate of 3% each year. Assume that productivity ($\Delta a$) grows by 1% each year.

What will be the effect of a permanent increase of money supply growth rate to 4%? According to estimates, stationary inflation rate will change from 2% to 3%. Unemployment rate in the long run will remain however constant, equal to 7%. In the short run, demand shock pushes inflation and unemployment in opposite directions.
Transitory dynamics – monetary expansion
Transitory dynamics – monetary expansion
Supply side policies

We have just seen, that demand side policies have only transitory impact on unemployment as in the long run, NAIRU is not affected by policies or shocks of this type.

On the contrary, shocks or policies affecting supply can have important influence on the NAIRU. In particular, the slowdown of productivity growth in the beginning of the 1970s exerted upward pressure on the natural rate of unemployment. This contributed to both temporary increase of unemployment rate and inflation rate.
Supply side policies

Assume in period 0 we have growth rate of productivity at 2% (average for 1960-1973). Then in period 1 productivity growth decreases to 1% (average for 1974-1998). The slowdown of productivity changes the NAIRU according to (10) from initially 5.2% to 7%. Assume that money supply growth is constant at 4%. This means that the slowdown of productivity changes stationary inflation rate from 2% to 3%.

Transitional dynamics are shown on the next slide. Supply shocks push inflation and unemployment in the short run in the same direction.
Transitory dynamics – negative supply shock
Transitory dynamics – negative supply shock
Demand and supply shocks

▶ 1960s and 1970s, demand was stoked up (in part because of Vietnam War), $\Delta m$ rose, inflation increased and unemployment fell

▶ Oil shock – NAIRU rose – both inflation and unemployment rose

▶ Inflation and unemployment move in opposite direction when there is a demand shock

▶ With a supply shock – they move together
Friedman (1968) and Lucas (1972) sought to establish that the Phillips curve was compatible with competitive functioning of the labour market in which agents observe prices imperfectly. That being the case, wage earners are incapable of correctly assessing their real wage, and there can be a lag in the adjustment of nominal wages when prices change. The essential contribution of this approach was to show that the impact of demand side policies is conditioned by expectations of agents. Assuming that agents have adaptive expectations of the inflation rate, Friedman (1968) emphasized that the real wage ought to be perfectly indexed to the general level of processes in the long run. That makes it possible to account for the inefficiency of macroeconomic policies at that horizon.
Nominal rigidities: the critique by Friedman and Lucas

Friedman’s message was stated even more radically by Lucas (1972). Adopting the hypothesis of rational expectations, he showed that demand side policies, when systematically applied, and thus 'foreseen' by agents, have no real effect even in the short run. Only unexpected demand side policies have real effects in the short run.

The idea is that agents learn how the economic system works, and after some time they make no systematic error in forecasting.
Unemployment persistence and hysteresis

Some economists have suggested that the labor market exhibits a form of 'hysteresis' (Blanchard and Summers, 1986). In physics, hysteresis refers to the failure of an object to return to its original value after being changed by an external force, even after the force is removed. In the labor market, a similar phenomenon would arise if the natural rate $\bar{u}$ depended on past unemployment $u$. In this case, a change in aggregate demand would first influence unemployment by causing $u$ to deviate from $\bar{u}$, but then would have a persistent effect on unemployment as $\bar{u}$ changed.
Unemployment persistence and hysteresis

Several theories have been proposed to explain why this might be the case. The most popular emphasize long-lasting damage suffered by workers who experience unemployment. These workers lose human capital, become less attractive to employers, and reduce their job search as they become accustomed to being unemployed (Layard et al., 1991). All these effects make workers less likely to be employed in the future. A recession that raises unemployment leaves a permanent scar on the economy, as $\bar{u}$ is higher even after the initial shock that caused the recession has disappeared. These theories of hysteresis were first developed to explain the large rise in the NAIRU in Europe during the 1980s: the increase in $u^*$ came immediately after the disinflationary recession that started the decade.
Unemployment persistence and hysteresis

Let us adopt the idea of hysteresis to our model. If the long-term unemployed become 'unemployable', it is like they were no longer participating in the labour market. Only the number of short-term unemployed will have an influence on wage variations. Assume that a fall or rise in wages depends both on the level of unemployment and the variation of unemployment rate. The latter is simply the indicator of short term unemployment. The Phillips curve (7) than becomes:

\[
\Delta w_t = \lambda_0 + (1 - \lambda_1)\Delta p_t + \lambda_1\Delta p_{t-1} - \lambda_2 u_t - \lambda_2' \Delta u_t + \lambda_3 a_t \tag{14}
\]
Since, following (3) we always have $\Delta p_t = \Delta w_t - \Delta a_t$, we get:

$$\lambda_1 (\Delta p_t - \Delta p_{t-1}) = \lambda_2 (\bar{u}_t - u_t) - \lambda'_2 \Delta u_t$$

(15)

where $\bar{u}_t = \frac{\lambda_0 - (1-\lambda_3) \Delta a_t}{\lambda_2}$ is the long-run NAIRU. If we take the Phillips curve given by (15), we can calculate so called short-run or instantaneous NAIRU ($\hat{u}_t$), given by setting $\Delta p_t = \Delta p_{t-1}$:

$$\hat{u}_t = \frac{\lambda_2}{\lambda_2 + \lambda'_2} \bar{u}_t + \frac{\lambda'_2}{\lambda_2 + \lambda'_2} u_{t-1}.$$  

(16)
Unemployment persistence and hysteresis

\[ \hat{u}_t = \frac{\lambda_2}{\lambda_2 + \lambda_2'} u_t + \frac{\lambda_2'}{\lambda_2 + \lambda_2'} u_{t-1} \]

This \( \hat{u}_t \) lies between last period’s unemployment and the long-run NAIRU \( \bar{u}_t \). The higher the effect \( (\lambda_2') \) of the change in unemployment relative to the effect \( (\lambda_2) \) of the level, the nearer the short-run NAIRU to the last period’s unemployment. This implies strong persistence of unemployment.
Transitory dynamics with hysteresis – negative supply shock

\( \Delta m = 4\% \), \( \Delta a = 2\% \) then decreases to 1\% for 10 periods and goes back to 2\%. Other variables as before.

![Graph showing time series data with different values of lambda_2']
Transitory dynamics with hysteresis – negative supply shock

\[ \lambda_0 = 0.03, \ \lambda_1 = 0.48, \ \lambda_2 = 0.1, \ \lambda_3 = 0.38, \ \Delta m = 4\%, \ \Delta a = 2\% \]
then decreases to 1\% for 10 periods, increases back to 2\% for next 10 periods and then goes back to 2\%.
Unemployment persistence and hysteresis

- In terms of policy, hysteresis means that once unemployment has risen, it cannot be brought back at once to the long-run NAIRU without a permanent rise in inflation.
- But it can be reduced gradually with inflation rising.
- It helps us to understand why inflation didn’t fell in Europe in the late 1980s.
- Another effect: extra unemployment has a smaller effect on wages when unemployment is already high.
- Large extra unemployment in 1980s had a small deflationary effect.
- By contrast, small excess demand of the early 1970s produced a large increase in inflation.
Next time

- Basic model of efficiency wages
  - Why do firms pay higher wages than the value of marginal product of labor?