The Solow model

1. Consider the following statement: 'According to the Solow model higher consumption rate implies automatically translate to higher consumption in the future. Increasing the consumption rate would not have any important effects on the economy.' Explain why you agree or disagree with this statement.

2. We saw earlier that an increase in the saving rate can lead to a recession in the short run (i.e. the paradox of saving). We examined the issue in the medium run according to the IS-LM-PC model. We can now investigate the long-run effects of an increase in savings. Using the Solow model, what is the impact of an increase in the saving rate on output per worker likely to be after one decade? After five decades?

3. Discuss how the level of output per person, in the long run, would likely be affected by each of the following changes:
    a. The right to exclude saving from income when paying income taxes.
    b. A higher female participation rate in the labour market (but constant population).

4. Suppose the United States moved from the current pay-as-you-go social security system to a fully funded one and financed the transition without additional government borrowing. How would the shift to a fully funded system affect the level and the rate of output growth per worker in the long run?

5. Suppose that the production function is given by: $Y = 0.5\sqrt{K}\sqrt{N}$
    a. Derive the steady-state levels of output per worker and capital per worker in terms of the saving rate, s, the depreciation rate, δ.
    b. Derive the equation for steady-state output per worker and steady-state consumption per worker in terms of s and δ.
    c. Suppose that d = 0.05. With your favourite spreadsheet, compute steady-state output per worker and steady-state consumption per worker for s = 0; s = 0.1; s = 0.2; s = 1. Explain the intuition behind your results.
    d. Use your favourite spreadsheet to graph the steady-state level of output per worker and the steady-state level of consumption per worker as a function of the saving rate (i.e. measure the saving rate on the horizontal axis of your graph and the corresponding values of output per worker and consumption per worker on the vertical axis).
    e. Does the graph show that there is a value of s that maximises output per worker? Does the graph show that there is a value of s that maximises consumption per worker? If so, what is this value?

6. The Cobb–Douglas production function and the steady state. Suppose that the economy's production function is given by:

$$Y = K^{\alpha}L^{1-\alpha}$$, and assume that α = 1/3.

   a. Is this production function characterised by constant returns to scale? Explain.
   b. Are there decreasing returns to capital?
   c. Are there decreasing returns to labour?
   d. Transform the production function into a relation between output per worker and capital per worker.
   e. For a given saving rate, s, and depreciation rate, d, provide an expression for capital per worker in the steady state.
   f. Give an expression for output per worker in the steady state.
   g. Solve for the steady-state level of output per worker when s = 0.32 and d = 0.08.
   h. Suppose that the depreciation rate remains constant at d = 0.08, while the saving rate is reduced by half, to s = 0.16. What is the new steady-state output per worker?

7. Continuing with the logic from Problem 6, suppose that the economy's production function is given by $Y = K^{\frac{1}{3}}N^{\frac{2}{3}}K$ and that both the saving rate, s, and the depreciation rate, d, are equal to 0.10.
   a. What is the steady-state level of capital per worker?
   b. What is the steady-state level of output per worker? Suppose that the economy is in a steady state and that, in period t, the depreciation rate increases permanently from 0.10 to 0.20.
   c. What will be the new steady-state levels of capital per worker and output per worker?
   d. Compute the path of capital per worker and output per worker over the first three periods after the change in the depreciation rate.
   e.

ADDITIONAL QUESTIONS:
Label each of the following statements as true, false or uncertain. Explain briefly.

1. The saving rate is always equal to the investment rate.
2. A higher investment rate can sustain higher output growth forever.
3. If capital never depreciated, growth could go on forever.
4. The higher the saving rate, the higher consumption in a steady state.
5. We should transform social security from a pay-as-you-go system to a fully funded system. This would increase consumption both now and in the future.
6. The US capital stock is far below the golden-rule level. The government should give savings tax breaks because the US capital stock is far below the golden-rule level.
7. Education increases human capital and thus output. It follows that governments should subsidise education.