## Dynamic Panel Data Part One

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## Stata przypomnienie

- Penn World Table
- PWT 6.3
- Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.3, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, August 2009.
- http://pwt.econ.upenn.edu/php_site/pwt_index.php


## Stata przypomnienie

pwt63.dta na stronie.
use "pwt63.dta", clear
Countries as a string converted into double:
encode country, generate(cty)
Panel dimension:
xtset cty year

## Stata przypomnienie

Population growth rate:
g popg=(pop-l.pop)/l.pop
Jump variable for every 5 years: gen num5=int((year-1950)/5+1)

## Stata przypomnienie

Mean averaging for every 5 years and collapsing:
collapse (mean) rgdpch ki grgdpch popg, by(cty num5)
New panel:
xtset cty num5
Decode:
decode cty, generate(Country)

## Stata przypomnienie

Logarithm of GDP:
g lpkb= log(rgdpch)
Growth rate:
g dpkb= $\log ($ rgdpch $)-\log (l . r g d p c h)$
Technological progress rate plus population growth rate plus depreciation rate:
g pop=popg+0.07

```
Contains data
    obs: 2,280
        vars: 15
size: 223,440 (99.9% of memory free) (_dta has notes)
storage display value
variable name type format label variable label
\begin{tabular}{|c|c|c|c|c|}
\hline cty & long & \(\% 24.0 \mathrm{~g}\) & Kraj & Country \\
\hline num5 & float & \(\% 9.0 \mathrm{~g}\) & & \\
\hline rgdpch & double & \(\% 10.0 \mathrm{~g}\) & & (mean) rgdpch \\
\hline ki & double & \(\% 10.0 \mathrm{~g}\) & & (mean) ki \\
\hline grgdpch & double & \(\% 10.0 \mathrm{~g}\) & & (mean) grgdpch \\
\hline popg & float & \(\because 9.0 \mathrm{~g}\) & & (mean) popg \\
\hline Country & str24 & \(\because 24 \mathrm{~s}\) & & Country \\
\hline lpkb & float & \(\% 9.0 \mathrm{~g}\) & & \\
\hline dpkb & float & \(\div 9.0 \mathrm{~g}\) & & \\
\hline pop & float & \(\because 9.0 \mathrm{~g}\) & & \\
\hline lagpkp & float & \(\div 9.0 \mathrm{~g}\) & & \\
\hline lpop & double & \(\% 10.0 \mathrm{~g}\) & & \\
\hline lki & double & \(\% 10.0 \mathrm{~g}\) & & \\
\hline _est_rDFE & byte & \(\% 8.0 \mathrm{~g}\) & & esample() from \\
\hline est DFE & byte & \%8.0g & & esample() from \\
\hline
\end{tabular}
```

Sorted by cty num5
Note: data has changed since last saved

| Variable \| | Obs | Mean | Std. Dev. | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| cty \| | 2280 | 95.5 | 54.85955 | 1 | 190 |
| num5 \| | 2280 | 6.5 | 3.45281 | 1 | 12 |
| rgdpch \| | 1780 | 8804.549 | 10935.33 | 279.9274 | 90095.22 |
| ki \| | 1780 | 21.46645 | 12.74437 | -. 7724877 | 91.59714 |
| grgdpch \| | 1769 | 2.267196 | 4.504332 | -31.73853 | 49.83998 |
| popg \| | 2280 | . 0191774 | . 0142226 | -. 0358202 | . 1578099 |
| Country \| | 0 |  |  |  |  |
| lpkb \| | 1780 | 8.465461 | 1.136879 | 5.634531 | 11.40862 |
| dpkb \| | 1590 | . 0916539 | . 1695351 | -1.380931 | 1.258495 |
| pop \| | 2280 | . 0891774 | . 0142226 | . 0341798 | . 2278099 |

(już z wygenerowanymi, przekształconymi zmiennymi)
xtdescribe

```
    cty: 1, 2, ..., 190
num5: 1, 2, ..., 12
Delta(num5) = 1 unit
Span(num5) = 12 periods
(cty*num5 uniquely identifies each observation)
```

n $=$
190
$\mathrm{T}=$
12

-xtline dpkb if cty<=10, overlay

xtdescribe

```
    cty: 1, 2, ..., 190
num5: 1, 2, ..., 12
Delta(num5) = 1 unit
Span(num5) = 12 periods
(cty*num5 uniquely identifies each observation)
```

n $=$
190
$\mathrm{T}=$
12


## Cross section 1955-1960

- reg dpk.b l.lpk.b pop ki if num5==2



## Cross section 2000-2005

. reg dpkb l.lpk.b pop ki if num5==12

| Source \| | SS | df | MS |
| ---: | :---: | ---: | ---: |
| Model \| | .173363788 | 3 | .057787929 |
| Residual \| | 3.20406699 | 184 | .017413408 |
| Total \| | 3.37743078 | 187 | .018061127 |

## Pooled regression for every cross section:

. reg dpkb l.lpkb pop ki

| Source | SS | df MS |  |  | Number of obs $=1590$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 3, 1586) | $=$ | 46.18 |
| Model | 3.6690368 | 31.2 | 01227 |  | Prob > F | $=$ | 0.0000 |
| Residual | 42.0022271 | 1586.02 | 83119 |  | R-squared | $=0.0803$ |  |
|  |  |  |  |  | Adj R-squared <br> Root MSE | $=0.0786$ |  |
| Total | 45.6712639 | 1589.02 | 42142 |  |  | $=$ | . 16274 |
| dpkb | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. Interval] |  |  |
| lpkb |  |  |  |  |  |  |  |
| L1. | -. 0124278 | . 0040023 | -3.11 | 0.002 | -. 0202781 | -. 0045776 |  |
|  |  |  |  |  |  |  |  |
| pop | -1.427988 | . 3252937 | -4.39 | 0.000 | -2.066039 | -. 7899376 |  |
| ki | . 0033681 | . 0003479 | 9.68 | 0.000 | . 0026856 | . 0040505 |  |
| _cons | . 2498058 | . 04974715.02 |  | 0.000 | $.1522287$ | $.3473828$ |  |

## Error clustering? Double the errors.

```
regdpkbl.lpkbpop ki, vce(cluster cty)
```

Linear regression

| Number of obs | $=1590$ |
| ---: | :--- | ---: |
| $\mathrm{~F}($ 3, 187) | $=13.69$ |
| Prob $>\mathrm{F}$ | $=0.0000$ |
| R-squared | $=0.0803$ |
| Root MSE | $=.16274$ |

(Std. Err. adjusted for 188 clusters in cty)

| dpkb | Coef. | Robust Std. Err. | t | $p>\|t\|$ | [95\% Conf | Interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| lpkb \| |  |  |  |  |  |  |
| L1. | -. 0124278 | . 0062808 | -1.98 | 0.049 | -. 0248182 | -. 0000374 |
| 1 |  |  |  |  |  |  |
| pop | -1.427988 | . 6651244 | -2.15 | 0.033 | -2.7401 | -. 1158769 |
| ki | . 0033681 | . 000809 | 4.16 | 0.000 | . 0017721 | . 004964 |
| _cons | . 2498058 | . 0965514 | 2.59 | 0.010 | . 0593357 | . 4402758 |

Error clustering? Double the errors.

- Estimating the model with OLS yields biased results because the estimator is inconsistent.
- There are unobservable constant effects for countries (factors constant over time, not included in the model), which causes the dependent variable to change faster for some observation units than others.

$$
E\left[x_{i t}^{\prime}\left(\eta_{i}+v_{i}\right)\right] \neq 0
$$

## Panel data estimators

- Why cross-sectional-time estimators?
- Panel data allow to analyze the phenomenon simultaneously in time and in cross-section or spatial dimensions. These estimators allow to isolate the individual specificity of individual objects.
- The use of data panels allows for greater heterogeneity (greater diversity) of study units.
- Provides more degrees of freedom and increases the efficiency of estimation.
- Extracting periodic effects makes it easier to study the dynamics of adjustment.
- Panel data allows you to isolate the influence of unobservable variables or effects.


## Panel data estimation

## Standard panel:

$$
\Delta y_{i t}=\gamma_{t}+(\alpha-1) y_{i, t-1}+\sum_{j=1}^{k} \beta_{j} x_{i t j}+\varepsilon_{i t} \text { dla } i=1, \ldots, N \mathrm{i} t=1, \ldots, T .
$$

$$
\varepsilon_{i t}=\eta_{i}+\gamma_{t}+v_{i t}
$$

Panel estimators are more efficient over OLS because they use unused information - the panel dimension. Fixed Effects, Between Effects, Random Effects global xlist l.lpkb pop ki quietly regress dpkb \$xlist, vce(cluster cty) estimates store OLS quietly xtreg dpkb \$xlist, be estimates store BE quietly xtreg dpkb \$xlist, re vce(robust) estimates store RE quietly xtreg dpkb \$xlist, fe vce(robust) estimates store FE

## Hausmann test

hausman fe re
Strong fixed effect!
xtreg dpkbl.lpkb pop ki, fe

One way or two way?
Wald test
xtregar dpkb l.lpkb pop ki, fe rhotype(dw) lbi
xi: xtregar dpkb l.lpkb pop ki i.num5, fe rhotype(dw) lbi

```
test ( _Inum5_3 _Inum5_4 _Inum5_5 _Inum5_6 _Inum5_7
\[
\text { _Inum5_ } \overline{8} \text { _Inum5_ } \overline{9} \text { _Inum5_10 _Inum5_11 _Inum } \overline{5} \overline{1}^{-12} \text { _Inum5_2) }
\]
```


## NOTE ON THE TWO WAY MODEL IN MACROECONOMICS

ALGORITHM

- 1. Panel>MNK, very rare if not.
- 2. FE versus RE, BE - Hausmann test. Very rare if not FE.
- 3. Determine whether one or two-way model.
- Nickell (1981) - In FE there is still a correlation between the lagged dependent variable and the transformed error expression, which makes these estimators have the desired properties purely asymptotically, i.e. when the number of observations over time tends to infinity.
- This is not the case of a typical growth model where usually there are significantly less than 50 observations over time (due to averaging, it is usually 5-10 observations).
- By definition, this method limits the analysis to looking for the mean within countries, perhaps ignoring significant differences between countries.
This method does not help in any way to solve the problem of causality, measurement error and omitted variables, variables over time. It also does not allow for estimating the impact of variables that are constant over time, such as the impact of geography or history, on economic growth.

Let us move forward

## Estimators:

- Anderson-Hsiao,
- Arellano-Bond,
- Blundell-Bonda,
- PMG,
- Kiviet's.


## Differencing

$$
\Delta y_{i t}=\alpha \Delta y_{i, t-1}+\Delta \mathbf{x}_{i t}^{\prime} \boldsymbol{\beta}+\Delta \varepsilon_{i t}
$$

- Getting rid of fixed effect, not much else.
- This transformation uses $y_{i, t-1}$ and therefor causes endogeneity, because
$y_{i, t-1}$ in $\Delta y_{i, t-1}=y_{i, t-1}-y_{i, t-2}$ is correlated with $\varepsilon_{i, t-1}$ in $\Delta \varepsilon_{i t}=\varepsilon_{i t}-\varepsilon_{i, t-1}$
-However, if there is no autocorrelation, the lagged variables may be exogenous, they may be used as instruments.
- After differentiating the fixed effects, a natural estimator of the Instrumental Variable Method is available.
- We can construct instruments from the lagged dependent variable, lagged twice, three times, etc.
- The solution to the problem of measurement error and opposite causality is the 2SLS estimator by Anderson and Hsiao (1981)
- It assumes estimating the model on the first differences and using the past GDP level in the second lag as an instrument for lagging first GDP differences.


## 2SLS (Anderson i Hsiao 1981)

- Assuming the absence of AR () in $\varepsilon_{i t}$, natural instruments for $\Delta y_{i, t-1}$ are $\Delta y_{i, t-2}$ and $y_{i, t-2}$
- Very close to $\Delta y_{i, t-1}$. Maybe collinearity?
- $y_{i, t-2}$ is more sensible: starting from $t=3$
- Nevertheless, we lose a lot of observation when $T$ is small.
- Similarly, for other variables


## 2SLS (Anderson il Hsiao 1981)

ssc install xtivreg28
xi: xtivreg2 dpkb pop lki i.num5 (lpkb
= l.lpkb), fd
help xtivreg2

Estimates efficient for homoskedasticity only
Statistics consistent for homoskedasticity only
[del]


Source: Stock-Yogo (2005). Reproduced by permission.

| Sargan statistic (overidentification test of all instruments): | 0.000 |
| ---: | ---: |
| (equation exactly identified) |  |

## 2SLS (Anderson i Hsiao 1981)

- Assuming the absence of AR () in $\varepsilon_{i t}$, natural instruments for $\Delta y_{i, t-1}$ are $\Delta y_{i, t-2}$ and $y_{i, t-2}$
- Very close to $\Delta y_{i, t-1}$. Maybe collinearity?
- $y_{i, t-2}$ is more sensible: starting from $t=3$
- Nevertheless, we lose a lot of observation when $T$ is small.
- Similarly, for other variables
- It allows to isolate the part of the dependent variable variation that is not related to the opposite causality, omitted variables and the measurement error.

This method leads to consistent estimates, but they may be ineffective when the random term is nonspherical due to the lack of use of all moment conditions (Hansen, 1982).

## One option: 2SLS (Anderson il Hsiao 1981)

- Need for further lags undesirable as it:
o Reduces T.
o Problem with short panels
- After differentiation, errors not i.i.d.
o differences in errors correlated
o 2SLS ineffective



## Hansen (1982)

The sensibility of introducing an instrument in the form of a lag of the dependent variable can be written in the form of an moment identifying assumption:

$$
E\left[\left(u_{i t}-u_{i, t-1}\right) y_{i, t-2}\right]
$$

To increase the efficiency of the estimator, Arellano and Bond (1991) use all possible instruments in the form of lags and differences.
The sensibility of introducing these instruments should be written in the form of conditions related to moments, identifying assumptions that are used to build the estimator of the Generalized Method of Moments.


## Solution: IV \& GMIM instruments

## (Holtz-Eakin, Newey, and Rosen 1988)

- Use of a lot of lags. In the absence, use zero in the matrix.
- Instruments for each delay and period have been created.
Instruments IV: $\left[\begin{array}{c}. \\ \vdots \\ y_{n} \\ \vdots \\ y_{i,-2}\end{array}\right]$ GMM: $\left[\begin{array}{ccccccc}0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ y_{n} & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & y_{i n} & y_{n} & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & y_{B} & y_{i n} & y_{y_{n}} & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots\end{array}\right]$.
-Result: Arellano-Bond (1991) difference GMM


## Solution: IV \& GMM instruments (Holtz-Eakin, Newey, and Rosen 1988)

The moment conditions were created with the assumption that the lagged levels of the dependent variable are orthogonal to the differentiated shock are known as GMM moment conditions.

The moment conditions created using strictly exogenous variables are simply the standard conditions of the instrumental variables (IV) method, they are also called standard moment conditions.

## Solution: IV \& GMM instruments

- Number of instruments:
- $\mathrm{p}=\mathrm{T}-2$ (one period for differences, one for lagged difference)
- $\mathrm{k}+\mathrm{p} *(\mathrm{p}+1) / 2$
- Where k is the number of exogenous variables.


```
Instruments for differenced equation
```

```
GMM-type: L(2/.).dpkb
```

GMM-type: L(2/.).dpkb
Standard: LD.lpkb D.pop D.ki
Standard: LD.lpkb D.pop D.ki
Instruments for level equation

```

\section*{Solution: IV \& GMM instruments}
- Number of instruments:
- \(\mathrm{p}=12-3\)
- \(4+9 *(9+1) / 2\)
- \(=49\)

\section*{Problem: the errors appear too small}
\begin{tabular}{|c|c|c|}
\hline & (1) & (2) \\
\hline & step & step \\
\hline L.dpkb & \[
\begin{aligned}
& -0.0784 * * \\
& (-3.16)
\end{aligned}
\] & \[
\begin{aligned}
& -0.118 * * * \\
& (-4.50)
\end{aligned}
\] \\
\hline L2.dpkb & \[
\begin{aligned}
& -0.176 * * * \\
& (-7.27)
\end{aligned}
\] & \[
\begin{aligned}
& -0.112 * * * \\
& (-6.41)
\end{aligned}
\] \\
\hline L. lpkb & \[
\begin{aligned}
& -0.443 * * * \\
& (-25.45)
\end{aligned}
\] & \[
\begin{aligned}
& -0.444 * * * \\
& (-19.57)
\end{aligned}
\] \\
\hline pop & \[
\begin{gathered}
-0.358 \\
(-0.63)
\end{gathered}
\] & \[
\begin{gathered}
-1.103 \\
(-1.84)
\end{gathered}
\] \\
\hline ki & \[
\begin{gathered}
0.00468 * * * \\
(5.77)
\end{gathered}
\] & \[
\begin{gathered}
0.00538 * * * \\
(6.05)
\end{gathered}
\] \\
\hline cons & \[
\begin{aligned}
& 3.734 * * * \\
& (23.94)
\end{aligned}
\] & \[
\begin{aligned}
& 3.789 * * * \\
& (19.93)
\end{aligned}
\] \\
\hline N & 1026 & 1026 \\
\hline
\end{tabular}

\section*{Problem, cont"d}
- Problem appears to be one of overfitting - Efficient GMM deemphasizes moments with high variance (high second moments)
- Feasible efficient GMM in small samples may deemphasize outliers (high first moments)
- Spurious precision

\section*{Problem: finite sample}

Let us compare:
xi: xtabond dpkb l.lpkb pop ki i.num5, lags (2)
eststo AB_ONESTEP
xi: xtabond dpkb l.lpkb pop ki i.num5, lags(2) two
eststo AB TWOSTEP
xi: xtabond dpkb l.lpkb pop ki i.num5,
lags(2) two
eststo AB_TWOSTEP_WIND
esttab

\section*{Solution Windmeijer correction (2005)}

Oszacowanie 1-stopniowe: \(\hat{\boldsymbol{\beta}}_{1}=f(\mathbf{Y})\) (warunkowo względem \(\mathbf{X}\), Z)

Oszacowanie 1-stopniowe błędów do \(\hat{\boldsymbol{\Omega}}\) :
\(\hat{\boldsymbol{\beta}}_{2}=\left(\mathbf{X}^{\prime} \mathbf{Z}(\mathbf{Z} \hat{\boldsymbol{\Omega}} \mathbf{Z})^{-1} \mathbf{Z} \mathbf{X}\right)^{-1} \mathbf{X} \mathbf{Z}(\mathbf{Z} \hat{\boldsymbol{\Omega}} \mathbf{Z})^{-1} \mathbf{Z} \mathbf{Y} \equiv g(\mathbf{Y}, \hat{\boldsymbol{\Omega}}) \equiv g(\mathbf{Y}, f(\mathbf{Y}))\)
Standardowe oszacowanie \(\operatorname{Var}\left\lfloor\hat{\boldsymbol{\beta}}_{2}\right\rfloor_{\text {uznaje }} \hat{\boldsymbol{\Omega}}\) za stałą, obserwowaną i dokładną - pomimo zależności od losowego \(\mathbf{Y}\) Roszerzenie Taylora \(g\) wokół prawdziwego \(\boldsymbol{\beta}\) :
\[
\hat{\boldsymbol{\beta}}_{2}=g\left(\mathbf{Y}, \hat{\boldsymbol{\Omega}}_{\hat{\boldsymbol{\beta}}_{1}}\right) \approx g\left(\mathbf{Y}, \hat{\boldsymbol{\Omega}}_{\beta}\right)+\frac{\partial}{\partial \hat{\boldsymbol{\beta}}} g\left(\mathbf{Y}, \hat{\boldsymbol{\Omega}}_{\hat{\beta}}\right)_{\hat{\boldsymbol{\beta}}=\boldsymbol{\beta}}\left(\hat{\boldsymbol{\beta}}_{1}-\boldsymbol{\beta}\right)
\]
"Korekta" bierze się z drugiego wyrazu:
\(\mathrm{E}\left|\hat{\boldsymbol{\beta}}_{1}-\boldsymbol{\beta}\right|=0\) zatem \(\left.\mathrm{E} \mid \hat{\boldsymbol{\beta}}_{2}\right\rfloor\)-brak obciążeń współczynników Wpływ jedynie na błędy.
\begin{tabular}{|c|c|c|c|}
\hline L. dpkb & -0.0784 ** & -0.118*** & -0.118* \\
\hline & (-3.16) & (-4.50) & (-2.18) \\
\hline \multirow[t]{2}{*}{L2. dpkb} & -0.176*** & \(-0.112^{* * *}\) & -0.112 *** \\
\hline & (-7.27) & (-6.41) & (-3.41) \\
\hline \multirow[t]{2}{*}{L. lpkb} & -0.443*** & -0.444*** & \(-0.444 * * *\) \\
\hline & (-25.45) & (-19.57) & ( -10.65 ) \\
\hline \multirow[t]{2}{*}{pop} & -0.358 & -1.103 & -1.103 \\
\hline & (-0.63) & (-1.84) & (-0.70) \\
\hline \multirow[t]{2}{*}{ki} & \(0.00468 * * *\) & \(0.00538 * * *\) & \(0.00538 * * *\) \\
\hline & (5.77) & (6.05) & (3.66) \\
\hline \multirow[t]{2}{*}{_cons} & 3.734 *** & 3.789 *** & \(3.789 * * *\) \\
\hline & (23.94) & (19.93) & (11.23) \\
\hline
\end{tabular}
,
1026
1026
1026
- t statistics in parentheses
\(\Rightarrow \quad * \mathrm{p}<0.05, * * \mathrm{p}<0.01, * * * \mathrm{p}<0.001\)

\section*{Problem: Weak instruments}

If \(y\) is nearly a random walk, \(y_{i, t-1}\) is a poor instrument for \(\Delta y_{i t}\), mathematical relationship notwithstanding

\section*{10 Random Wallks}


\section*{Problem: weak instruments}
- sort cty num5
. correlate lpk.b L.lpk.b L2.lpk.b L3.lpk.b L4.lpk.b L5.lpkb L6.lpkb


Solution: Instead of purging fixed effects, find instruments orthogonal to them (Arellano and Bover 1995)
- If \(\mathrm{E}\left[y_{i t} \mu_{i}\right]\) stationary, then \(\mathrm{E}\left[\Delta y_{i t} \mu_{i}\right]=0\)
- \(\Delta y_{i, t-1}\) uncorrelated with fixed effects, thus with \(v_{i t}=\mu\) good instrument in levels (if no AR)
- Make system of difference and levels equations
- Concretely, make a stacked data set, with differenc up top, levels below. Treat as single estimation pro
- Instrument differences with levels and v.v.
-"System GMM" (Blundell and Bond 1998)

\section*{Problem: too many instruments}
- In difference and system GMM, \# instruments ( \(j\) ) quadratic in \(T\)
- Analogy:
- In 2SLS, if \(j=\#\) of regressors, first-stage \(R 2\) 's=1.0 and 2SLS=OLS (biased)
- Too many instruments overfit endogenous variables
- And \# of cross-moments in \(\operatorname{Var}[\mathbf{Z} \mathbf{E} \mid \mathbf{X}, \mathbf{Z}]^{-1}\) to be estimated for efficient GMM quadratic in \(j\)-quartic in \(T\) !
- Estimate of \(\operatorname{Var}[\mathbf{Z} \mathbf{E} \mid \mathbf{X}, \mathbf{Z}]^{-1}\) degrades
- Hansen test very weak - \(p\) values of 1.000 not uncommon
- Little guidance on how many is too many
- xtabond2 warns if \(j>N\)

\section*{Solution: consider limiting instruments}
- Limit number of lags of variables used as instruments
- Or "collapse" instruments:
\[
\begin{aligned}
& {\left[\begin{array}{ccccccc}
0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
y_{i 1} & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & y_{i 2} & y_{i 1} & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & y_{i 3} & y_{i 2} & y_{i 1} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right] \longrightarrow\left[\begin{array}{cccc}
0 & 0 & 0 & \cdots \\
0 & 0 & 0 & \cdots \\
y_{i 1} & 0 & 0 & \cdots \\
y_{i 2} & y_{i 1} & 0 & \cdots \\
y_{i 3} & y_{i 2} & y_{i 1} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{array}\right] .} \\
& \sum_{i} y_{i, t-2} \Delta \hat{e}_{i t}=0 \text { for each } t \geq 3
\end{aligned} \longrightarrow \sum_{i, t} y_{i, t-2} \Delta e_{i t}=0 .
\]

\section*{Arellano-Bond \(\operatorname{AR}()\) test}
- Expect \(\operatorname{AR}()\) in \(v_{i t}=\mu_{i}+\varepsilon_{i t}\)
- To check for \(\operatorname{AR}(1)\) in \(\varepsilon_{i t}\), test for \(\operatorname{AR}(2)\) in \(\Delta e_{i t}\)
- E.g., compare \(e_{i t}-e_{i, t-1}\) and \(e_{i, t-2}-e_{i, t-3}\) to detect \(e_{i, t-1} \sim e_{i, t-2}\)
- Test statistic for \(\operatorname{AR}(l)\) in differences: \(\sum_{i, t} \Delta e_{i t} \Delta e_{i, t-l}\)
- Normal under null of no \(\operatorname{AR}(l)\)
- Arellano and Bond calculate its standard deviation
- \(z\) test for AR()
- More general than other AR() tests in Stata.
- abar: post-estimation command for regress, ivreg, ivreg2
-estat abond

MAKE YOUR "POWERPOINT* PRESENTATION SO BORING THAT OUR CEO WILL SLIP INTO A TRANCE.


```

xtunitroot fisher lpkb, dfuller lags(0)

```
Fisher-type unit-root test for lpkb
Based on augmented Dickey-Fuller tests

Ho: All panels contain unit roots
Ha: At least one panel is stationary

AR parameter: Panel-specific
Panel means: Included
Time trend: Not included
Drift term: Not included

Number of panels \(=190\)
Avg. number of periods \(=9.37\)

Asymptotics: T -> Infinity

ADF regressions: 0 lags
Statistic p-value
\begin{tabular}{llrr} 
Inverse chi-squared(376) & P & 680.9705 & 0.0000 \\
Inverse normal & Z & 2.6933 & 0.9965 \\
Inverse logit t(924) & \(\mathrm{L}^{*}\) & -0.8559 & 0.1962 \\
Modified inv. chi-squared & Pm & 11.1211 & 0.0000
\end{tabular}

P statistic requires number of panels to be finite.
othenstatistics are suitable for finite or infinite number of panels.

\section*{Problem: random walk}
- Blundell and Bond (1998) proposed to use, in addition to regression on differences, additional regression at levels with delayed variables as instruments.
This requires the fulfillment of additional momentum conditions that are based on stationarity conditions relative to the initial observation:
\[
E\left[\Delta y_{i, t-s}{ }^{\prime}\left(\eta_{i}+v_{i t}\right)\right]=0
\]

\section*{Problem: random wallk}
- These conditions are met when the data generation process is mean-stationary:
\[
\begin{aligned}
& y_{i, 1} \frac{n_{i}}{(1-\alpha)}+\varepsilon_{i} \operatorname{przy} E\left(\varepsilon_{i}\right)=E\left(\varepsilon_{i} \eta_{i}\right)=0 \\
& x_{i, 1} \frac{n_{i}}{(1-\alpha)}+\varepsilon_{i} \operatorname{przy} E\left(\varepsilon_{i}\right)=E\left(\varepsilon_{i} \eta_{i}\right)=0
\end{aligned}
\]

\section*{Problem: random wallk}
- Blundell and Bond (2000) show that this condition is not really a necessary condition. Considering the equation in the first, it can be shown that if:
\[
E\left(\eta_{i}^{\prime} \Delta x_{i, t}\right)=0
\]

\section*{Problem: random walk}
- and assuming that the same data generation process resulted in GDP per capita data in a given data series in the sample for a sufficiently long period before the selected sample, that the impact of the baseline conditions (in this case, the initial capital level) can be considered negligible, then:
\[
E\left(\eta_{i}^{\prime} \Delta y_{i, t}\right)=0
\]

\section*{Problem: random wallk}
- It can be seen that if the first differences of these variables were correlated with the fixed effects for a given country, it would have incredible long-term implications.
This does not mean that, for a given country, the constant effects do not play any role in determining growth. Their influence is one of the determinants of the steady state of the production level per unit of labor productivity, depending on other conditions in the steady state. The essence of these assumptions is that there is no correlation between the increase in production and the fixed effect, with no control for the presence of other variables.

\section*{Problem: random walk}
- As shown in Monte Carlo simulations (e.g. (Blundell and Bond, 1998, Blundell, et al. 2000), when these conditions are met, the resulting UMM estimator on differences and levels (hereinafter BB, the GMM System) has better finite load and RMSE properties than Arelllano and Bond's differential estimator.

\section*{Problem: heteroscedastisity \& \(\mathbb{A R}\)}
- In the presence of heteroscedasticity and autocorrelation in the model, it is possible to use a two-stage UMM estimator using the first step (Davidson and MacKinnon, 2004) to estimate the residual weight matrix of the estimate. We want it to be directly proportional to the inverse of the variance and covariance matrices of the instruments, i.e. the matrix:
\[
\begin{gathered}
V\left\{Z_{i}{ }^{\prime} \Delta \varepsilon_{i}\right\}=E\left\{Z_{i}{ }^{\prime} \Delta \varepsilon_{i} \Delta \varepsilon_{i}{ }^{\prime} Z_{i}\right\} \\
p \lim W_{N}=E\left\{Z_{i}{ }^{\prime} \Delta \varepsilon_{i} \Delta \varepsilon_{i}{ }^{\prime} Z_{i}\right\}^{-1}
\end{gathered}
\]

\section*{Problem: heteroscedastisity \& AR}
- Using the mean:
\[
\hat{\mathbf{w}}_{N}^{\text {opt }}=\left(\frac{1}{N} \sum_{\mathrm{i}=1}^{N} \mathbf{Z}_{\mathrm{i}}{ }^{\prime} \hat{\mathbf{x}}_{\mathbf{i}} \Delta \hat{\mathbf{s}}_{\mathbf{i}} \mathbf{Z}_{\mathrm{i}}\right)^{-1}
\]
- However, there are no estimates of the residual values. Solution: the two-step method.
- The model is estimated using the instrumental variable method by substituting the unit matrix for the WN matrix, obtaining estimates of the residuals
The obtained estimator is unbiased and consistent, but it is not effective because the selected matrix is not optimal
- We use the obtained estimates of the error term from the first step to estimate the optimal WN matrix, which we then use in the second step to estimate the final parameters.

\section*{Solution: two-step procedure}
- While asymptotically more efficient, the two-stage GMM estimator in finite samples provides estimates of standard errors that are heavily biased downwards. It is possible to solve this problem by means of a twostep covariance correction in a finite sample proposed by Windmeijer (2005). This adjustment makes the robust two-stage GMM estimator on differences and levels more effective than the robust one-stage estimators, even when the panel is relatively short (correction already discussed in the context of the \(A B\) estimator)

Problem: measurement error and causality
- Additionally, the estimator solves the problem of measurement error and opposite causality. Bond et al. (2001) indicate that thanks to the use of binary variables corresponding to successive time periods, the time-varying measurement error in a given observed series in the sample will have no consequences for the model estimation and this does not affect the validity of the GMM instruments used.
In turn, lags in levels help reduce the problem of opposite causality.
The coefficient thus estimated takes into account Granger causality.


\section*{Problem: exogeneity}
- In the methods presented so far, endogenous instruments are used. For example, in the case of GMM levels and differences in empirical models, most often in the equation of the first differences in growth, the differences of the explanatory variables and the second lags in the level of the dependent variable are used, and in the case of the equation of levels, these are the delayed first differences of the dependent variable.
It is possible to include exogenous instrumental variables in the model, which allows for taking into account variables that may have the opposite causality, or torastas a third variable.
xtdpdsys dpkb l.lpkb pop ki, lags(1) vce(robust) artests(2)

- artests not computed for one-step system estimator with vce (gmm)
- Arellano-Bond test for zero autocorrelation in firstdifferenced errors

- HO: no autocorrelation
p estat sargan
- Also problems. To solve the problem proceed with the same operation as xtabond, we increase the lags parameter, we switch to the two-step method.
Own exercises - come to the correct form of the BlundellBond model analogically to Arellano-Bond. Let me just hint that it will be easier with dummy variables.

\section*{Problem: sooo, \(B B\) or \(A B ?\)}
- Additional conditions for the BB estimator can be tested with the differential Sargan test known as the Hansen C test or J test.
The easiest way to download the xtabond2 module:
- net install xtabond2
- And repeat the estimates using this module. Syntax available on:
- Roodman (2006) How to do xtabond2

\section*{Problem: small \(\mathbb{N}\) size}
- This is often the case for research of a regional nature. The estimator proposed by Kiviets (1995), which considers the correction of the model of the first differences in a balanced panel, where the number N is necessarily small.

\section*{Problen: small \(\mathbb{N}\) size}
- This creates a revised estimate of the fixed effects that is more effective than the estimates of Anderson and Hsiao (1981), Arellano and Bond (1995) and Blundell and Bond (1998) with small T and N. Bruno (2005) presents a modified version of this estimator for unbalanced panels, which is important in the case of growth models, when the length of the time series is different for different countries.

\section*{Problem: small \(\mathbb{N}\) size}
- The disadvantage of this methodology is the assumption of the strict exogeneity of the explanatory variables and the inability to take into account the opposite causality and measurement error, which undermines the use of this estimator in dynamic growth models in applications other than small (regional) country samples.

Installation of the Kiviets estimator in Bruno's version (2005):
net install xtlsdvc
The need to install an additional package for the GMM System: net install xtabond2

Syntax
```

xtlsdvc lpkb pop ki, initial (b.b) vcov(50)

```

The effective estimator in the parentheses:
```

B.b - Blundell bond

```
AB - Arellano Bond
FD - Anderson Hsiao
xtlsdvc lpkb pop ki, initial (bb)
LSDVC dynamic regression
(SE not computed)
lpkb | Coef. Std. Err.
Conf. Interval]
lpk.b |
L1. | . 9631341
    pop | . 6083637
        ki | . 0051197

\section*{Problem: heterogeneity}
- Another problem raised in the econometric literature on growth estimation is the heterogeneity of countries. Until now, it was assumed that for all countries the estimated coefficients are the same, and therefore for each \(j\) and \(i\) :
\[
\beta_{i j}=\beta_{j}
\]
- Is it so? Are all objects from the same distribution? If we increase the period of education by one year, will the effect be the same in Japan, Poland and Burkina Faso?

\section*{Problem: heterogeneity}
- This type of problem cannot be solved with country samples. There are too few of them. Nevertheless, one can move towards methods with heterogeneous coefficients. Condition: \(\mathrm{N}>500\). However, it is possible to solve the heterogeneity between short-run factors, assuming that in the longrun case they are converging the same \(C E\).

\section*{Problem: heterogeneity}
- The Pooled Mean Group (PMG) estimator when applied to estimating economic growth can be described by the following equation :
\[
\Delta y_{i, t}=\sum_{z=1}^{p-1} \gamma_{i} \Delta y_{i, t-z}+\sum_{z=0}^{q} \tau_{i} \Delta x_{i, t-z}+\varphi_{i}\left(y_{i, t-1}-\alpha_{i}-\sum_{j=1}^{k} \beta_{j} x_{i, j, t-1}\right)+\varepsilon_{i, t} .
\]
- This equation makes it possible to separately estimate the short-term dynamics of the explained variable and the long-term dynamics, thanks to the inclusion of a cross-sectional time error correction mechanism in the sample, different for different countries.
xtpmg dpkb d.lpop d.lki, lr(l.lpkb lki lpop) pmg

Pooled Mean Group Regression: Estimated Error Correction Form (Estimate results saved as PMG)


\section*{Problem: heterogeneity}
- Ok. all the most common methods are discussed. It remains to be discussed what is less frequently used in economics due to the short panel problems. xtrc - allows you to estimate with the assumption of variable coefficients. This corresponds to a different coefficient for each country for each variable. xtmixed - hierarchical models.
So far linear models. There are also extensions of nonlinear models to panels.

\section*{Zakończenie}
\begin{tabular}{llll}
\hline Model & Przekształcenie danych & Zmienne objaśniające & Zgodność \\
\hline FE & Wewnątrzobiektowe & \(y_{i, t-1}, x_{i, t}\) & nie \\
FEDW & Wewnątrzobiektowe & \(y_{i, t-1}, x_{i, t}\) & tak \\
AH & \(\Delta\) & \(\Delta y_{i, t-1}, \Delta x_{i, t}\) & tak \\
AB & \(\Delta\) & \(\Delta y_{i, t-1}, \Delta x_{i, t}\) & tak \\
BB & \(\Delta y_{i, t-1}, \Delta x_{i, t}, y_{i, t-1}, x_{i, t}\) & tak \\
Kiviets & \(\Delta y_{i, t-1}, \Delta x_{i, t}, y_{i, t-1}, x_{i, t}\) & tak \\
PMG & \(\Delta\) & \(\Delta y_{i, t-1}, \Delta x_{i, t}, y_{i, t-1}, x_{i, t}, E C M\) & tak \\
\hline
\end{tabular}

\section*{ALGORITHM}
1. Panel>MNK, very rare if not.
2. FE versus RE, BE - Hausmann test. Very rare if not FE.
3. Determine whether one or two-way model.

\section*{ALGORITHM}
- 4. Anderson-Hsiao, Craigg-Donald whether instruments exogenous on the first stage.
- 5. Arellano-Bond - one or two step - Sargan test \& Arellano-Bond AR test.
- 6. If the tests are pointing to problems, increase the number of instruments, switching from the one-stage method to the two-stage method, increasing the number of instruments.

\section*{ALGORITHM}
- 7. Blundell Bond - as with Arellano Bond, but JHansen test to see whether additional GMM-sys constraints are viable.
- If not \& small \(N\) size - Kiviet's
- If not \& heterogeneity, long \(T\) - PMG.
- If you choose Kiviet's or PMG it is nice to show robustness! You can always run the Hausman test against the FE!

\section*{ALGORITHM}
- The biggest problem of empirical research in macroeconomics is the uncertainty of model parameters and explanatory variables. Trivializing - what's on the right?

\section*{Last but not least problems}

Outliers, missing data?
Many tests for outliers:
net install grubbs
But first, it's best to draw figures like in the first class and see if any observations are exceptionally different. Missing data:
- ipolate
- epolate


\section*{Dziękuję za uwagę.}```

