Lecture 1

History of general equilibrium theory

Adam Smith: The Wealth of Nations, 1776

many heterogeneous individuals with diverging interests
many voluntary but uncoordinated actions (trades)
results in a balanced situation (invisible hand)
this state is optimal (today this corresponds to the 1. fundamental theorem of welfare economics)

Leon Walras, 1874 (more than 100 years later!)

discovers general equilibrium theory
consumers (households) and producers (firms)
households endowed with initial wealth (labour)
firms described by their production possibilities
equilibrium described by a vector of market clearing prices
only relative prices matter, one of them can be normalized to 1 (numeraire)
Walras' law
stability of equilibrium

Edgeworth, 1881

discovers the relationship between general negotiation concept and the market.
2 individuals with initial endowments can perform arbitrary transactions (barter).
results in a set of remaining allocations, called contract curve
equilibrium is an element of this set
the core of an economy
if the number of individuals goes to infinity, the core converges to equilibrium.

Pareto (1909)

formulates general concept of efficiency of an allocation (Pareto optimal allocation)
recognized (without proof) that for appropriate initial endowments the market mechanism can single out a given efficient allocation (today this is called the 2. fundamental theorem of welfare economics).

History of general equilibrium modeling

Existence and uniqueness of equilibria starts with German language literature - they could show existence for some special cases and recognized that existence is not as easy to solve as Walras thought (by counting variables):

Cassel, 1924
Zeuthen, 1932
Neisser, 1932
von Stackelberg, 1933
Schlesinger, 1934

Interaction with game theory which was invented at this time:

v.Neumann, 1937, was the first to discover the importance of fixed point theorems for equilibrium existence theorems

v.Neumann & Morgenstern, 1944, proved existence of equilibria for two person 0-sum games

Formal proof of existence

Wald, 1934, 1951

McKenzie, 1954 & Arrow und Debreu, 1954, simplified and generalized of the results of Abraham Wald by using the fixed point theorems; present general equilibrium theory model in its current formulation.

Debreu, 1959, complete systematic treatment of the basic model, presents further generalizations

Core-equivalence (generalisation of Edgeworth to large economies)

Debreu, Scarf, 1963
Aumann, 1964
Hildenbrand, 1970, 1974

Countless modern subfields of economics based on the general equilibrium model:

dynamics and growth
rationing
overlapping generation, modern macro
modern finanse

…

Abbreviation	Name	Description
x(p,m)	Marshallian (Walrasian or ordinary or market) demand function	The commodity bundle that maximizes utility subject to the budget constraint
u(x)	utility function	it summarize a consumer's behavior
v(p,m)	indirect utility function	A function of prices and income, not commodities
e(p,u)	expenditure function	the minimum amount of money an individual needs to achieve some level of utility, given a utility function and prices
h(p,u)	Hicksian (compensated) demand function	it specifies what consumption bundle achieves a target level of utility and minimizes total expenditures
m(p,x)	compensated demand function	consumer's income adjusted to compensate for income effect of price changes

Properties for the convex preferences

Utility f	Indirect utility f	Walrasian Demand f	Hicksian Demand f	Expenditure f
-	Homogeneity of degree zero in (p,m)	Homogeneity of degree zero in (p,m)	Homogeneity of degree zero in p	Homogeneity of degree one in p
Locally nonsatiated preferences	Strictly increasing in m & nonincreasing in p	Walras' law	No excess utility	Strictly increasing in u & nondecreasing in p
Quasiconcave	Quasiconvex in (p,m)	Convex set	Convex set	Concave in p
Continuous	Continuous in (p,m)	Continuous in (p,m)	Continuous in p	Continuous in p

Important properties of consumer preferences

Properties	Formulation	Description
Rationality	Completeness & Transitivity	Any two bundles can be compared & If preferences were not transitive, there might be sets of bundles which had no best elements.
Monotonicity	$x \geq y \Rightarrow x \succeq y$	At least as much of everything is at least as good.
Strong monotonicity	$x \geq y$ & $x \neq y \Rightarrow x \succ y$	At least as much of everything and strictly more of some goods is strictly better
Local nonsatiation	$\lvert x-y \rvert \leq \varepsilon > 0$ such that $y \succ x$	For any consumption bundle and any arbitrary small distance *ε* away from *x*, it is another bundle *y* within this distance that is strictly preferred to *x*
Convexity (it means that utility is quasiconcave)	$x \succeq z$ & $y \succeq z \Rightarrow$ $ax + (1-a)y \succeq z$ for $0 \leq a \leq 1$	Averages are at least as good as extremes.
Strict convexity	$x \succeq z$ & $y \succeq z$ & $x \neq y \neq z \Rightarrow$ $ax + (1-a)y \succ z$ for $0 < a < 1$	A consumer prefers averages to extremes.

Well-behaved preferences: rational, continuous, convex, locally nonsatiated

Concavity is an assumption about how the numbers assigned to indifference curves change as you move outward from the origin (cardinal concept).

Quasiconcavity talks only about the shape of indifference curves, not the curvature or the numbers assigned to them (ordinal concept).

There are many utility functions that can represent the same preferences. Utility and preferences have to do with the shape of indifference curves. The difference between the utility of two bundles doesn't mean anything.

Important identities

Identities	Description
$u(x(p,m)) \equiv v(p,e(p,u))$	If we give to consumer the minimal income to get utility *u* at prices *p*, then the maximal utility she can get is *u*
$h_i(p,u) \equiv x_i(p,e(p,u))$	The minimum income necessary at the given prices to achieve the desired level of utility
Samuelson lemma $e(p,u(x)) \equiv m(p,x)$	Money metric utility function (or minimum income function) specifies how much money would consumer need at the prices *p* to be as well off as she could be by consuming the bundle *x*.
Hoelling-Shepard's lemma $h_i(p,u) = \frac{\partial e(p,u)}{\partial p_i}$	If utility function *u(.)* is continuous and represents a locally nonsatiated and strictly convex preferences, then the cost minimizing point of a given good *i* with price $p_i > 0$ is unique.
Antonelli-Allen-Roy's lemma $x_i(p,m) = -\frac{\frac{\partial v(p,m)}{\partial p_i}}{\frac{\partial v(p,m)}{\partial m}}$	If utility function *u(.)* is continuous and represents a locally nonsatiated and strictly convex preferences, $p_i > 0$, $m > 0$, then the Marshallian demand function *x(.)* relates to the derivative of the indirect utility function *v*(.)

Envelope theorem:

$$maximize_x\ f(x,a),$$

where *x* is the choice variable and *a* is a parameter.

Define an *indirect objective function f** when the choice variable *x* is chosen optimally $x=x^*(a)$:

$$f^*(a) = f(x^*(a),a).$$

Note that *f** is a function of the parameter *a* but is not a function of *x*. In the indirect objective function, the choice variable is not free; *x* is always set to maximize the objective function.

The envelope theorem states that **$df^*(a)/da = \partial f(x^*(a),a)/\partial a$**

Compare UMP with EMP

The process of finding the optimal point is different in the UMP and EMP, but they both pick out the same point because they are looking for the same basic relationship $\frac{u_i(x)}{u_j(x)} = \frac{p_i}{p_j}$

Duality of the EMP and UMP:

- If x* solves the EMP when prices are p and wealth is m, then x* solves the EMP when prices are p and the target utility level is u(x*).
- Maximal utility in the UMP is u(x*) and minimum expenditure in the EMP is m.
- If we know the consumer's expenditure function e(p,u) $\Rightarrow$ we may define indirect utility function v(p,m) because e(p,u) contains exactly the same information as the v(p,m) $\Rightarrow$ we may derive the Walrasian demand function using Roy's identity

Homework:

Consider U(A,B) = A·B subject to a budget constraint that $P_A A + P_B B = M$, where P_A and P_B are strictly positive prices and M is strictly positive income.

a) Find the Walrasian demand correspondence and the indirect utility function

b) Verify that Roy's Identity holds for good A.

c) Specify the money metric utility function